Analytical Study of Finite Amplitude Capillary Waves Stability

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Abstract: Using the Lyapunov method, we find analytically the equations of wave surface of capillary waves in fluid of infinite depth and compare them to the solution, obtained by Crapper. We prove analytically the stability of Crapper waves with respect to symmetric and non-symmetric disturbances.

Key–Words: Direct Lyapunov method, capillary waves, Lyapunov function

1 Introduction

In [1] the exact solution of the problem of the potential plane-parallel flow of an ideal fluid in the domain $-\infty < x < \infty$, $-\infty < y < \eta(kx)$ was constructed, where the function $\eta(kx)$ is periodic $\eta(kx) = \eta(kx + 2\pi)$, the wave number k is related to the wave length as follows $\lambda = 2\pi/k$. The Laplace condition $p - p_0 + \sigma/r = 0$ is satisfied on the wave surface $\eta(kx)$, where r is the curvature radius of the cylinder, p and p_0 are the fluid pressure inside and outside the cylinder, σ is the surface tension coefficient. Crapper found the exact solution, but used numerical methods to prove it. The solution was expressed through elliptical functions.

Later Kinnersley extented the Crapper waves to the case of finite depth [2], and Crowdy studied the Crapper waves, using conformal maps [3]. The Crapper wave stability was studied in [4]. The infinite chain of linear differential equations for perturbations was obtained and the eigenvalues of these equations were studied. Such a solution requires an excessively large number of computations. The Lagrange method of generalized coordinates may be proposed as an alternative. An analogic method was used in [5] to study the stability of the McLeod plane-parallel flow [6], which models the motion of a drop with surface tension.

The direct Lyapunov method reduces the stability problem to the efficient potential energy minimum condition. The potential energy minimum condition is used to prove the stability of the stationary motion of capillary waves in the frames of the weakened Lyapunov stability definition [7].

In this work we present the analytic solution for the capillary waves stability problem (earlier numerical methods were used). We use the second variation of the Lyapunov function to prove the stability of capillary waves with respect to symmetric and non-symmetric disturbances.

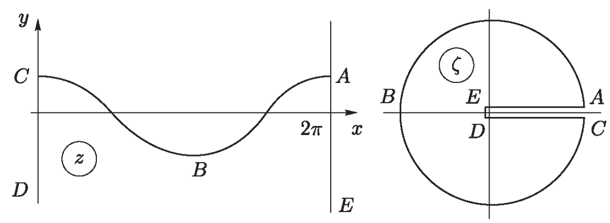

Figure 1: Mapping of one wave period on a disc.

2 The Hamilton Principle and the Euler-Lagrange equation

To describe the dynamics of capillary waves, we use the wave parametrization, introduced by Stokes [8, 9]. We seek the conformal mapping of the disc $|\zeta| < 1$ of the complex plane ζ with a cut on the positive part of abscissa (see Fig. 1) on the domain of one wave period on the complex plane $z = x+iy$ in the following form

$$z(\zeta) = \frac{\lambda}{2\pi}\left[i\ln\zeta + \sum_{n=1}^{\infty} z_n\zeta^n\right] . \qquad (1)$$

The circle $\zeta = e^{i\gamma}$ corresponds to the surface of the wave $z = x_s + i\eta$. We consider the real and imaginary parts of the Laurent series coefficients $z_n = x_n + iy_n,\ n = 1, 2, \ldots$ to be the generalizaed coordinates of the wave $q_i,\ i = 1, 2, \ldots$. The motion equations will be the Lagrange equations with the Lagrange function L, which equals the difference between the kinetic and the potential wave energy $L = E_{\rm kin} - E_{\rm pot}$.

3 Lyapunov Function

The kinetic energy of the wave is the quadratic function of generalized velocities $\dot{x}_0,\ \dot{q}_i,\ i = 1, 2, \ldots$, where x_0 is the cyclic coordinate that determines the horizontal movement of the wave, $\dot{x}_0$ – the wave propagation velocity.

