Unsteady viscous flow with non linear free surface around oscillating SWATH ship sections

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Ship motion problem is approached exploiting the assumption of linearity and subdividing forces acting on the hull in different components; in this paper the focus is on the radiation problem. A fully viscous solver, based on the numerical solution of the Navier-Stokes equation through the finite volume technique, is used to calculate radiation forces on hull surface, with non linear free surface evaluated through the volume of fluid method. Radiation forces are the most influenced by viscous effects, especially for particular hull geometries as SWATH ships. OpenFOAM libraries are used to solve the unsteady flow around one circular and two SWATH two dimensional sections forced to heave in calm water. The comparison with experiments evidence a very good agreement of the forces measured over a wide frequency range including irregular frequencies.

Key–Words: Unsteady viscous solvers, free surface, OpenFOAM, SWATH

1 Introduction

Hydrodynamics design of a ship hull is normally based on the determination of forces induced on the hull which advances in a non realistic calm water. Most ships require to ensure good performance also in rough seas; for those kind of vessels seakeeping studies allow the comparison among different hull geometries on the basis of the their motions in waves, with the aim to extend their operational capability in the highest sea states. The core problem of the motion prediction in waves is the estimation of added mass and damping i.e. the forces acting on a oscillating body at different frequencies.

Traditional seakeeping methods based on the strip theory and developed in the past exploits the potential flow assumption, to solve a Laplacian equation for the velocity potential together with boundary condition for the hull and for the free surface. The first approaches were based on conformal mapping, among which Lewis's method [1], they were proposed to find the velocity potential in the case of complex geometry [2]. More recently the potential flow problem is solved using boundary element methods, placing singularities on the hull boundaries; the method developed by Frank [3] was the first.

The assumption that the greatest part of the energy is dissipated in waves generation, makes the potential flow theory a valid simplification in the development of seakeeping predictions [4]. For particular ship hulls, such as SWATH or Semi-SWATH typologies, the energy losses due to viscous effects are non-negligible in comparison with that due to wave generation. Assuming valid the superimposition principle, the solution of the problem can be achieved splitting it into different parts, and recovering the viscous effects only where the eddies generation affects in a non negligible way the results of the calculation, in order to develop a method which can be still based on potential flow solution, so to maintain the computational efforts as limited as possible. The present study describes the development of a time domain model to solve the viscous flow around oscillating sections. The aim is to predict amplitude and phase of of a section forced to oscillate in calm water. The motion of a ship as a rigid body and the external forces acting on the hull can be described with the following system of six equations in six unknowns:

$$\mathfrak{F}_j(t) = \sum_{k=1}^{6} M_{jk}\ddot{\eta}_k \quad j = 1, 2, ..., 6 \tag{1}$$

With k as the motions index and j as forces index.
Non-linear effects are important in severe sea states however an incident regular wave of amplitude ζ_a, far from breaking, can be considered in a linear theory approach. In such a way the wave induced motions can be well described considering them linearly proportional to ζ_a.
The hydrodynamic problem is normally dealt splitting it into two sub-problems:

1. The forces and the moments on the body when the ship is restrained from oscillating and subjected to incident regular waves. The hydrodynamic loads are called *wave excitation loads*; they can be splitted in the *Froude-Kriloff* and the *diffraction* forces and moments.

2. The forces and moments on the body when the structure is forced to oscillate with the wave excitation frequency in any rigid-body motion mode. There are no incident waves. The hydrodynamic loads are identified as *added mass*, *damping* and *restoring* terms.

Due to linearity the forces obtained in 1 and in 2 can be added to give the total hydrodynamic forces [5]. Using the hypothesis of small amplitude motions, the wave exciting forces ($\mathfrak{F}_{Ej}$) can be expressed in terms of radiation and restoring forces as follow:

$$\mathfrak{F}_{Ej} = \sum_{k=1}^{6}(M_{jk}\ddot{\eta}_k) - \mathfrak{F}_{RADj}(t) - \mathfrak{F}_{RESj}(t) \quad (2)$$

where:

$$\mathfrak{F}_{RADj} = -\sum_{k=1}^{6}(A_{jk}\ddot{\eta}_k + B_{jk}\dot{\eta}_k) \quad (3)$$

The term $A_{jk}\ddot{\eta}_k$ (A_{jk} added mass matrix) indicates the component of the hydrodynamic forces in phase with the acceleration and the term $B_{jk}\dot{\eta}_k$ (B_{jk} damping matrix) the one in phases with the velocity.

