Hydrodynamic traffic flow models and its application to studying traffic control effectiveness

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*Abstract:-*The present research was motivated by the arising difficulties in rational traffic organization the Moscow Government was faced in the recent years. The decrease of the roads handling capacity in the centre of the city (the Boulevard ring, the Gardens ring) caused by the increase of traffic density brought to necessity of re-organizing the traffic flows and constructing of the third ring road around the centre of the city.

The present paper contains the results of investigations on mathematical modeling of essentially unsteady-state traffic flows on ring roads. The developed mathematical model is not limited only by the substance conservation equation and empirical relationships between density and velocity, but contains both the continuity (substance conservation) and momentum equations. It presents a closed form mathematical model and suggests the procedure to develop the model parameters based on technical characteristics of the vehicles. Thus, the available experimental data on the traffic flows observations is used only for validation of the model.

Key-Words:- Traffic, flow, model, continuous, handling capacity, traffic jam, regulation strategy

1 Introduction

First attempts of developing the mathematical models for traffic flows were undertaken by Lighthill and Whitham [1], Richards [2], Greenberg [3], Prigogine [4]. A detailed analysis of the results obtained could be found in the book by Whitham [5]. The authors regarded the traffic flow from the position of continuum mechanics applying empirical relationships between the flow density and velocity of vehicles. Development of computer technique gave birth to continua models application not only for analytical solutions [6] but also in numerical simulations of traffic flows [7-10]. Another approach using discrete approximation for public traffic and passengers interaction is illustrated by [11-12].

The presence of traffic regulation in the roads affect essentially flow dynamics. In particular, on changing the traffic light for red the line of standing cars near the traffic light begins to grow. Cars approaching from behind come to a stop. The rate of its growth is determined by the mean traffic flux and density on the road. It is, actually, the velocity of a compression wave described in [13-18]. The growth of the standing lane is limited by the time of approaching of the rarefaction wave propagating from the traffic light, when it becomes green. The first characteristics of this wave moves at a speed of $\boldsymbol{k}$ - the speed of weak disturbances. Then cars start to move slowly and the fast cars approaching from behind need to slow down to the low velocity of the traffic flow. That gives birth to a traffic jam, which moves counter flow. One border of the jam moves at a speed of compression wave, the other is an expanding rarefaction wave. The size of the jam gradually decreases. Depending on the distance of the traffic regulating section from the tunnel, the standing line or traffic jam propagating counter flow could enter the tunnel. As it was shown, cars slowing down, the increase of density, then acceleration bring to an increase of pollutants emission and decrease of its removal thus increasing air pollution in the tunnel.

The present paper aims at investigating the peculiarities of standing lines growth and traffic jams formation in the sections of roads with regulated traffic. Contrary to the first mathematical models [1-5], the model proposed in [15-18] scoped not only the continuity equation but the differential equation for velocity dynamics which takes into account limits for velocity and acceleration of the vehicles, their technical features and the peculiarities of the drivers' reaction on the road situation. This model has no analogue in the classical hydrodynamics. The modeled devices of traffic control scope street traffic lights and "laying policemen".

2 Autoroute traffic flow model

We consider a uni-directional traffic flow. Crossings and presence of traffic lights should be described by specific boundary conditions. We introduce the Euler's co-ordinate system with the Ox axis directed along the autoroute and time denoted by t. The average flow density $\rho(x,t)$ is defined as the relation of the surface of the road occupied by vehicles to the total surface of the road considered:

$$\rho = \frac{\mathrm{S}_{\mathrm{tr}}}{S} = \frac{hn\mathrm{l}}{hL} = \frac{n\mathrm{l}}{L},$$

where h is the lane width, L is the sample road length, l is an average vehicle's length plus a minimal distance between jammed vehicles, n is the number of vehicles on the road. With this definition, the density is dimensionless changing from zero to unit.