The summands in the kinetic energy may be separated into three groups: quadratic in $\dot{x}_0$, linear in x_0 and independent of $\dot{x}_0$

$$E_{\rm kin} = \tfrac{1}{2}M\dot{x}_0^2 + M_1\dot{x}_0 + M_2 = \\ = \tfrac{(M\dot{x}_0+M_1)^2}{2M} + M_* \ , \\ M_* = M_2 - \tfrac{M_1^2}{2M} \ . \tag{2}$$

Here M is independent of velocities, M_1 and M_2 are the linear and quadratic function of velocities $\dot{q}_i$. As $E_{\rm kin}$ is positively definite, then M_* is also a positively definite quadratic form of $\dot{q}_i$.

Suppose that the system of Lagrange equations has a stationary solution, for which

$$\dot{x}_0 = u, \quad \dot{q}_i = 0, \quad i = 1, 2, \ldots \ .$$

In this solution the surface of the wave moves with velocity u, without changing its form. If one considers now the disturbed motion of the wave, the momentum conservation law holds

$$\frac{\partial E_{\rm kin}}{\partial \dot{x}_0} = M\dot{x}_0 + M_1 = M_0 u \ ,$$

where M is the function of generalized coordinates q_i, M_0 is the value of function M at the stationary point $q_i = q_i^0,\ i = 1, 2, \ldots$. Thus, using 2, we write the energy conservation law as

$$\frac{(M_0 u)^2}{2M} + E_{\rm pot} + M_* = E \ .$$

For stationary motion $M_* = 0$ and, thus, the energy value is

$$\frac{(M_0 u)}{2} + E^0_{\rm pot} = E_0 \ ,$$

where $E^0_{\rm pot}$ is the value of potential energy at a stationary point. The function E is a Lyapunov function if it is positively definite. As M_* is positively definite, we consider only the functional

$$U = \frac{(M_0 u)^2}{2M} + E_{\rm pot} \ ,$$

If the stationary point is the minimum of U, the Lyapunov Theorem implies that the stationary motion is stable.

4 Kinetic and Potential Energy of Capillary Waves

Consider the system of coordinates in which the fluid is at rest at infinity. The kinetic energy of one period of the stationary wave in this system is expressed through the Stokes coefficients y_n as follows [10]

$$E_{\rm kin} = \tfrac{1}{2}M\dot{x}_0^2, \quad M = \tfrac{\rho\lambda^2}{2\pi}\tfrac{S}{2} \ , \\ S = \sum_{n=1}^{\infty} n(x_n^2 + y_n^2) \ . \tag{3}$$

The capillary potential energy is proportional to the arc length l of one wave period

$$E_{\rm pot} = \sigma l,$$

where σ is the surface tension coefficient. On the complex plane ζ the arc length is calculated as follows

$$l = \oint ds = \oint \left|\frac{dz}{d\zeta}\right| \frac{d\zeta}{i\zeta} \ , \tag{4}$$

where the integral of the differential of the arc length ds is taken along the circle $|\zeta| = 1$.

The Stokes coefficients are not very suitable for arc length calculation l.

So, an analytical function, expressed through parameters q_i, is introduced

$$Q(\zeta) = 1 + \sum_{i=1}^{\infty} q_i \zeta^i \ . \tag{5}$$

This function is found using the equality

$$\tfrac{dz}{d\zeta} = \tfrac{\lambda}{2\pi}\tfrac{i}{\zeta}Q^2(\zeta) \ , \\ \left|\tfrac{dz}{d\zeta}\right| = \tfrac{\lambda}{2\pi}Q(\zeta)\bar{Q}(1/\zeta) \ , \\ \zeta = e^{i\gamma} \ . \tag{6}$$

Substituting (6) in (4), the ark length can be written as an integral of this function using the Residue Theorem

$$l = \frac{\lambda}{2\pi}\oint Q(\zeta)Q(1/\zeta)\frac{d\zeta}{i\zeta} = \lambda\left(1 + \sum_{n=1}^{\infty}|q_n|^2\right) \ .$$

