# Resonance Theory of Stationary Longitudinal Structures in the Boundary Layer

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*Abstract:* - On the basis of resonance theory the possibility of the longitudinal structures generation was fixed in the compressible boundary layer by an external vorticity. It takes place under a condition when parameters of externally vortex wave become the close to parameters of eigen stationary perturbations of a boundary layer. Researches are conducted as in case of subsonic numbers of the Mach, and in case of a supersonic flow at M=2. Data of the resonance theory agree with direct calculations of an interaction external vorticity with boundary layer satisfactorily. Parameters of two-dimensional stationary perturbations of a subsonic boundary layer completely match with data of Grosch C. E., Jackson T. L., Kapila A. K. (1992). In particular, the infinite set of eigen functions is installed, which are damped by a power law of the longitudinal coordinate, $x^{-\lambda_k}$. Researches of three-dimensional perturbations showed, that the damping degree of perturbations down a flow depends on a wave number in the lateral direction poorly. However, there are the optimal values of the wave number in the lateral direction, in which perturbations damped down by a stream the most poorly. If in case of subsonic speeds decrements of perturbations of the first mode doesn't depend neither on a Reynolds number, nor on value of a lateral wave number, then in case of M=2 the nature of a perturbations reduction on longitudinal coordinate depends both on a wave number, and on a Reynolds number.

*Key-Words:* - Mach number, turbulence, supersonic flow, boundary layers, disturbances, waves, transition

## 1 Introduction

The first works on the development of small two-dimensional stationary disturbances in the incompressible boundary layer on a flat plate were [1, 2]. In them it was shown that perturbations of flow parameters in the boundary layer fade down on a flow under the degree law. They had shown that the parameters disturbances of the flow $q_i$ in the boundary layer damped down the flow of power law, $\sim 1/(x)^{\lambda_k}$. Exponents $\lambda_k$ were eigen numbers of the formulated problem on eigen values. Interest in a study of longitudinal structures in many respects was defined by [3] where they were found experimentally. Their nature could be associated both with a nonlinear interaction of perturbations, and linear interaction of an external turbulence with boundary layer. An interaction of external turbulence with a subsonic interface on a flat plate was researched experimentally in [4-7] and in some other papers which review can be found in [8]. It was noted in all these works that in the interaction result of an external turbulence with boundary layer in stable region relatively small perturbations in the subsonic boundary layer longitudinal structures were observed.

Longitudinal velocity profile of stationary disturbances excited by an external turbulence, at least in the low frequency range, has a bell-shaped type, maximum which is located at a distance from the wall $Y_{\max} \approx 2.5\delta$, where δ — Blasius thickness. For the first time theoretically an interaction of an external longitudinal vorticity with a subsonic boundary layer is explored in [9]. There it was set that under the influence of the periodic external vorticity in the lateral direction the amplitude of longitudinal structure increased linearly down a flow and was inversely proportional to its period.

It completely matched with the dependence of the perturbation amplitude on the boundary layer thickness given in [4]. At the same time the dependence of the perturbations amplitude of the longitudinal velocity on the normal coordinate was coincided with data [1-3] which were obtained for two-dimensional perturbations.

It should be noted that the theory [9] is applicable only in case of enough large periods of an external vorticity. More exact results, using parabolized stability equations, were obtained in [10]. The calculations revealed that the form of longitudinal velocity perturbations profiles does not depend on the structure period practically. Its amplitude increased in proportion to the thickness of the boundary layer.

In the subsequent theoretical and numerical study of the stationary disturbances generation in the supersonic boundary layer by external waves were continued in [9-19].

To some extent the external turbulence interaction with a boundary layer can be described by means of a continuous spectrum of the stability task. For the first time a connection between the continuous spectrum and the task of an interaction of external disturbances (acoustic) with a parallel flow in the boundary layer was specified in [20]. Perhaps the most actively the possibility of the description of vortex perturbations interaction of an external flow with a boundary layer by means of a continuous spectrum began to be used after appearance of papers [22, 22].

Another explanation for a generation of the intense perturbations in the boundary layer by external waves is qusi-resonant interaction of external disturbances with their eigen waves. Apparently in [23] for the first time an attention was paid to quasi-resonant generation of oscillations within the considered flow area under the influence of harmonic perturbations in the time. There the system of the differential equations of first order with harmonic in time the right part had been considered. Homogeneous system has set of eigen frequency $\omega_i$. Therefore disturbance amplitude will significantly exceed values of the right part if its frequency ω differs a little from eigen frequency $\omega_i$.