$$\mathfrak{F}_{RESj} = -\sum_{k=1}^{6} C_{jk}\eta_k \quad (4)$$

$C_{jk}\eta_k$ (C_{jk} restoring matrix) indicates the hydrostatic forces.
Using the definition given in (3) and (4) into the equation of motion (1), this last can be expressed as:

$$\mathfrak{F}_{Ej}(t) = \sum_{k=1}^{6}[(M_{jk}+A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k] \quad (5)$$

According to the strip theory the problem can be reduced in a series of two dimensional problems for which it is to study motions of transverse sections with three degrees of freedom: sway, heave and roll. Assuming linear sinusoidal motion and response, it is possible to pass from time domain to frequency domain and using Euler's formula for complex notation, the radiation forces can be written as follow:

$$F_{RADj} = \sum_{k=1}^{6}\xi_k[-\omega_e^2 a_{jk} + i\omega_e b_{jk}] \quad (6)$$

In the case of heave motion:

$$F_{RAD3} = \xi_3[-\omega_e^2 a_{33} + i\omega_e b_{33}] \quad (7)$$

2 Numerical methods

Imposing an harmonic law of motion to a transverse section, the two dimensional radiation forces can be found by the integration over the section of the hydrodynamic pressure and the tangential strain fields which the incompressible viscous fluid exerts on the hull. Pressure and velocity fields can be described writing the conservation principles for the mass and the momentum:

$$\frac{dm}{dt} = 0 \qquad \frac{d(m\mathbf{v})}{dt} = \sum \mathbf{f} \quad (8)$$

Expressing the mass as an extensive property, its rate of change can be expressed as the sum of the rate of change of the mass in the original control volume and the net flux of it through the boundaries (convective term); applying Gauss divergence theorem to the convective term it is possible to express the surface integral as a volume integral; considering an infinitesimal small control volume, the conservation law can be expressed as follow:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (9)$$

Momentum associated with a given mass can be changed through the action of all forces acting on the control mass. Expressing the momentum in terms of an extensive property and considering that the forces which can act on the fluid can be exerted on the surface of the control mass (*surface forces*) or throughout the volume of the control mass (*body forces*), the momentum conservation can be expressed by the following relation:

$$\frac{\partial}{\partial t}\int_{\Omega}\rho\mathbf{v}d\Omega + \int_{S}\rho\mathbf{v}\mathbf{v}\,.\,\mathbf{n}dS = \int_{S}\mathbf{T}\,.\,\mathbf{n}ds + \int_{\Omega}\rho\mathbf{b}d\Omega \quad (10)$$

For an incompressible fluid the mass and the momentum conservation can be written as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (11)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j}(\nu \frac{\partial u_i}{\partial x_j}) - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + g_i \quad (12)$$

The integral form of conservation equations is at the basis of the *Finite Volume* technique. The computational domain is subdivided into a finite number of control volumes (*cells*) and the conservation equations can be applied to each cell as well as to the solution domain as a whole.
Considering a generic scalar quantity ϕ, it is possible to define the integral form of the equation for the conservation of ϕ as:

$$\frac{\partial}{\partial t}\int_S \rho\phi d\Omega + \int_S \rho\phi \mathbf{v}\cdot\mathbf{n} dS = \sum f_\phi \quad (13)$$

Where f_ϕ represents the transport of the quantity ϕ by diffusion effects:

$$f_\phi^d = \int_S \Gamma\nabla\phi\cdot\mathbf{n} dS \quad (14)$$

Where Γ is the diffusivity for ϕ.
Including also the effect of sources or sinks (q_ϕ):

$$\frac{\partial}{\partial t}\int_S \rho\phi d\Omega + \int_S \rho\phi \mathbf{v}\cdot\mathbf{n} dS = \int_S \Gamma\nabla\phi\cdot\mathbf{n} dS + \int_\Omega q_\phi d\Omega \quad (15)$$