From (1) and (6) we deduce that

$$1+\sum_{k=1}^{\infty} k\frac{z_k}{i}\zeta^k = Q^2(\zeta) =$$

$$= 1+2\sum_{k=1}^{\infty} q_k\zeta^k + \sum_{k=2}^{\infty}\zeta^k\sum_{n=1}^{k-1} q_n q_{k-n}$$

and, therefore

$$z_1 = 2iq_1 \ , \qquad kz_k = 2iq_k + i\sum_{n=1}^{k-1} q_n q_{k-n}, \quad k=2,3,\dots \tag{7}$$

Using substitution

$$M=\rho\frac{\lambda^2}{2\pi}\frac{S}{2}, \quad u^2 = 2\pi\frac{\sigma}{\rho\lambda}\frac{c^2}{2\pi}, \quad l=\frac{\lambda}{2\pi}\bar{l} \ ,$$

the Lyapunov function can be expressed in the dimensionless form as follows

$$U=\sigma\tfrac{\lambda}{2\pi}\bar{U}, \quad \bar{U}=\tfrac{S_0^2}{4S}c^2+\bar{l} \ , \qquad \bar{l}=\tfrac{l}{\lambda}=1+\sum_{k=1}^{\infty}|q_n|^2 \ , \tag{8}$$

where $\bar{U}$ and $\bar{l}$ is a dimensionless Lyapunov function and the arc length of one wave period and c is the dimensionless wave velocity.

The assertion that the first variation of $\bar{U}$ equals zero allows us to find the parameters q_n of the wave and its propagation velocity c.

5 Stationary Capillary Waves

Let us show that he solution of the variational equation $\delta\bar{U}=0$ may be presented as follows

$$q_i = 2b^i \ , \tag{9}$$

where b is a parameter of a family of solutions. To prove this we consider small disturbances of coordinates with respect to stationary values

$$q_n = 2b^n + \varepsilon(\xi_n + i\eta_n), \quad n=1,2,\dots \ . \tag{10}$$

We substitute them into function $\bar{l}$ (8) and expand by parameter ε

$$\bar{l} = 1+\sum_{k=1}^{\infty}\left((2b^n+\varepsilon\xi_n)^2+\varepsilon^2\eta_n^2\right) = 1+\varepsilon\delta\bar{l}+\varepsilon^2\delta^2\bar{l} \ , \tag{11}$$

where $\delta\bar{l}$ and $\delta^2\bar{l}$ are the first and second variation of $\bar{l}$.

Substituting (10) in (11) and separating the real and imaginary parts $z_n = x_n + iy_n$, we will find the expansion for the Stokes coefficients

$$x_k = -\frac{2\varepsilon}{k}\left(\eta_k + 2\sum_{n=1}^{k-1} b^n\eta_{k-n}\right), \quad k=1,2,\dots$$

$$y_k = 4b^k + \frac{2\varepsilon}{k}\left(\xi_k + 2\sum_{n=1}^{k-1} b^n\xi_{k-n}\right) + \frac{\varepsilon^2}{k}\sum_{n=1}^{k-1}\left(\xi_{k-n}\xi_n - \eta_{k-n}\eta_n\right) \ . \tag{12}$$

Substituting these expression in (3), we obtain the expansion by parameter ε for the functional S

$$S = S_0 + \varepsilon\delta S + \varepsilon^2\delta^2 S \ , \tag{13}$$

where δS and $\delta^2 S$ are the first and second variation of S.

Substituting expansions (11) an (13) in functional (8) we obtain the expansion

$$\bar{U} = U_0 + \varepsilon\delta U + \varepsilon^2\delta^2 U \ , \tag{14}$$

The stationary solution is found using the fact that the first variation equals zero

$$\delta\bar{U} = -\tfrac{1}{4}c^2\delta S + \delta\bar{l} = 0 \ , \qquad \delta\bar{l} = \tfrac{d\bar{l}}{d\varepsilon}\Big|_{\varepsilon=0} = 4\Sigma_1, \quad \sum_1 = \sum_{k=1}^{\infty} b^k\xi_k \ , \qquad \delta S = \sum_{k=1}^{\infty} 8b^k\delta(k\,y_k) = 16(\Sigma_1+2\Sigma_2) \ , \qquad \Sigma_2 = \sum_{k=1}^{\infty} b^k\sum_{n=1}^{k-1} b^n\delta q_{k-n} \ . \tag{15}$$