The aim of this work was to study the development of internal stationary perturbations in the compressible boundary layer on a flat plate and to describe theirs on the basis of resonance theory.

## 2 Problem statement and basic equations

The linear statement is considered. The flow of a compressible gas in a boundary layer on a flat plate is taken as an initial undisturbed flow. Disturbances in a boundary layer we shall consider in orthogonal coordinate system $(\xi,\psi,z)$ [15] connected with stream-surfaces of basic flow and look like $\tilde{a}(\xi,\psi)\exp(i\alpha\xi+i\beta z-i\omega t)$. Here $\psi$ - flow function; for a plate $\xi = x+O(\mathrm{Re}^{-2})$; $\mathrm{Re}=\sqrt{u_\infty x/\nu_\infty}$ ; $u_\infty,\nu_\infty$ - speed and kinematical viscosity of a ram airflow; $x,y,z$ - longitudinal, normal to a wall and transversal co-ordinates of the Cartesian system with the beginning on an edge of a plate. Gas is perfect with a constant Prandtl number, Pr. Resulting a set of Navier-Stokes equations to a linear view, using estimations on the whole degrees of a Reynolds number, Re, rejecting the members order $\mathrm{Re}^{-2}$ respect to the main ones, the properties of a critical layer [15] and neglecting by a deformation of a perturbations distribution with changing of coordinate $x$ it is possible to receive the dimensionless equations:

$$
\begin{aligned}
&\tilde{v}' = \rho T'\tilde{v} - T\left(f_0 u'\right)\tilde{u} - \tilde{u}_w - i_c T\tilde{r} - \\
&-\left(f_2\rho T'\right)\tilde{T} - f_1 T\tilde{u}' + f_2\tilde{T}',\\
&\tilde{p}' = -\left(i_c + r_h u\right)\tilde{v} + i_x\tilde{\tau}_{12} + i_z\tilde{\tau}_{23} - 2\mu_r\tilde{u}_w',\\
&\tilde{\tau}_{12}' = i_x\tilde{p} + \left(i_c + f_1 u' + u\partial\right)\tilde{u} + \\
&f_2 u'\tilde{r} - \tilde{i}_T + f_2\tilde{u}' + \rho u'\tilde{v}\\
&\tilde{u}' = -i_x\tilde{v} - u'\mu_t\tilde{T} + \tilde{\tau}_{12}/\mu_r ,\\
&\tilde{\tau}_{23}' = i_z\tilde{p} + \left(i_c - \mu_a\right)\tilde{w} - i_z\mu_r\tilde{u}_w + f_2\tilde{w}',\\
&\tilde{w}' = -i_z\tilde{v} + \tilde{\tau}_{23}/\mu_r ,\\
&\tilde{q}' = i\omega RT\,\tilde{p} + \rho H'\tilde{v} + f_2 H'\tilde{r} - u\tilde{i}_t + f_2 u\tilde{u}'\\
&+\left(i_c u + f_1 H' + f_2 u'\right)\tilde{u} + f_2\tilde{h}' + \left(i_c - \mu_a/\mathrm{Pr}\right)\tilde{h},\\
&\tilde{h}' = -\mathrm{Pr}\,u'\tilde{u} - h'\mu_t\tilde{T} + \mathrm{Pr}\left(\tilde{q} - u\tilde{\tau}_{12}\right)/\mu_R ,
\end{aligned}
\tag{1}
$$

where the stroke means a derivative on $Y$; $dY = d\psi/u\,\mathrm{Re}$; $\tilde{p},\tilde{v},\tilde{u},\tilde{w},\tilde{h}$ - amplitudes of pressure; normal to a surface of a plate, longitudinal and transversal speeds; enthalpy disturbances. The expressions for $\tilde{\tau}_{12},\tilde{\tau}_{23},\tilde{q}$ can be found in [15].