This equation applies to each cell and the sum of all the equations for all values gives the global conservation equation since surface integrals over inner faces mutually cancel out.
To obtain an algebraic equation for a particular cell, the surface and volume integrals need to be approximated using quadrature formulae. The net flux through the cells boundary is the sum of integrals over the control volume faces:

$$\int_S f dS = \sum_k \int_{S_k} f dS \quad (16)$$

Where f is generally representative of convective ($\rho\phi\mathbf{v}\cdot\mathbf{v}$) or the diffusive ($\Gamma\nabla\phi\cdot\mathbf{n}$) flux vector in the direction normal to control volume face.
To calculate the surface integral in (16) it is necessary to know the integrand f everywhere on the surface S_k; this information is not available, as the only known values are the nodal ones, so approximations must be introduced: first of all the integral is approximated in terms of the variable values at one or more locations on the cell face and then the cell-face values are approximated interpolating between the nodal values of the cells which share the face.
If the midpoint rule is applied:

$$F_e = \int_{S_e} f dS = \overline{f_e} S_e \approx f_e S_e \quad (17)$$

Using a linear interpolation (*Central Differential Scheme*), an expression for the value of the quantity f at the cell face e can be obtained using the values of two nearest nodes (f_E and f_P).

$$f_e = f_E \frac{x_e - x_P}{x_E - x_P} + f_P(1 - \frac{x_e - x_P}{x_E - x_P}) \quad (18)$$

In the same way the volume integrals can be expressed through the product of the mean value of the integrand in the volume and the volume of the cell itself.
If the mean value it is approximated as the value of the integrand in the only point in which it is known:

$$Q_P = \int_\Omega q d\Omega = \overline{q}\Delta\Omega \approx q_P \Delta\Omega \quad (19)$$

where q_p stands for the value of q at the cell center: since all the variables are known at the centroid, no interpolation is required.
By summing all the flux approximations and source terms, it is possible to produce an algebraic equation; the number of equations and unknowns are both equal to the number of cells and the matrix of the system is well conditioned [6].
The divergence of the momentum equation for an incompressibile newtonian fluid:

$$\frac{\partial}{\partial x_i}\left(\frac{\partial p}{\partial x_i}\right) = -\frac{\partial}{\partial x_i}\left[\frac{\partial(\rho u_i u_j)}{\partial x_j}\right] \quad (20)$$

gives an equation for the pressure, solved with the momentum equation through the PISO scheme (Pressure Implicit with Splitting of Operators) [7].
As already confirmed by many authors as for instance [8], one of the most difficult problem in the application of computational method in flows around ships, is the implementation of the fully non linear free surface condition. Generally the kinematic and dynamic conditions must be applied on a surface that has to be computed as part of the solution. In the volume of fluid technique, Navier-Stokes equations are solved for a single fluid mixture whose density and viscosity are calculated on the basis of an indicator function α that represents the concentration of air and water for each cell, normalized from 0 to 1 all over the domain. The indicator function can be considered as a time dependent scalar field that is found through the solution of a transport equation:

$$\frac{d\alpha}{dt} = 0 \quad \Rightarrow \frac{\partial\alpha}{\partial t} + \nabla\cdot\alpha U = 0 \quad (21)$$

The efficiency of this method, when coupled with a good grid refinement in the free surface region, was shown by many authors in the past [9].

3 Results

The computational effort required to obtain a solution for the hydrodynamic problem grows with the increase of the number of cells used in the discretization. in order to limit the number of cells used to discretize the computational domain, a body fitted structured mesh will be used for the regions where the viscous effects are supposed to be more important: this includes the wavy free surface and the viscous boundary layer near the body. For regions other than the ones previously defined, a triangular unstructured mesh is adopted. Mesh morfing is used to describe the oscillatory motion of the body.
Vugt's experiments made on a circular section forced to oscillate with a sinusoidal law of motion were used to validate the numerical model [10] (Fig.1).

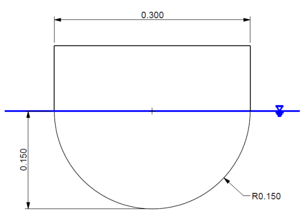

Fig. 1: Circular section geometry

OpenFOAM *interDyMFoam* library was used to the calculation of forces in each time step (Fig.2).