The double sum Σ_2 may be modified by changing the order of summation

$$\Sigma_2 = \sum_{n=1}^{\infty} b^n \sum_{k=n+1}^{\infty} b^k\delta q_{k-n} =$$

$$= \sum_{n=1}^{\infty} b^{2n}\sum_{k_1=1}^{\infty} b^{k_1}\delta q_{k_1} = \tfrac{b^2}{1-b^2}\Sigma_1 \ .$$

From this we find

$$\delta S = 16\frac{1+b^2}{1-b^2}\Sigma_1$$

Substituting $\delta\bar{l}$ and δS in equation (15), we obtain the equality

$$-c^2 4\left(\frac{1+b^2}{1-b^2}\right)\Sigma_1 + 4\Sigma_1 = 0 \ ,$$

from which we find the wave velocity

$$c^2 = \frac{1-b^2}{1+b^2} \ . \qquad (16)$$

Let us show that formulas (9) and (16) determine the wave found in [1]. We substitute (9) in function (5)

$$Q(\zeta) = 1 + 2\frac{b\zeta}{1-b\zeta}, \quad Q^2(\zeta) = \left(\frac{1+b\zeta}{1-b\zeta}\right)^2$$

Thus, by integrating equation (6) we find the

$$z(\zeta) = \frac{\lambda}{2\pi}\left(i\ln\zeta - \frac{4}{1-b\zeta} - 4i\right) \qquad (17)$$

The parameter b may be expressed through the dimensionless amplitude a, which is defined as

$$\tfrac{\lambda}{2\pi}a = \tfrac{1}{2}\left(z(-i) - z(i)\right) =$$

$$= \tfrac{\lambda}{2\pi}\tfrac{1}{2}\left(\tfrac{4}{1-bi} - \tfrac{4}{1+bi}\right) \ .$$

Thus we find the connection between the amplitude and parameter b

$$a = \frac{4b}{1-b^2}, \quad b = \frac{\sqrt{4+a^2}-2}{a}$$

We obtain the parametric equation of the surface of the wave from (17) for $z = x + iy, \ \zeta = e^{i\alpha}$

$$x = -\frac{\lambda}{2\pi}\left(\alpha + \frac{4b\sin\alpha}{1-2b\cos\alpha+b^2}\right) \ ,$$

$$y = \frac{\lambda}{2\pi}\left(\frac{4(1-b\cos\alpha)}{1-2b\cos\alpha+b^2} - 4\right) \ .$$

These are the same expressions for x and y that were obtained by Crapper. Therefore, a new deduction method for the known exact solution for the capillary wave [1] is presented. The values $b = b_0 = 0.454, \ a = 2.280$ corresponds to the maximum wave development. In Fig. 2 the graphs of waves with values $b := 0.1; \ 0.3$ and maximum wave development $b = 0.454$ are presented.

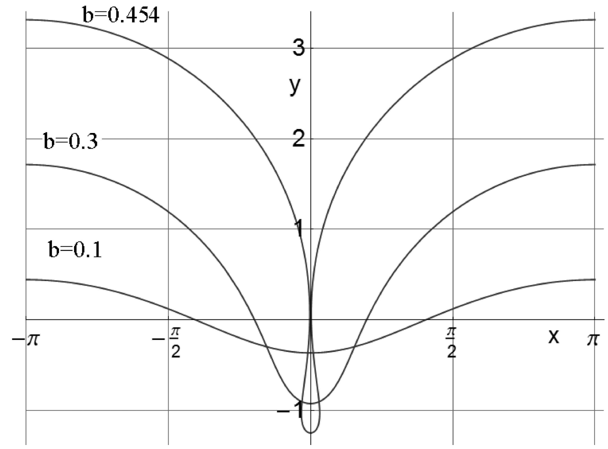

Figure 2: Capillary waves at different values of parameter b.

