Radiation and Mass transfer Effects on nonlinear MHD boundary layer flow of liquid metal over a porous stretching surface embedded in porous medium with heat generation

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Abstract: - The present paper analyzes the effects of mass transfer on steady nonlinear MHD boundary layer flow of a viscous incompressible fluid over a nonlinear porous stretching surface embedded in a porous medium in presence of nonlinear radiation and heat generation. The liquid metal is assumed to be gray, emitting, and absorbing but non-scattering medium. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. The resulting nonlinear ordinary differential equations are solved numerically using Runge-Kutta fourth order method along with shooting technique. Comparison with previously published work is obtained and good agreement is found. The effects of various governing parameters on the liquid metal fluid dimensionless velocity, dimensionless temperature, dimensionless concentration, skin-friction coefficient, Nusselt number and Sherwood number are discussed with the aid of graphs.

Key-Words: - Radiation, Mass transfer, MHD, Stretching surface, porous medium, heat generation.

1 Introduction

Liquid metals are a specific class of coolants. Their basic advantage is a high molecular thermal conductivity which, for identical flow parameters, enhances heat transfer coefficients. Another distinguishing feature of liquid metals is the low pressure of their vapors, which allows their use in power engineering equipment at high temperature and low pressure, thus alleviating solution of mechanical strength problems. The most widespread liquid metals used in engineering are alkali metals. Among them sodium is first and foremost, used as a coolant of fast reactors and a working fluid of hightemperature heat pipes. Liquid metal heat transfer plays an important role in modern life, since the liquid metals are used as coolant in nuclear reactor and as working fluids in space power plants. Therefore it is essential to consider the variable thermal properties of liquid metals heat transfer problem (Arunachalam and Rajappa [1], Lubarsky and Kaufman [2], Lyon [3]).

In recent years, a great deal of interest has been generated in the area of heat and mass transfer of the boundary layer flow over a nonlinear stretching sheet, in view of its numerous and wide-ranging applications in various fields like polymer processing industry in particular in manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution. Sakiadis [4] initiated the study of boundary layer flow over a continuous solid surface moving with constant speed as result of ambient fluid movement; this boundary flow is generally different from boundary layer flow over a semi-infinite flat plate. Erickson [5] studied this problem to the case in which the transverse velocity at the moving surface is nonzero with the effects of heat and mass transfer being taken in to account. Danberg and Fansler [6], using nonsimilar solution method, studied the flow inside the boundary layer past a wall that is stretched with a velocity proportional to the distance along the wall. Gupta and Gupta [7], using similar solution method, analyzed heat and mass transfer in the boundary layer over a stretching sheet subject to suction or blowing. The laminar boundary layer on an inextensible continues flat surface moving with a constant velocity in a non-Newtonian fluid characterized by a power-law model is studied by Fox et al. [8], using both exact and approximate

methods. Rajagopal et al. [9] studied the flow behavior of viscoelastic fluid over stretching sheet and gave an approximate solution to the flow field. Troy et al. [10] presented an exact solution for Rajagopal problem. Vajravelu and Roper [11] studied the flow and heat transfer in a viscoelastic fluid over a continues stretching sheet with power law surface temperature, including the effects of viscous dissipation, internal heat generation or absorption, and work due to deformation in the energy equation. Vajravelu [12] studied the flow and heat transfer characteristics in a viscous fluid over a nonlinearly stretching sheet without heat dissipation effect. Cortell [13], [14] has worked on viscous flow and heat transfer over a nonlinearly stretching sheet. Cortell [15] have investigated the influence of similarity solution for flow and heat transfer of a quiescent fluid over a nonlinear stretching surface.

The deep interest in the porous medium is easily understandable since porous medium is used in vast applications, which covers many engineering disciplines. For instance, applications of the porous media includes, thermal insulations of buildings, heat exchangers, energy collectors, solar geophysical applications, solidification of alloys, nuclear waste disposals, drying processes, chemical reactors, energy recovery of petroleum resources etc., More applications and good understanding of the subject is given in the recent books by Nield and Bejan [16], Vafai [17], Pop and Ingham [18].

The radiation effects have important applications in physics and engineering particularly in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects on the boundary layer may play important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Moreover, when radiative heat transfer takes place, the fluid involved can be electrically conducting since it is ionized due to the high operating temperature. Accordingly, it is of interest to examine the effect of the magnetic field on the flow. Studying such effect has great importance in the application fields where thermal radiation and MHD are correlative. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are examples of such fields. Elbashbeshy [19] free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. Raptis and Perdikis [20] studied viscous flow over a nonlinear stretching sheet in the presence of a chemical reaction and magnetic field. Awang and Hashim [21] obtained the series solution for flow over a nonlinearly stretching sheet with chemical reaction and magnetic field. Abbas and Hayat [22] addressed the radiation effects on MHD flow due to a stretching sheet in porous space. Effect of radiation on MHD steady asymmetric flow of an electrically conducting fluid past a stretching porous sheet has been analysed analytically by Ouaf [23], Mukhopadhyay and Layek [24] investigated the effects of thermal radiation and variable fluid viscosity on free convection flow and heat transfer past a porous stretching surface. Gururaj and Pavithra [25] investigated nonlinear MHD boundary layer flow of a liquid metal over a porous stretching wurface in presence of radiation. The effects of variable viscosity and nonlinear radiation on MHD flow over a stretching surface with power-law velocity was reported by Anjali Devi and Gururaj [26].