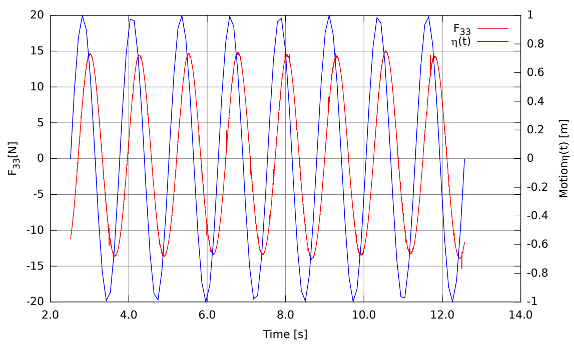

Fig. 2: circular section @ $\omega = 5rad/s$

For each frequency the added mass and the damping coefficients are obtained passing form time to frequency domain through the Fourier transform.
Considering the numerically predicted force signal in the interval $[t_0; t_1]$, with $(\frac{t_1-t_0}{T})$ integer number, the real and the imaginary part of the radiation force for the kth harmonic is given:

$$\Re F_{33}(k\omega_k) = \int_{t_0}^{t_1} F_{33}(t)cos(2\pi k\overline{\omega}t)dt \qquad (22)$$

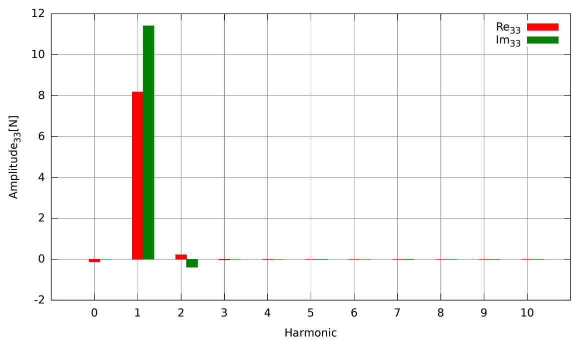

Fig. 3: $\Re F_{33}$ and $\Im F_{33}$ in frequency domain (circular section @ $\omega = 5rad/s$

$$\Im F_{33}(k\omega_k) = \int_{t_0}^{t_1} F_{33}(t)sin(2\pi k\overline{\omega}t)dt \qquad (23)$$

Where $\overline{\omega}$ represent the forced oscillation frequency.
In the linear approach the added mass and damping coefficient are obtained from the 1st harmonic, i.e. the force component characterized by the same frequency of the motion. Considering a sinusoidal motion amplitude of ξ_k, added mass and damping coefficient are expressed as follow:

$$a_{33}(\omega_k) = -\frac{\Re F_{33}(\omega_k)}{\omega_k^2 \xi_3} \quad b_{33}(\omega_k) = \frac{\Im F_{33}(\omega_k)}{\omega_k \xi_k} \qquad (24)$$

The comparison between the hydrodynamic coefficients obtained with the fully viscous method and the experiments shows a good agreement which confirm the good accuracy of the method.

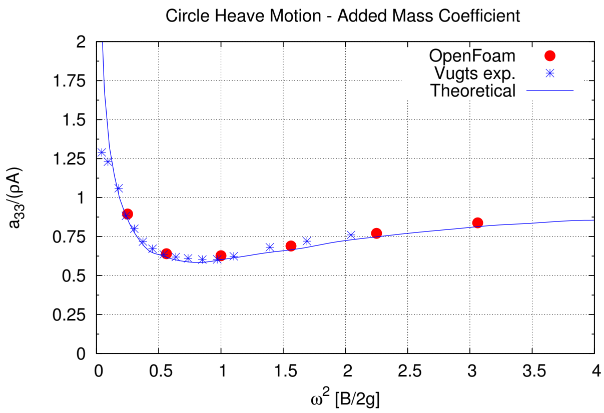

Fig. 4: Added Mass of a circular section - a_{33}

The interest on SWATH ships comes from their markedly reduced motion characteristics in rough seas. A large impulse for the development of SWATH ships happened in late eighties with the design of the T-AGOS 19 for the US Navy, since then different kind of cross-sections have been proposed. The hydro-