6 Second Variation

The second variation

$$\delta^2\bar{U} = \frac{1}{2}\frac{d^2\bar{U}}{d\varepsilon^2}\Big|_{\varepsilon=0}$$

is the quadratic form of variations $\xi_i, \ \eta_i, \ i = 1, 2, \ldots$. It is expressed through the first and second variations of functionals S and $\bar{l}$

$$\delta^2\bar{U} = \frac{c^2}{4}\left(\frac{(\delta S)^2}{S_0} - \delta^2 S\right) + \delta^2\bar{l} \ ,$$

$$S_0 = \frac{16b^2}{(1-b^2)^2},$$

The second variations are calculated with the help of (3), (8), (10) and (12)

$$\delta^2 l = \tfrac{1}{2}\tfrac{d^2\bar{l}}{d\varepsilon^2}\Big|_{\varepsilon=0} = \delta^2 l_1 + \delta^2 l_2 \ ,$$

$$\delta^2 l_1 = \sum_{n=1}^{\infty}\xi_n^2, \quad \delta^2 l_2 = \sum_{n=1}^{\infty}\eta_n^2 \ ,$$

$$\delta^2 S = \tfrac{1}{2}\tfrac{d^2 S}{d\varepsilon^2}\Big|_{\varepsilon=0} = \delta^2 S_1 + \delta^2 S_2 \ ,$$

$$\delta^2 S_1 = \sum_{k=1}^{\infty}\left(\tfrac{4}{k}\left(\xi_k + 2\sum_{n=1}^{k-1}\xi_n b^{k-n}\right)^2 + \right.$$
$$\left. + 8b^k\sum_{n=1}^{k-1}\xi_n\xi_{k-n}\right),$$

$$\delta^2 S_2 = \sum_{k=1}^{\infty}\left(\tfrac{4}{k}\left(\eta_k + 2\sum_{n=1}^{k-1}b^n\eta_{k-n}\right)^2 - \right.$$
$$\left. - 8b^k\sum_{n=1}^{k-1}\eta_n\eta_{k-n}\right) \ .$$

The variables ξ_n and η_n of the second variation $\delta^2\bar{U}$ may be separated and the second variation $\delta^2\bar{U}$ may be presented as the sum of two quadratic forms $\delta^2\bar{U} = \delta^2\bar{U}_1(\xi)+\delta^2\bar{U}_2(\eta)$. The first $\delta^2\bar{U}_1(\xi)$ depends only on ξ and is expressed through $\delta^2 S_1$ and $\delta^2\bar{l}_1$, which depend only on ξ. The second is expressed through $\delta^2 S_2$ and $\delta^2\bar{l}_2$, which depend only on η.

The first quadratic form defines the stability of the wave with respect to the symmetric disturbances ξ_n, the second one defines the stability with respect to the asymmetric disturbances η_n.

7 Stability of the Wave with respect to Symmetric Disturbances

Let us first consider the quadratic forms of second variations for symmetric disturbances

$$\delta^2\bar{U}_1 = \frac{c^2}{4}\left(\frac{(\delta S)^2}{S_0} - \delta^2 S_1\right) + \delta^2\bar{l}_1 \ .$$

For $\delta^2 U_1(\xi)$ the following inequality holds

$$\delta^2\bar{U}_1 > \lambda_{\min}\sum_{n=1}^{\infty}(\xi_n)^2 \ , \qquad (18)$$

where $\lambda_{\min}$ is the smallest eigenvalue of the quadratic form.

The matrix a_{mn} of the quadratic form $\delta^2\bar{U}_1$ as $b = 0$ is diagonal and its diagonal elements are $a_{11} = 4,\ a_{nn} = (n-1)/n,\ n = 2,3,\ldots$. The eigenvalues that correspond to the adjoint linear operator are $\lambda_n = a_{nn}$. The smallest eigenvalue is equal to the second diagonal element $\lambda_{\min} = a_{22} = 1/2$. For eigenvalues the expansion in powers of b may be obtained. For the first five eigenvalues the expansions are

$$\lambda_{min} = \lambda_1 = \tfrac{1}{2} - \tfrac{11}{7}b^2 + \tfrac{2489}{343}b^4 \ ,$$