Additional members of the system are of the form:

$\tilde{u}_w = i_x\tilde{u} + i_z\tilde{w}$; $\tilde{i}_t = i_x\mu_r\tilde{u}_w + \mu_a\tilde{u}$ ;

$\tilde{r} = \tilde{\rho}/\rho = g_m\tilde{p} - \rho\tilde{T}$ ; $i_c = \mathrm{Re}\,u_c = i\,\mathrm{Re}(u\alpha-\omega)$ ; $i_x = i\alpha\,\mathrm{Re}T$ ; $i_z = i\beta\,\mathrm{Re}T$ ; $\mu_a = \left(i_x^2 + i_z^2\right)\mu_r$

$r_h = \mathrm{Re}\,h_1 = f_0 u' + f_1\rho T'$ ; $f_0 = -f_1/u$ ;

$f_2 = f_1 u$ ; $\mu_r = \mu\rho/\mathrm{Re}$ ; $\mu_T = d\ln\mu/dT$ , , ,

$f_1 = -\psi/\left(2\,\mathrm{Re}^2 u\right)$, $\mathrm{Re} = \sqrt{\xi}$ .

The system (1) was normalized with the help of following scales: $\nu_\infty/u_\infty$ - length, $\nu_\infty/u_\infty^2$ - time, $\mu_\infty$ - viscosity and flow function, $u_\infty$ - velocity and its disturbances, $T_\infty$ - temperature, $\rho_\infty$ - density , $u_\infty^2$ - enthalpy, $\rho_\infty u_\infty^2$ - pressure and disturbances of viscous stresses, $\rho_\infty u_\infty^3$ - value $\tilde{q}$ , $u_\infty^2/T_\infty$ - specific heat (the index ∞ corresponds to values in the incident airflow). In this case: $g_m = \gamma \mathrm{M}^2$, $g_{m1} = (\gamma-1)\mathrm{M}^2$, where $\gamma = c_p/c_V$ - relation of heat capacities, M – Mach number.

The system (1) can be represented in the form:

$$
\mathbf{Z}' = A(\eta,\mathrm{Re},M,\omega,\beta,\alpha)\mathbf{Z}\,. \tag{2}
$$

$\mathbf{Z} = \left(\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}, \tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q}\right)$, A—quadratic matrix of given functions of $\mathrm{Re}$ and $Y$.

In the absence of external perturbations the system of equations (3) is solved with the following boundary conditions. The disturbances of speeds and a temperature on a surface and in infinity are equals to zero:

$$z_2(0) = z_3(0 = z_4(0) = z_5(0) = 0,\ (z_5'(0) = 0)$$
$$z_2(\infty),\ z_3(\infty),\ z_4(\infty),\ z_5(\infty) = 0 \quad (3)$$

Thereby the task on own values (the task of stability) is formulated. For example, at given values of Re, M, ω and β, the value of a wave number α is searched. In case of positive values of an imaginary part of a wave number $\alpha_i$ the flow is stability and vice versa.

In the presence of external perturbations of a type of $\tilde{a}_0(\xi,\psi)\exp(i\alpha_0\xi + i\beta z - i\omega t)$ in a matrix of $A$ it is necessary to replace $\alpha$ on $\alpha_0$. Boundary conditions take the form:

$$\tilde{v}(0) = \tilde{u}(0 = \tilde{w}(0) = \tilde{T}(0) = 0,\ (\tilde{T}'(0) = 0);$$
$$\mathbf{Z}(\infty) = \mathbf{Z}_0 \quad (3)$$

## 3 Results

### 3.1 Egen stationary disturbance

In this paper the case of stationary perturbations is researched, which parameters differ from parameters of nonstationary perturbations a little in case $\omega = 2\pi f \nu_\infty / u_\infty^2 < 10^{-6}$, $f$ — the frequency in Hertz. All distributions given below were normalized on maximum values of the longitudinal velocity which located in the range: 2.5<*Y*<*3.5*.

For the greater confidence in calculations, comparison of our results with data of [3], received at M=0 and β = 0, was carried out. There within of boundary layer equations, the stationary and nonstationary perturbations are researched. There has been shown, that longitudinal logarithmic derivative of $x^{-\lambda_k}$ on longitudinal coordinate has the form $\dfrac{\partial \ln(\bar{u}_k)}{\partial x} = i(\alpha_r + i\alpha_i) = -\lambda_k / x$.

$\lambda_k$ - an infinite set of numbers. The first four of them with a precision of three digit are equal: 1.0, 1.89, 2.81, 3.76.