A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation was carried out by Samad and Mohebujjaman [27]. Singh [28] analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. Kesavaiah et al. [29] reported that the effects of the chemical reaction and radiation on MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Mohammed Ibrahim and Bhaskar Reddy [30] studied the effects of thermal radiation on steady MHD free convective flow past along a stretching surface in presence of viscous dissipation and heat source. MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation was studied by Samad and Mobebujjaman. [31]. Ramana Reddy et.al [32] have studied the Thermal diffusion and radiation effects on unsteady MHD Free convection heat and mass transfer flow past a linearly Accelerated vertical porous plate with variable temperature and mass diffusion. Hitesh kumar [33] have analysed the problem of radiative heat and mass

transfer over an inclined plate at prescribed heat flux with chemical reaction. Turkyilmazoglu and I.Pop [34] were discussed the effect of heat and mass transfer of unsteady natural convection flow of some nanofluids past a vertical infinite flat plate with radiation.

However the interaction of nonlinear radiation with mass transfer of an electrically conducting and diffusing fluid past a nonlinear stretching surface has received little attention. Hence an attempt is made to investigate the nonlinear radiation effects on a steady convective flow over nonlinear stretching surface in presence of magnetic field, porous medium and heat generation. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in Figs.

2. FORMULATION OF THE PROBLEM

Consider coupled radiation and forced convection along a horizontal porous stretching surface which is kept at uniform temperature T_w and moving with velocity $u_w = u_0 x^m$ through and stationary liquid metal. The liquid metal is assumed to be a gray, emitting, absorbing and electrically conducting, but non-scattering medium at temperature T_∞ . A variable magnetic field is applied normal to the horizontal surface $B(x) = B_0 x^{\frac{m-1}{2}}$ in accordance with Chiam [32].

The following assumptions are made

- 1. Flow is two-dimensional, steady and laminar.
- 2. The fluid has constant physical properties.
- 3. The usual boundary layer assumptions are made [Ali [35]].
- 4. The *x* axis runs along the continuous surface in the direction of motion and *y* axis perpendicular to it. and
- 5. The radiation dissipation in the x-axis is negligible in comparison with that in the yaxis following the lines of Michael F. Modest (Radiative Heat Transfer. Page 696) [36].

The continuity, momentum, energy conservation and mass conservation equations under the above assumption are written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B^2(x)}{\rho}\right)u - \frac{v}{K'}u$$
(2)

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q_0 T$$
(3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}$$
(4)

with the associated boundary conditions

$$u_{w} = u_{0}x^{m}, v = v_{0}(x), T = T_{w}, C = C_{w} \text{ at } y = 0$$

$$u = 0, T = T_{\infty}, C = C_{\infty} \text{ as } y \to \infty$$
 (5)

where u, v are velocity component of fluid in x and y direction, ρ is the density of the fluid, ν is the kinematic viscosity, σ is the electric conductivity, K is the thermal conductivity, B_0 is constant applied magnetic induction, c_p is specific heat at constant pressure, B(x) is the variable applied magnetic induction, $v_0(x)$ is the variable injection velocity

$$\left(v_0(x) = c \sqrt{\frac{v u_w}{x}}\right)$$
, *T* is temperature of the fluid, *C*

is the concentration of the fluid, T_w is the temperature of the heated surface, T_∞ is the temperature of the ambient fluid, C_w is the concentration of the heated surface, C_∞ is the concentration of the ambient fluid, q_r is the component of radiative flux, u_0 is constant, *m* velocity exponent parameter,

The radiative heat flux term is simplified by using the Roseland diffusion approximation (Hossian et al [37]) and accordingly

$$q_r = -\frac{16\sigma^* T^3}{3\alpha^*} \frac{\partial T}{\partial y} \tag{6}$$

where σ^* is Stefan-Boltzmann constant, α^* is the Rosseland mean absorption coefficient.