The flow velocity denoted $\mathrm{v} = \mathrm{v}(x,t)$ can vary from zero to $\mathrm{v}_{\max}^{0}$, where $\mathrm{v}_{\max}^{0}$ is maximal permitted road velocity out of the traffic control devices zones. From definitions, it follows that the maximal density $\rho = 1$ relates to the case when vehicles stay bumper to bumper. It is naturally to assume that the traffic jam with $\mathrm{v} = 0$ will take place in this case.

Determining the "mass" distributed on a road sample of the length L as:

$$m = \int_0^L \rho dx,$$

one can develop a "mass conservation law" in the form of continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \mathrm{v})}{\partial x} = 0 \tag{1}$$

Then, we derive the equation for the traffic dynamics. The traffic flow is determined by different factors: drivers reaction on the road situation, drivers activity and vehicles response, technical features of the vehicles. The following assumptions were made in order to develop the model:

- It is the average motion of the traffic described and not the motion of the individual vehicles, that is modelled. Consequently, the model deals with the mean features of the vehicles not accounting for variety in power, inertia, deceleration way length, etc.
- The "natural" reaction of all the drivers is assumed. For example, if a driver sees red lights, or a velocity limitation sign, or a traffic jam ahead, he is expected to decelerate until full stop or until reaching a safe velocity, and not to keep accelerating with further emergency braking.
- It is assumed that the drivers are loyal to the traffic rules. In particular, they accept the velocity limitation regime and try to maintain the safe distance depending on velocity.

The velocity equation is then written as follows:

$$\frac{d\mathrm{v}}{dt} = a;$$

$$a = \max\left\{-a^-; \min\left\{\mathrm{a}^+; \mathrm{a}'\right\}\right\}; \tag{2}$$

$$a' = \sigma_0 a_\rho + (1-\sigma_0)\int_0^Y \omega(y) a_\rho(t, x+y)\, dy + \frac{V(\rho) - \mathrm{v}}{\tau}$$

$$a_\rho = -\frac{k^2}{\rho}\frac{\partial \rho}{\partial x}.$$

Here, a is the acceleration of the traffic flow; a^+ is the maximal positive acceleration, a^- is the emergency braking deceleration; a^+ and a^- are positive parameters which are determined by technical features of the vehicle. The parameter $k > 0$ is the small disturbances propagation velocity ("sound velocity"), as it was shown in [16-17]. The parameter τ is the delay time which depends on the finite time of a driver's reaction on the road situation and the vehicle's response. This parameter is responsible for the drivers tendency to keep the vehicles velocity as close as possible to the safe velocity depending on the traffic density $V(\rho)$ [15-17]:

$$V(\rho)=\begin{cases}-k\ln\rho, & \mathrm{v}<\mathrm{v}_{\max}^0,\\ \mathrm{v}_{\max}^0, & \mathrm{v}\ge\mathrm{v}_{\max}^0\ .\end{cases}$$

The velocity $V(\rho)$ is determined from the dependence of the traffic velocity on density in the "plane wave" when the traffic is starting from the initial conditions $\rho_0=1$ and $\mathrm{v}=0$, with account of velocity limitation from above $\left(\mathrm{v}\le\mathrm{v}_{\max}^0\right)$. The value of τ could be different for the cases of acceleration or deceleration to the safe velocity $V(\rho)$:

$$\tau=\begin{cases}\tau^+, & \mathrm{V}(\rho)<\mathrm{v}\\ \tau^-, & \mathrm{V}(\rho)\ge\mathrm{v}\end{cases}$$

The remaining parameters in (2) are the following. $Y=\min\{Y_0,L-x\}$ is the characteristic visibility ahead depending on the weather, $\omega(y)$ is the "weight" (in the mathematical sense) of the traffic ahead of the vehicle, which influences on the drivers decision of changing the velocity and which could be determined as follows, for example:

$$\omega(y)=\frac{\omega_0(y)}{\int\limits_0^Y\omega_0(y)\,dy},\quad \omega_0=\begin{cases}1, & 0\le y\le Y_0\\ 0, & y<0,\ y>Y_0\end{cases},$$

σ_0 is a dimensionless parameter $\left(0\le\sigma_0\le1\right)$ characterising the "weight" of local situation compared to the situation ahead.