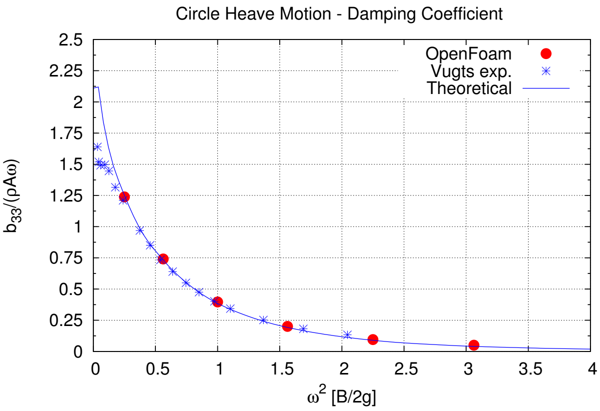

Fig. 5: Damping of a circular section - b_{33}

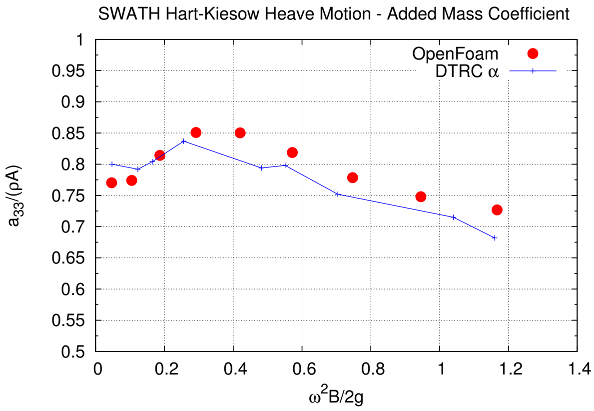

Fig. 7: a_{33}: $\xi = 0.0254m$ @ $T = 0.252m$

dynamic coefficients prediction is validated comparing the fully viscous calculations performed by OpenFOAM with the experiments carried out at DTMB. The first section to be presented is the B-model whose experiments were performed by Hart and Kiesow in 1988 [11]. This section is the typical cross section analyzed in the framework of T-AGOS 19 (Fig.6).

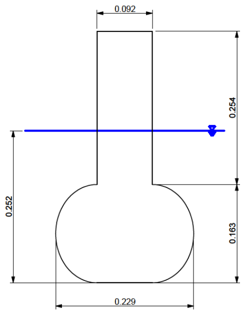

Fig. 6: SWATH B model geometry

The simulations were performed imposing a siusoidal heave motion of 0.0254 m of amplitude, varying the frequency from 2 rad/s to 15 rad/s for a draught of 0.252 m, according to the experiments.
The added mass and damping coefficients are presented in figure 7 and 8, in the following, nondimensionalized form:

$$\alpha = \frac{a_{33}}{\rho A} \qquad \delta = \frac{b_{33}}{\rho A \omega} \tag{25}$$

Numerical results show good agreement with the experiments in almoast all the frequency range.
The "*golf club*" shape has the strut outboard of the underwater body's center-line such that the outside edge of the strut is tangent to the outside edge of the lower hull (Fig.9).

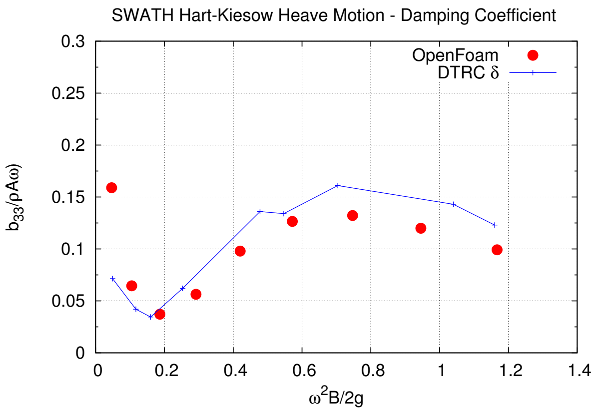

Fig. 8: b_{33}: $\xi = 0.0254m$ @ $T = 0.252m$

This shape has been of interest to ship operators since the offset strut might facilitate over the side operations, especially required for naval and oceanographic purpose.
The oscillation frequency was varied in the range from $3\frac{rad}{s}$ to $15\frac{rad}{s}$. The model was tested at two different drafts and different oscillation amplitudes: the design draft (0.226 m) was tested with an oscillation amplitude of 0.0127m and the deep draft (0.277 m) was tested with an amplitude of 0.0127 m and 0.0381 m. The comparison between the results obtained solving the Navier Stokes equations and the experiments shows that the viscous solver can replicate with high fidelity the resonance both in terms of peak value of the forces and frequency values (Fig.10 - Fig-15). The overestimation of added mass at the lower frequencies could be due to mesh related problems (cell skewness change due to mesh morfing), which may introduce some errors in the solution of the vorticity field. At these high values of oscillation period the experimental measurments can be affected by same errors as well: as the oscillation frequency increases, the forces

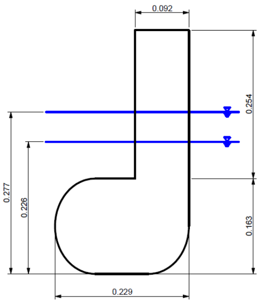

Fig. 9: SWATH G model geometry

measurement can be influenced by motions and vibrations due to high mechanical stresses on the structure used for the experiments.