The equation of continuity is satisfied if we choose a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}$$
 and $v = -\frac{\partial \psi}{\partial x}$

Introduction the usual similarity transformation [Ali [34]].

$$\eta(x, y) = y \sqrt{\frac{m+1}{2}} \sqrt{\frac{u_0 x^{m-1}}{\nu}},$$

$$\psi(x, y) = \sqrt{\frac{2}{m+1}} \sqrt{\nu u_0 x^{m+1}} f(\eta)$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \theta_w = \frac{T_w}{T_{\infty}}$$
(7)

Equations (2), (3) and (4) can be written as

$$f''' + ff'' - \left(\frac{2m}{m+1}\right)f'^{2} - \left(M^{2} + \frac{1}{K}\right)f' = 0$$
(8)
$$\left[1 + \frac{4}{3R}\left(1 + \left(\theta_{w} - 1\right)\theta\right)^{3}\right]\theta''$$

$$+\frac{4}{R}\left(\left[1+\left(\theta_{w}-1\right)\theta\right]^{2}\right)\theta^{\prime 2}$$
(9)

 $+\Pr f \theta' + \Pr Q\theta = 0$ $\phi'' + Scf \phi' - Scf' \phi = 0$ (10)

where $M = \sqrt{\frac{2\sigma B_0^2}{\rho u_0(m+1)}}$ is the magnetic

interaction parameter, $R = \frac{K\alpha^*}{4\sigma^* T_{\infty}^3}$ is radiation

parameter, $K = \frac{K'u_0}{v}$ is permeability parameter,

$$Q = \frac{Q_0}{u_0}$$
 is heat generation parameter, $Sc = \frac{v}{D}$ is the

Schmidt number.

with the boundary conditions

 $f(0) = -S, f'(0) = 1, \theta(0) = 1, \phi(0) = 1$ $f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0$ (11) where $G = \sqrt{\frac{2}{2}}$ is the Deresity perpendence of (11)

where $S = \sqrt{\frac{2}{m+1}}c$ is the Porosity parameter, *c* is non dimensional constant. (for Injection S > 0

non dimensional constant. (for Injection S > 0 and for Suction S < 0).

3. Solution of the problem

The set of coupled non-linear governing boundary layer equations (8) – (10) together with the boundary condition (11) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order nonlinear differential equations (8) – (10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et. al[38]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta \eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to f''(0), $\theta'(0)$ and $\phi'(0)$, are also display in graphs.

4. Results and discussion

As a result of the numerical calculations, the $f'(\eta)$, dimensionless dimensionless velocity temperature $\theta(\eta)$ and dimensionless concentration $\phi(\eta)$ distributions for the flow under consideration are obtained and their behavior have been discussed for variations in the governing parameters viz., the magnetic interaction parameter M, velocity exponent parameter m, permeability parameter K, Porosity parameter S, Radiation parameter R, Prandtl number Pr, surface temperature parameter θ_w , heat generation parameter Q and Schmidt number Sc. In the present study, the following default parametric values are adopted. M = 1.0, m =1.0, K = 1.0, S = 0.1, Pr = 0.71, R = 1.0, $\theta_w = 1.1$, Q = 0.05, Sc = 0.6. All graphs therefore correspond to these unless specifically indicated on the appropriate graph.

In order to ascertain the accuracy of our numerical results, the present study is compared with the previous study. The temperature profiles are compared with available theoretical solution of Elbashbeshy [19], free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field in Fig.1 and Fig.2. It is observed that the present results are in good agreement with that of Elbashbeshy [19].

Fig 3(a) displays the plot of dimensionless velocity $f'(\eta)$ for different values of M. It is noted that as magnetic interaction parameter M increases, transverse velocity $f'(\eta)$ decreases elucidating the fact that the effect of magnetic field is to decelerate the velocity. The effect of magnetic interaction parameter M over the dimensionless temperature $\theta(\eta)$ and dimensionless concentration $\phi(\eta)$ is shown with the help of Figs. 3(b) and 3(c). It is observed that temperature and concentration increases with an increase in the magnetic interaction parameter M.

The effect of velocity exponent parameter *m* over the dimensionless velocity field $f'(\eta)$ is shown in the graph of Fig. 4(a). It is observed that the effect of velocity exponent parameter is to increase the velocity. Figs 4(b) and 4(c) show the dimensionless temperature profiles and dimensionless concentration profiles for different values of exponent parameter m. It is noticed that the temperature and concentration reduce with an increase of exponent parameter m.

Fig. 5(a) shows the effect of permeability parameter K on the velocity. It is noticed that as the permeability parameter K increases, the velocity is also increases. Fig 5(b) displays the variation of the thermal boundary-layer with permeability parameter K. It is found that the thermal boundary layer thickness decreases with an increase in the permeability parameter K. The influence the permeability parameter K on the concentration field is shown in Fig 5(c). It is observed that the concentration reduces with an increase of the permeability parameter K.

Prandtl number variation over the dimensionless temperature profile is elucidated through Fig. 6. As Prandtl number *Pr* increases, the temperature $\theta(\eta)$

decreases, illustrates the face that the effect of Prandtl number Pr is to decrease the temperature in the magnetic field. Furthermore, the effect of Prandtl number Pr is to reduce the thickness of thermal boundary layer.