Thus, in the expression for the traffic acceleration (right hand side of (2)) the first term corresponds to the local situation influence, the second – to the influence of the traffic situation ahead at the distance less or equal to Y, and the third term is responsible to the tendency to maintain the velocity close to the maximal safe velocity $V(\rho)$.

The estimate of the small disturbances propagation velocity k was made in [15-17] using the following considerations. Let the traffic flow start from a stationary state $\left(\mathrm{v}=0,\ \rho=1\right)$ and accelerate until it reaches the velocity $\mathrm{v}_{\max}^0$ at the maximal density ρ_* which still allows the safe driving. We will denote the safe density as the maximal density at which the distance between vehicles is equal or more than the deceleration way length $X(\mathrm{v})$. Then, we get the following expressions:

$$\rho_*=\left(1+X\left(\mathrm{v}_{\max}^0\right)/1\right)^{-1},$$
$$k=\mathrm{v}_{\max}^0\ln^{-1}\left(1+X\left(\mathrm{v}_{\max}^0\right)/1\right).$$

At $\mathrm{v}_{\max}^0=80$ kph the deceleration way of an automobile of Lada brand is 45 m, which gives $k=35$ kph when the mean length of a vehicle is 5 m (including the minimal distance between motionless vehicles). For this velocity v_{max}^0, the maximal safe density is $\rho_*=0.1$. The maximal accelerations of an automobile of this type are $a^+=1.63\ \mathrm{m/s^2}$ and $a^-=5.5\ \mathrm{m/s^2}$.

The "sound velocity" k estimated as above fits the experimental data [2, 3, 5] rather good.

Summarising the above, one comes to the model consisting of two PDE's (1) and (2) describing the automobile traffic dynamics on a single-lane road. The divergent form of those equations is:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho\mathrm{v})}{\partial x}=0,$$
$$\frac{\partial(\rho\mathrm{v})}{\partial t}+\frac{\partial\left(\rho\mathrm{v}^2\right)}{\partial x}=\rho a,$$

and the acceleration a is determined by three last expressions in (2).

We should consider the following boundary conditions on the endings of the motorway section $0\le x\le L$. Two variants of the conditions at the inflow $x=0$ are possible:

1) No traffic jam: the flow density is given, and the velocity is equal to the maximal safe for this density:

$$\rho(0,t)=\rho_0;\quad \mathrm{v}(0,t)=\mathrm{v}(\rho_0).$$

2) Traffic jam or jam at the inflow ending: zero density gradient is assumed, and the velocity is equal to the maximal safe velocity at the density at the inflow:

$$\left.\frac{\partial\rho}{\partial x}\right|_{x=0}=0;\quad \mathrm{v}(0,t)=V(\rho).$$

Presence or absence of the jam at the inflow boundary of the domain is determined each time after a time step calculation using the following criterion. The jam takes place, if two conditions are true at $x=0$:

$$\left.\frac{\partial\rho}{\partial x}\right|_{x=0}>0\ \text{ and }\ \rho>\rho_0.$$

The "free outflow" condition is set on the outflow ending of the domain at $x=L$:

$$\frac{\partial\rho}{\partial x}=0;\quad \frac{\partial\mathrm{v}}{\partial x}=0.$$

As for the initial conditions, we assume that the domain portion at $0 \le x \le x_0$ is occupied with vehicles with density ρ_0 moving at the velocity $V(\rho_0)$, and the rest part $x_0 < x \le L$ is free from the vehicles $(\rho = 0,\ \mathrm{v} = 0)$.

3 Modelling traffic control devices

We consider two most popular devices used to control automobile traffic on the urban roads: street traffic lights and "laying policemen". The last is a set of low jumps across the road surface which are placed at some distance from each other and force the drivers to decrease velocity essentially in the zone of their placement. The "laying policemen" are often used to provide friendly conditions for pedestrians crossing roads in the vicinity of schools, institutions, marketplaces, etc.