The distribution of the longitudinal velocity perturbation in a boundary layer is given in Fig.1: present calculations at M=*0.1*, *β=0*, Re= ***200***, $\alpha_i$=*2.50·10*$^{-5}$ (symbol) and data of [3] at M=0, $\lambda_1 = 1$ (solid line).

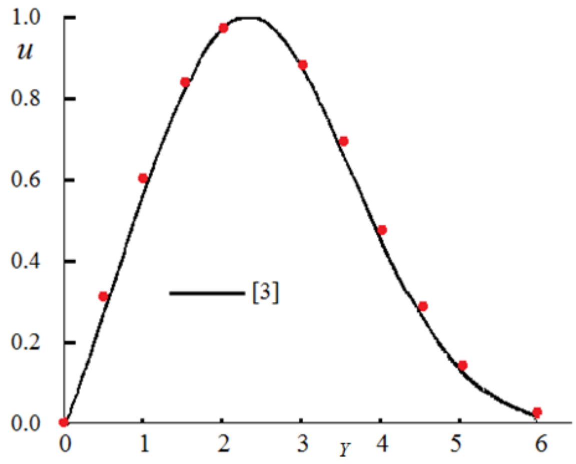


Fig. 1 Comparison of calculation results (M=0.2) of the longitudinal velocity distribution with data of [3] (M=*0*).

Dependence of a rate amplification $\alpha_i$ and $\lambda_1 = \alpha_i \mathrm{Re}^2$ on a Reynolds number is given in Fig. 2. Data show that $\alpha_i \sim 1/\mathrm{Re}^2$, and $\lambda_1 = 1$ (in the full accordance with the results of [2, 3]).

Along with two-dimensional perturbations in subsonic boundary layer (M=*0.1*) calculations carried out for three dimensional stationary disturbance, $\beta \neq 0$, in subsonic and supersonic boundary layer, unlike researches of [2, 3] (M=*0*, $\beta = 0$).

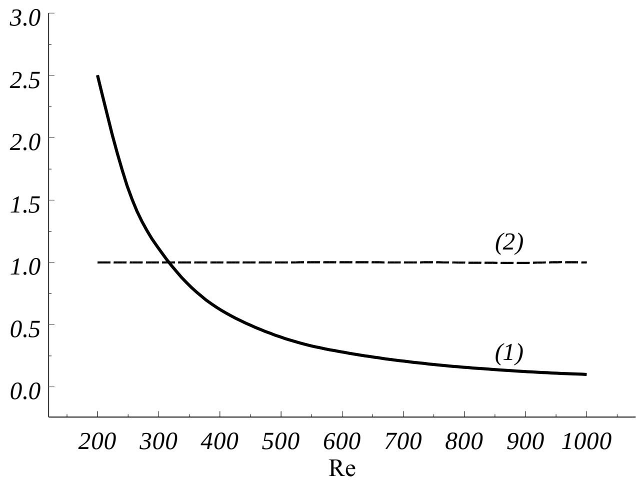


Fig. 2 Dependence of a rate amplification $\alpha_i$ *(1)* and $\lambda_1 = \alpha_i \mathrm{Re}^2$ *(2)* on a Reynolds number, M=*0.2*.

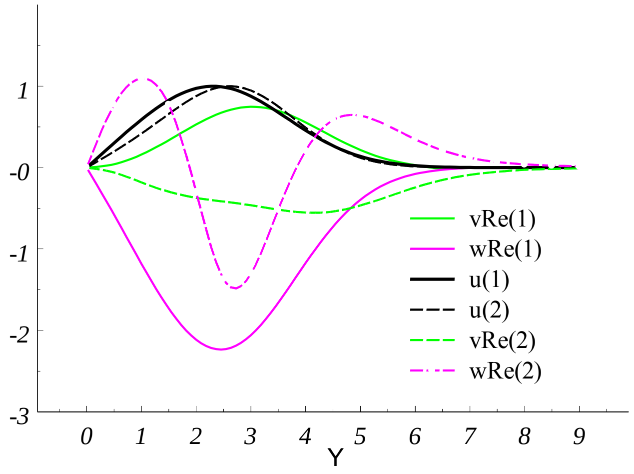


Fig. 3 Distributions of longitudinal, normal and lateral velocities perturbations and a phase shift on a boundary layer at Reynolds numbers *200*(1) и $10^3$ (2), (M=*0.2*, $\beta$ =*7·10*$^{-4}$).