Fig.7. illustrates the effects of radiation parameter R over the dimensionless temperature $\theta(\eta)$. It is observed that the effect radiation parameter is to reduce the temperature, elucidating the face that the thermal boundary layer thickness decreases as R increases.

The effect of surface temperature parameter θ_w over the dimensionless temperature $\theta(\eta)$ is shown in Fig

8. Increasing surface temperature parameter θ_w is to increase the temperature.

Fig 9 depict the dimensionless temperature profiles for different values of the heat generation parameter Q. It is observed that the temperature increase with an increase in Q. The influence of the Schmidt number Sc on the dimensionless concentration profiles is plotted in Fig 10. As the Schmidt number increases the concentration decreases.

The non-dimensionless velocity, temperature and concentration profiles for the different values of suction S are shown through Figs.11 (a), 11(b) and 11(c) respectively. From these Figs it is observed that velocity, temperature and concentration profiles are increases with an increase of suction parameter S.

Fig 12 displays the variation of skin-friction coefficient f''(0) against the magnetic interaction parameter M for different values of velocity exponent parameter m. It is seen that the skin-

friction coefficient f''(0) decreases with increase of velocity exponent parameter m and it decreases for increasing magnetic interaction parameter M.

It is observed from Fig 13 that the dimensionless rate of heat transfer $\theta'(0)$ increases with increase of velocity exponent parameter *m*. further, it is noted that the dimensionless rate of heat transfer $\theta'(0)$ increases in magnitude for increasing magnetic interaction parameter *M*.

Fig 14 shows the dimensionless rate of mass transfer $\phi'(0)$ increases with increase of velocity exponent parameter *m*. also it is observed that the dimensionless rate of mass transfer $\phi'(0)$ increases in magnitude for increasing magnetic interaction parameter *M*.

Fig 15 demonstrates the effect of magnetic interaction parameter M over the dimensionless rate of heat transfer $\theta'(0)$ for different values of radiation parameter R. It is seen that the dimensionless rate of heat transfer $\theta'(0)$ decreases with increase of radiation parameter R and increases with respect to magnetic interaction parameter M.

Fig 16 portrays the variation of dimensionless rate of heat transfer $\theta'(0)$ against the magnetic interaction parameter M for different values of heat generation parameter Q. it is apparent that increasing the dimensionless rate of heat transfer $\theta'(0)$ decreases with increase of heat generation parameter Q and increases with respect to magnetic interaction parameter M.

Fig 17 displays the variation of skin-friction coefficient f''(0) against the magnetic interaction parameter M for different values of permeability parameter K. It is seen that the skin-friction coefficient f''(0) decreases with increase of velocity exponent parameter m and it decreases for increasing magnetic interaction parameter M.

It is observed from Fig 18 that the dimensionless rate of heat transfer $\theta'(0)$ increases with increase of permeability parameter *K*. further, it is noted that the dimensionless rate of heat transfer $\theta'(0)$ increases in magnitude for increasing magnetic interaction parameter *M*.

Fig 19 shows the dimensionless rate of mass transfer $\phi'(0)$ increases with increase of permeability parameter *K*. also it is observed that the dimensionless rate of mass transfer $\phi'(0)$ increases in magnitude for increasing magnetic interaction parameter *M*.



Fig.1. Temperature profiles for different R



Fig.2. Temperature profiles for different θ_w



Fig. 3(a). Velocity Profiles for different M



Fig.3(b) Temperature profiles for different M



Fig.3(c). Concentration profiles for different M



Fig. 4(a). Velocity Profiles for different m



Fig. 4(b). Temperature Profiles for different m



Fig.4(c). Concentration profiles for different m



Fig. 5(a). Velocity Profiles for different k



Fig.5(b). Temperature profiles for different k



Fig.5(c). Concentration profiles for different k



Fig. 6. Temperature profiles for different Pr



Fig. 7. Temperature profiles for different R



Fig. 8. Temperature profiles for different θ_{w}



Fig. 9. Temperature profiles for different Q



Fig. 10. Concentration profiles for different Sc



Fig.11(a). Velocity Profiles for different S



Fig. 11(b). Temperature Profiles for different S



Fig. 11(c).Concentration Profiles for different S



Fig.12. Skin friction coefficient for different m



Fig.13. Dimensionless rate of heat transfer for different *m*



Fig.14. Dimensionless rate of mass transfer for different *m*



Fig.15. Dimensionless rate of heat transfer for different *R*



Fig.16. Dimensionless rate of heat transfer for different Q



Fig.17. Skin friction coefficient for different k



Fig.18. Dimensionless rate of heat transfer for different *k*



Fig.19. Dimensionless rate of mass transfer for different *k*

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