$$\lambda_2 = \tfrac{2}{3} - \tfrac{2}{3}b^2 - \tfrac{38}{15}b^4,$$

$$\lambda_3 = \tfrac{3}{4} - \tfrac{1}{2}b^2 + \tfrac{1}{2}b^4 \ , \qquad (19)$$

$$\lambda_4 = \tfrac{4}{5} - \tfrac{2}{5}b^2 + \tfrac{2}{5}b^4 \ ,$$

$$\lambda_5 = \tfrac{5}{6} - \tfrac{1}{3}b^2 + \tfrac{1}{3}b^4 \ .$$

In Fig. 3 the dependences of the first five eigenvalues on b are presented. The solid lines stand for the results of numeric calculations, the dashed lines stand for the expansions in powers of b (19). From the graphs we see that the greater the index of the eigenvalue is, the better it is approximated by its expansion. In the second variation $N = 20$ independent variations $\delta q_i,\ i = 1,2,\ldots 20$ are taken into consideration. The smallest eigenvalue $\lambda_1(b)$ decreases monotonously until it reaches the value $\lambda(b_0) = 0.03069$ and for $N > 15$ almost does not depend on N.

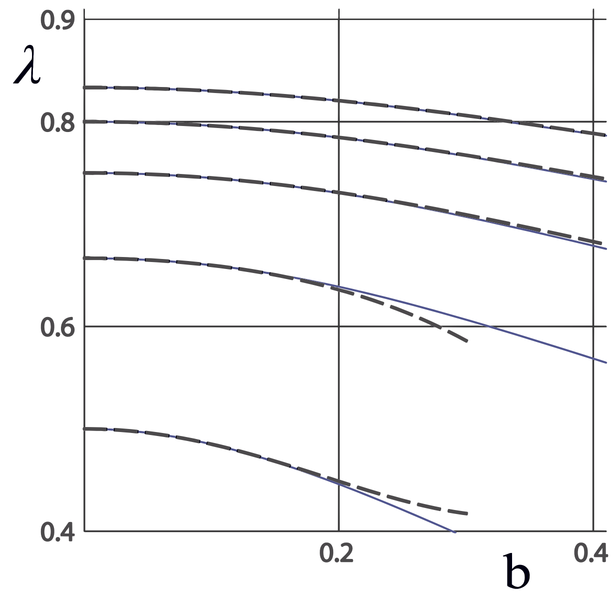

Figure 3: Eigenvalues (symmetric disturbances).

Thus, inequality (18) implies that the second variation $\delta^2 U > 0$ is strictly positive for all variations δq_i. By the Lyapunov Theorem the stationary motion of capillary wave is stable for all possible amplitude values.

The eigenvalues determine the main oscillation frequencies near stationary motion.

8 Stability of Capillary Waves with respect to Non-symmetric Disturbances

Consider now the quadratic form of the second variation for non-symmetric disturbances

$$\delta^2\bar{U}_2 = -\frac{c^2}{4}\delta^2 S_2 + \delta^2\bar{l}_2 \ . \qquad (20)$$

The matrix

$$b_{mn} = \frac{1}{2}\frac{\partial^2(\delta^2\bar{U}_2)}{\partial\eta_m\partial\eta_n}$$

for $b = 0$ is diagonal and $b_{nn} = (n-1)/n,\ n = 2,3,\ldots$. The eigenvalues are $\lambda_n = b_{nn}$.

The matrix b_{mn} is singular, its determinant equals zero. This is due to the linear dependence of the generalized $\eta_1, \eta_2, \eta_3, \ldots$, which is expressed as follows

$$r = \frac{1}{2}\sum_{k=1}^{\infty} kb^k \frac{\partial(\delta^2 U_2)}{\partial \eta_k} = 0 \ . \tag{21}$$

Let us prove this fact as follows. (20) implies that the series r in powers of b is the difference between the series r_1 and r_2

$$r_1 = \frac{1}{2}\sum_{k=1}^{\infty} kb^k \frac{\partial(\delta^2 l_2)}{\partial \eta_k} = \sum_{k=1}^{\infty} kb^k \eta_k,$$

$$r_2 = \frac{1-b^2}{8(1+b^2)}\sum_{k=1}^{\infty} kb^k \frac{\partial(\delta^2 S_2)}{\partial \eta_k} \ .$$