Distributions of longitudinal, normal and lateral velocities perturbations (u, v, w) on a boundary layer are shown in Fig. 3 and Fig. 4, when Re=*200* and $10^3$ (M=*0.2*, $\beta$=*0.7·10*$^{-4}$). In addition distributions of phases of longitudinal velocities are shown in Fig. 4.

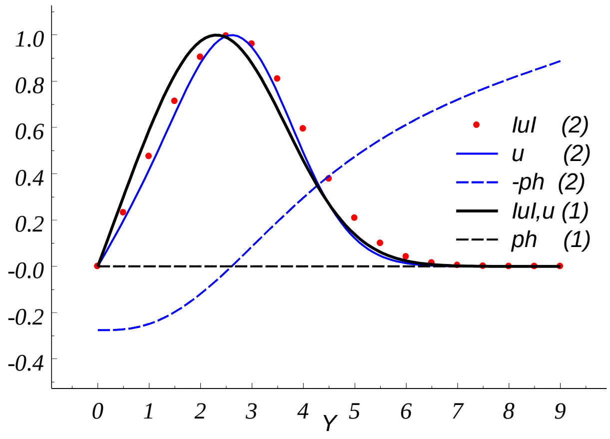


Fig. 4 Distributions of maximum, */u/,* and actual, *u,* values of velocity perturbations and a phase shift on a boundary layer at Re =*200(1)* and $10^3$*(2)* (M=*0.2*, $\beta$=*7·10*$^{-4}$).

If in case of low numbers of Renolds (Re=200) there is no influence of a wave number $\beta$ on the velocities perturbations distribution, then with increase in Re it not so. Especially it is visible on phase shift. If in case of Re=200 phase shift on a boundary layer is equal to zero, as well as in a case β =0, then in case of Re=$10^3$ it reaches about one radian, (about 60 °). At the same time the phase decreases with an increase of the coordinate *Y.* It is connected with the fact that a Reynolds number (coordinate *x*), boundary layer thickness, an effective wave number $\bar{\beta}=\beta\,\mathrm{Re}$ and a product $\bar{\beta}w$ increase. And it carries to more strong influence of a lateral velocity on longitudinal velocity through a continuity equation.

From presented data (Fig. 3) it is visible that normal and lateral velocities have an order 1/Re.

In case of supersonic speeds along with velocities perturbations disturbances of the mass flow play an important role. Moreover, at hot-wire anemometer using in experiments the mass flow perturbations are measured as a rule. Therefore in Fig. 5 the mass flow perturbation *m* distribution in a boundary layer is shown in case of M=2 not only velocity perturbations. Symbols show the distribution of the velocity perturbation for M = 0.1. It is interesting to note that for the small Reynolds number (Re=*200*) velocity perturbation distribution does not depend on the Mach number.

Fig. 4 and 5 show that deformation of a longitudinal velocity profile in a supersonic boundary layer in case of a Reynolds number change is similar to its change in a case of a subsonic boundary layer.

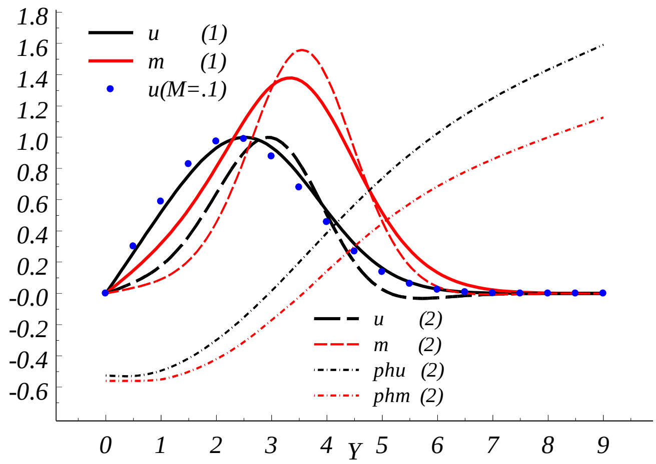


Fig. 5 Distributions *u*, *m* и *ph* on a boundary layer at M=2. Re=*200 (1)*, $10^3$ *(2)*; $\beta$ =7·10$^{-4}$

However, full phase shifts of velocity and mass flow perturbations on a boundary layer in case of M=2 are approximately equal π=3.14 and they are positive while in case of a subsonic interface they are much less and negative (Fig. 4). At small Reynolds numbers the influence parameter $\beta$ is insignificantly, the phase shift on boundary layer is absent.