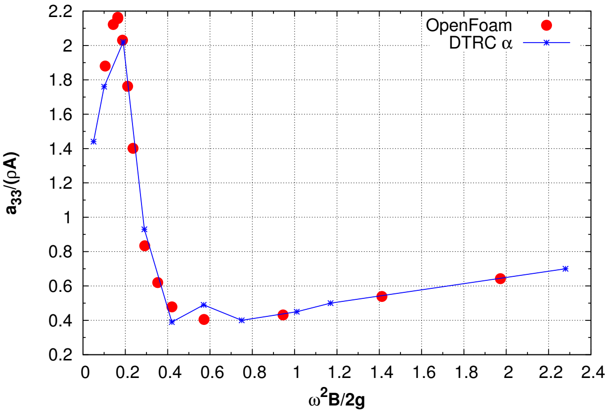

Fig. 10: a_{33}: $\xi = 0.0127m$ @ $T = 0.226m$

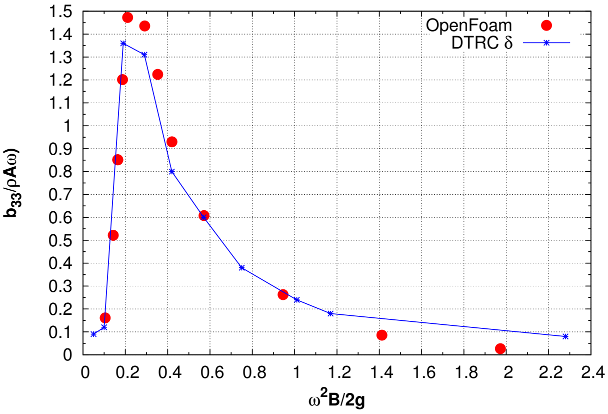

Fig. 11: b_{33}: $\xi = 0.0127m$ @ $T = 0.226m$

From the numerics, as the frequency increase, the wavelenght becomes shorter:

$$\omega^2 = \frac{2\pi g}{\lambda} \tag{26}$$

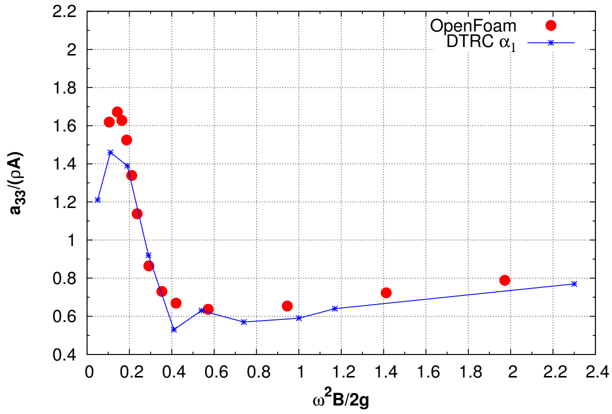

Fig. 12: a_{33}: $\xi = 0.0127m$ @ $T = 0.277m$

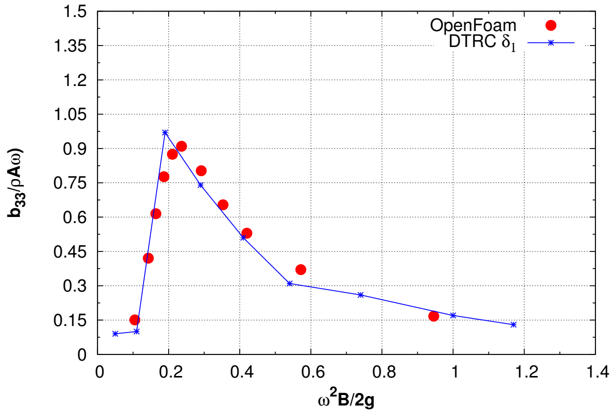

Fig. 13: b_{33}: $\xi = 0.0127m$ @ $T = 0.277m$

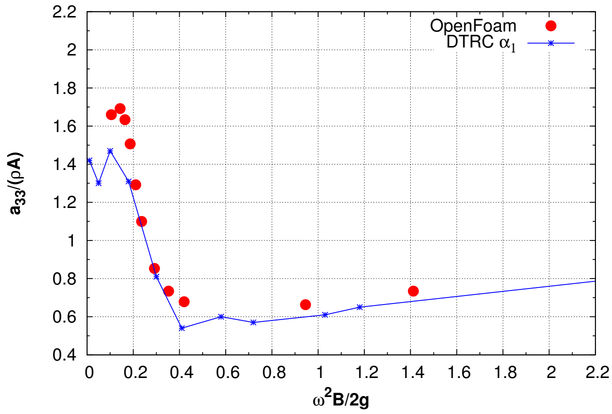

Fig. 14: a_{33}: $\xi = 0.0381m$ @ $T = 0.277m$

and mesh resolution should be properly adapted to capture the right wave profile with the volume of fluid method. Both these experimental and numerical issues can explain the discrepances found in the high frequency range.