It can be checked that in any finite number of coordinates $\eta_1, \ldots, \eta_n$ the difference $r_1 - r_2$ is small of power b^{n+1}

$$r_1 - r_2 = O(b^{n+1}) \ .$$

The series r_1 converges for admissible values of parameter $0 < b < 0.454$ and bounded values of η_k. The difference of partial sums of r_1 and r_2 tends to zero as $n \to \infty$. Thus the series r_2 also converges and the difference $r_1 - r_2$ equals zero. Thus we complete the proof.

Equality (21) is equivalent to the fact that the linear combination of the matrix columns $b_{\cdot n}$ satisfies the equality

$$\sum_{k=1}^{\infty} kb^k b_{km} = 0 \ ,$$

and, thus, the matrix (b_{mn}) is singular.

This can also be explained by the fact that the mapping (1) is multivalued. The mapping $\zeta' = e^{i\gamma_0}$ maps the circle $|\zeta| = 1$ into itself, therefore the form of the wave does not change. So, let us put the first coordinate η_1 equal to zero. Then all the other coordinates $\eta_n, \ n = 2, 3, \ldots$ are independent.

Such choice of coordinates implies that the matrix (b_{mn}) as $b = 0$ is diagonal $b_{mn} = (n-1)/n\delta_{mn}$, and the smallest eigenvalue equals the second diagonal element $\lambda_{\min} = b_{22} = 1/2$. For eigenvalues we may obtain expansion in powers of b. For the first four eigenvalues, set by the increasing index numbers, the expansions are

$$\begin{aligned} \lambda_{min} = \lambda_1 &= \tfrac{1}{2} - 3b^2 + 23b^4 \ , \\ \lambda_2 &= \tfrac{2}{3} - \tfrac{2}{3}b^2 - 16b^4 \ , \\ \lambda_3 &= \tfrac{3}{4} - \tfrac{1}{2}b^2 + \tfrac{1}{2}b^4 \ , \\ \lambda_4 &= \tfrac{4}{5} - \tfrac{2}{5}b^2 + \tfrac{2}{5}b^4 \ . \end{aligned} \tag{22}$$

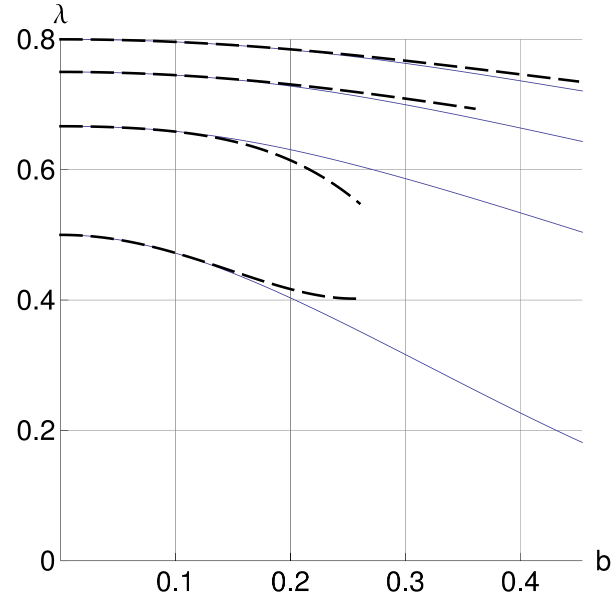

Figure 4: Eigenvalues (non-symmetric disturbances).

In Fig. 4 the dependences of the first four eigenvalues on parameter b are presented. The solid lines stand for the numeric calculations, the dashed ones for the expansion in powers of b. The smallest eigenvalue $\lambda_1(b)$ monotonously decreases until $\lambda(b_0) = 0.181408$ and for the number of generalized coordinates $N > 15$ almost does not depend on N.

Acknowledgements: The research was supported by the Russian Science Foundation (project N 14-19-01633) in IPMech RAS..

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