Dependences of decrements and their product on Re$^2$ for two-dimensional (*(01), (02)*), and also three-dimensional (*(b1),(b2)*) perturbations on the Reynolds number are shown in Fig. 6 (M=*2*). Here, for comparing, dependence $\alpha_i \mathrm{Re}^2$ on the Reynolds number is given for a case of subsonic speeds (M=*0.2*) at $\beta=7\cdot 10^{-4}$. These data show that the spatial decrements weakly dependent on the Mach number.

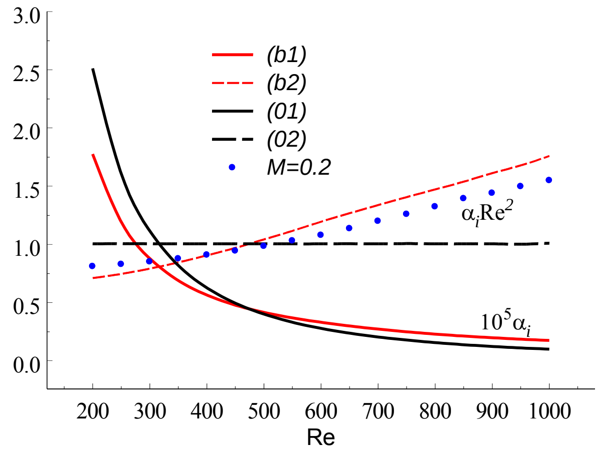


Fig. 6 Decrements dependences, $\alpha_i$, and product $\alpha_i \mathrm{Re}^2$ for $\beta=0$ ((*01*),(*02*)) and $\beta=7\cdot 10^{-4}$ ((*b1*),(*b2*)) on Reynolds number at M=*2*.

At Re<400 three-dimensional perturbations decrements ($\beta=7\cdot 10^{-4}$) are smaller than the two-dimensional perturbations decrements. At Re>*400* on the contrary, decrements of three-dimensional perturbations are greater than two-dimensional perturbations decrements perturbations. In case of Re=*1000* they are about 1.5 times more than decrements of two-dimensional perturbations.

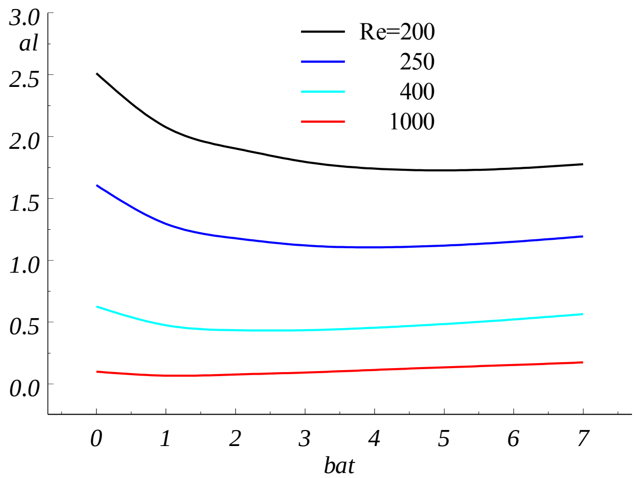


Fig. 7 Decrements dependences on a lateral wave number at M=*2* (*al* = $\alpha_i\cdot 10^5$, *bet* =$\beta\cdot 10^4$).

In Fig. 7 dependences of decrements on a wave number $\beta$ are shown. It is possible to note that in the given dependences there are minima at $\beta=\beta^*$.

Calculations show that if M= *2* then $\beta^*\mathrm{Re}=0.1$ at all Reynolds numbers. At subsonic speeds (M=*0.2*) value $\beta^*\mathrm{Re} \approx 0.08$ .