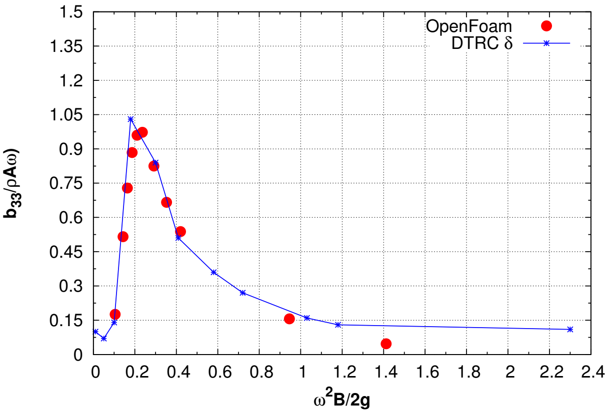

Fig. 15: b_{33}: $\xi = 0.0381m$ @ $T = 0.277m$

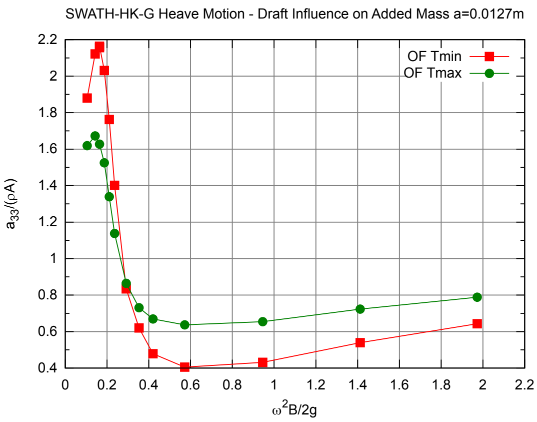

Fig. 16: G added mass: draught influence

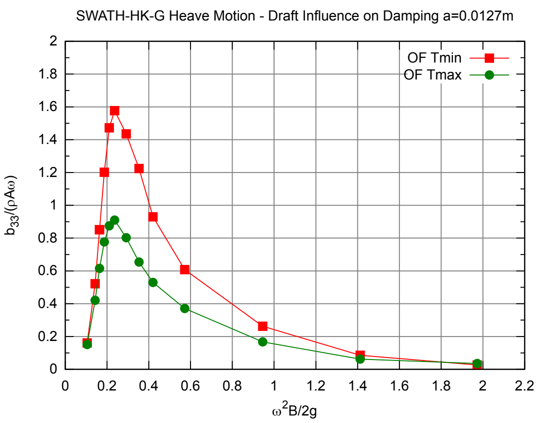

Fig. 17: G damping coefficient: draught influence

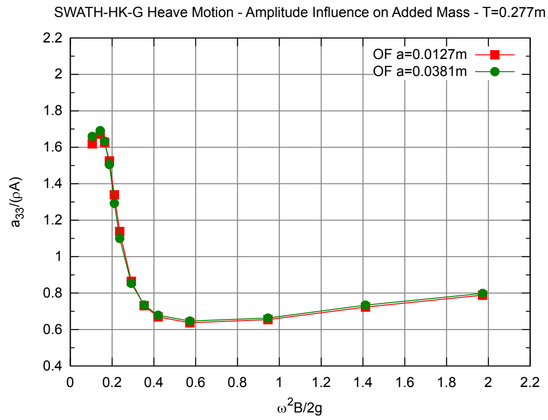

Fig. 18: G added mass: amplitude influence

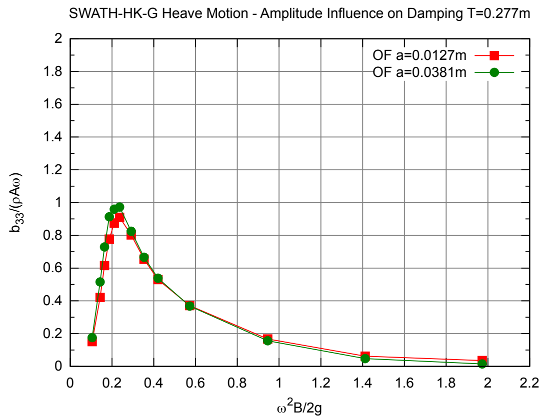

Fig. 19: G damping coefficient: amplitude influence

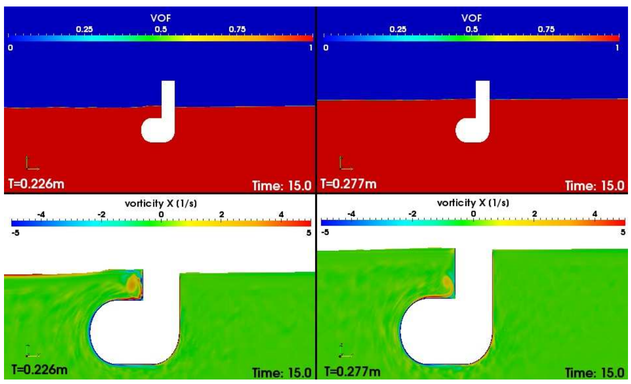

Fig. 20: Draugth Influence - $\omega = 4.5\frac{rad}{s}$

The influence of draught was investigated and the results obtained show that the proximity of the bulb to the free surface produces a radiated wave with higher amplitude in the mid range of frequency; when the sharp edge comes closer to the free surface the eddies

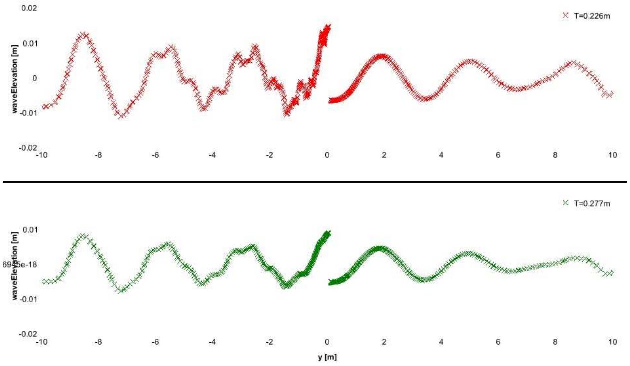

Fig. 21: Draugth Influence - $\omega = 4.5\frac{rad}{s}$

generation increases and with it also viscous damping. Changes in the phase angle of the forces at low frequencies lead the influence of draught to be more evident in the part of forces proportional to the acceleration; at high frequencies, as draught decrease, the fluid accelerated becomes smaller leading to smaller value for added mass coefficient.
The hydrodynamic coefficients are not influenced by changes of the oscillation amplitude. This confirms the linear relationship between forces and amplitude in this relatively low amplitudes range. Further investigations will be done at higher amplitudes.
The sharp edge of the SWATH section gives a contribution to the damping by generating a vorticity field which dissipates energy, although the energy lost in the radiated waves has the greater importance in this case. In fact, the circular section which has no corners but a larger breadth close to the free surface, is able to generate higher