### 3.2 Resonance theory of the interaction of external perturbations with boundary layer

In the presence of external perturbations, problem (2), (4) can be reduced to solving of inhomogeneous differential equation with zero boundary conditions similar to the conditions (4). Indeed, let's introduce a vector-function $\mathbf{W}=\mathbf{Z}-\varphi(Y)\mathbf{Z}_0$. The function $\varphi(\eta)$ must satisfy to conditions: $\varphi(0)=0$, $\varphi(\infty)=1$. Then instead of the system (2) we have a non-uniform system of equations

$$\mathbf{W}' - A(Y,\mathrm{Re},M,\omega,\beta,\alpha_0)\mathbf{W} \equiv L_0\mathbf{W} = \\ = \varphi(Y)A\mathbf{Z}_0 - \varphi'(Y)\mathbf{Z}_0 \qquad (5)$$

with boundary conditions similar to (4):

$$w_2(0)=w_3(0=w_4(0)=w_5(0)=0,\ (w_5'(0)=0) \\ w_2(\infty),\ w_3(\infty),\ w_4(\infty),\ w_5(\infty)=0 \qquad (6)$$

It is possible to show that in case of small values $|\alpha-\alpha_0|$ amplitudes any component of a vector **W** and components of a vector **Z** will be proportional to $1/|\alpha-\alpha_0|$. For example longitudinal speed perturbations can be taken as $u_n=\dfrac{u}{u^0}\left|\dfrac{\alpha^0-\alpha_0}{\alpha-\alpha_0}\right|$.

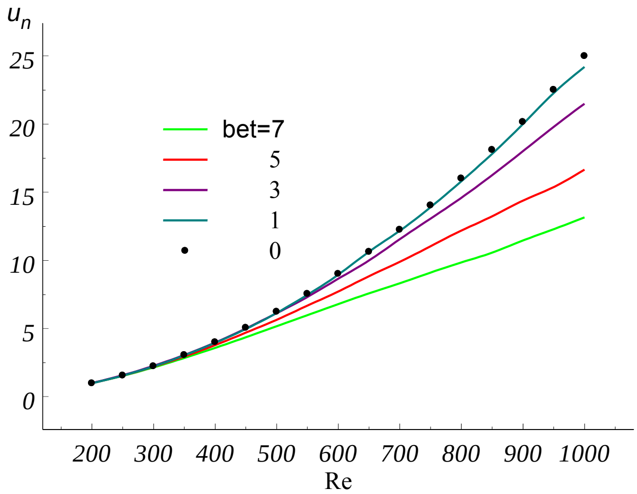


Рис. 8. Dependences of velocity perturbations amplitudes on the Reynolds number at M=*0.2* and different values *bet*=$\beta\cdot 10^4$.

Results of calculations on interactions of external perturbations of the form $\tilde{a}_0\exp(i\beta z)$ with a subsonic boundary layer are given below. Similar investigations were done in papers: [9, 10, 15, 16].

Dependences of amplitude peaks of a longitudinal velocity on a Reynolds number in case of M=*0.2* and different values of a wave number $\beta$ are given in Fig. 8. Here $u^0$ is the amplitude peak of a longitudinal velocity at Re=*200*. From the presented data it is visible that in case of small wave numbers (*bet =0; 1*) amplitudes of stationary perturbations, grows proportionally to longitudinal coordinate $x \sim \mathrm{Re}_x = \mathrm{Re}^2$ as it was predicted in [4, 9]. In case of large wave numbers (*bet=7*) it grows proportionally to $\mathrm{Re} = \sqrt{\mathrm{Re}_x}$ .

Dependences of the amplitude peaks of a longitudinal velocity on wave number $\beta = bet \cdot 10^{-4}$ at different Reynolds numbers are shown in Fig. 9. Here $u^0$ is the amplitude peak of a longitudinal velocity at $\beta=0$.These results show that perturbations amplitude of a longitudinal velocity rises with β linearly in the area of small values of wave number that are also consistent with the findings of [4, 9].

From fig. 9 it is possible to conclude that in case of the fixing Reynolds number the maximum of perturbations are achieved approximately at $\beta^*\mathrm{Re}=0.1$. Calculations of paper [10] showed that the amplitude of excited stationary perturbations, which were proportional to $\exp\left(i(\beta z+\beta y/3)\right)$, reached its maximum also when $\beta^*\mathrm{Re}\approx 0.1$.