radiated waves, and thus it has a higher damping coefficient than the SWATH's one. Since the sides of the SWATH section are perpendicular to the heave motion direction, the acceleration to the fluid is mostly generated by the top and the bottom walls of the bulb. Their large extention and normal orientation to the motion direction (in comparison with the bottom of the circular section) explains the higher values of added mass of the SWATH.

4 Conclusion

A fully viscous unsteady solver of Navier-Stokes equations with non linear free surface is presented and applied to study the radiation forces due to regular oscillating motions of ships like sections in calm water.
The final goal of the research project is to develop a method which can allow seakeeping performance as an optimization criteria during early ship design stage. For this reason the estimation of hydrodynamic coefficients should be more accurate and free from empirically derived viscous corrections (with respect to method based on potential flow) but faster with respect to a fully 3D viscous flow simulations.
The proposed method has been validated for a section representative of a monohull. The results obtained show the excellent accuracy of the method through the comparison of the numerical prediction with the experimental results.
The viscous component of radiation forces becomes more relevant when the waves generated by the oscillation decrease: this is the case of the SWATH section, whose numerical results are compared with experiments with good accuracy.
The excellent results obtained in the validation of this method for the 2D sections presented in this paper represent a step forward towards the final goal of a time computationally optimized method for the viscous non-linear simulation of ship motions in waves.

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