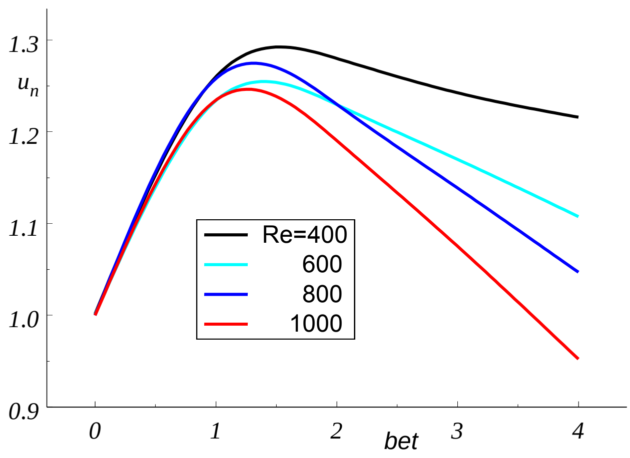


Рис. 9. Dependences of velocity perturbations amplitudes on wave number at M=*0.2* and different values Re.

Due to the fact that for the data, presented on Fig. 9, the value of $u_n$ changes slightly in the range $1 <$ bet $< 1.5$, for dependences of the disturbances amplitude excited by external waves on the Reynolds number it is possible to use the curve of Fig. 8, corresponding to *bet=1*. In papers [15, 16] it was obtained that the maximum values of the excited velocities amplitudes are observed at $\beta^*\mathrm{Re}\approx 0.5$. With regard to the dependencies shown in Fig. 8 it is different from the amplitude dependencies of disturbances on Reynolds number presented in [10, 15, 16], where there are peaks.

One reason is related to the exponential dependence of a amplitude of an external wave on $\beta^2 x = -\beta^2\mathrm{Re}^2$, $\exp(-\beta^2\mathrm{Re}^2)$. Taking into account such attenuation dependences $u = u_n\exp(-\beta^2\mathrm{Re}^2)$ on a Reynolds number at M=*0.2* for three values of a wave number β is shown in Fig. 10. Taking into account such correction it is possible to notice that $\beta^*\mathrm{Re}\approx 0.8$ what is coordinated with data [16].

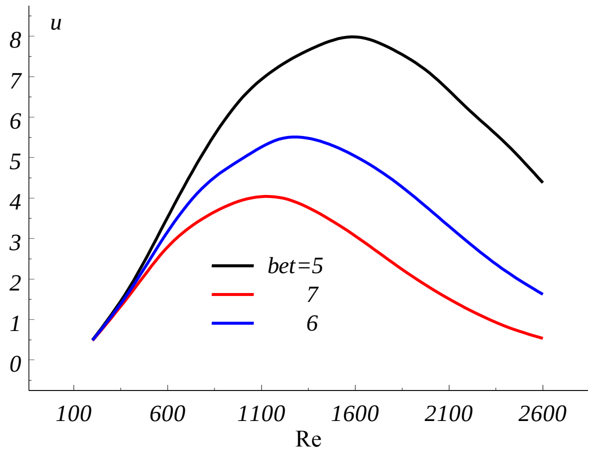


Fig. 10 Dependences of velocity perturbations amplitudes, *u*, on the Reynolds number at M=*0.2* and different values *bet*.

## 4 Conclusion

As a result of the conducted researches it was received:

1. Calculations results of Eigen two-dimensional stationary disturbances under the classical theory completely coincide with data [3].
2. Data on the eigenvalue problem of three-dimensional stationary disturbances as for subsonic and supersonic boundary layers are received. In case of low numbers of Renolds there is no influence of a wave number $\beta$ on the velocities perturbations distribution. With increasing of Reynolds number the longitudinal velocity phase shift on the boundary layer appears. Decrements of three-dimensional perturbations decrease in inverse proportion to longitudinal coordinate at small Reynolds numbers, and in inverse proportion to thickness of a boundary layer at large numbers of Reynolds.
3. The direct connection of an excitation of internal perturbations of the boundary layer by external waves with resonant theory of their

interaction with the boundary layer is established. Obtained on its basis amplitude dependences of internal perturbations excited by an external vorticity, on a wave number and a Reynolds number are coordinated with direct calculations which are available in the known literature.

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