3.1 Street traffic lights.

The main parameters of this device are the green, yellow and red signals duration, denoted t_g, t_y, t_r correspondingly. The following algorithm is proposed to model the street lights work.

1. In the moment when the signal changes to yellow, the following distance is calculated:

$$x_r = \frac{\left(\mathrm{v}_{\max}^0\right)^2}{2a_r},$$

where a_r is the usual deceleration less than the deceleration of the emergency braking a^-. The vehicles which are closer to the street lights at this moment than this distance, cannot stop and therefore they pass the street lights, this yields the traffic rules.

2. When the yellow signal is on, the maximal permitted velocity at the point $x_1(t)$ is set to be the following:

$$\mathrm{v}_{\max}^e(x_1) = \mathrm{v}_{\max}^0 \cdot \frac{t_{ess}}{t_e},$$

where t_{ess} is the time left for the yellow signal to be on; $x_1(t)$ moves towards the street lights like:

$$x_1 = L_1 - x_r \cdot \frac{t_{ess}}{t_y},$$

and L_1 is the co-ordinate of the street lights device. As the result, by the moment of red lights all the traffic flow is blocked at the street lights.

3. In the moment when the street lights change to green, the maximal permitted velocity change back to $\mathrm{v}_{\max}^0$ like in the other portions of the domain.

3.2 "Laying policemen"

This system of velocity limitation is modelled by maximal permitted velocity the position of the device which is much lower than one for the other portions of the road $\mathrm{v}_{\max}^0$. The present work deals with a system of two "policemen" laying at a given distance between them; this case is mostly used in practice. The position of the first device is denoted L_1. Then, the maximal permitted velocity along the domain $0 \le x \le L$ is given as follows:

$$\mathrm{v}_{\max} = \begin{cases} \mathrm{v}_{\mathrm{p}}, & x \in \{L_1, L_1 + d\}, \\ \mathrm{v}_{\max}^0, & x \in [0, L] / \{L_1, L_1 + d\}, \end{cases}$$

where $\mathrm{v}_{\mathrm{p}} < \mathrm{v}_{\max}^0$, and the parameter v_p (maximal velocity of passage) is one of the governing parameters.

4 Results of numerical investigations of traffic regulation strategies

The numerical calculations of the problems were processed using the TVD method with second order of approximations (Van Leer's flux limiter). The mesh had $N = 201$ grid nodes.

The following parameters values were used in simulations: $L = 1000$ m is the length of the domain; $x_0 = 100$ m is the length of the domain portion with vehicles at the first instance; $L_1 = 500$ m is the position of the traffic control devices (either street lights or the first "laying policemen"); $d = 50$ m is the distance between the "laying policemen"; $\rho_0 = 0.1 \div 0.5$ is the inflow density, $\mathrm{v}_{\max}^0 = 25$ m/s is the maximal permitted velocity on the main portion of the road; $\mathrm{v}_{\mathrm{p}} = 3$ m/s is the maximal velocity of the "laying policemen" passage; $k = 7.9$ m/s is the velocity of small disturbances propagation in the automobile traffic flow; $a^+ = 1.5$ m/s^2 is the maximal traffic flow acceleration; $a^- = 5$ m/s^2 is the maximal (emergency) deceleration of the flow; $a_r = 1.5$ m/s^2 is the standard deceleration; $Y_0 = 100$ m is the characteristic visibility ahead; $\sigma_0 = 0.7$ is the local situation "weight";

$\tau^{+} = 3.3$ s , $\tau^{-} = \infty$ are the delay times responsible to maintaining the safe velocity; $t_g = 40 \div 300$ s , $t_y = 5$ s , $t_r = 30$ s are durations of the street lights cycle.

Thus, the varying parameters were the inflow density ρ_0 (together with the inflow velocity) and the green lights duration t_g.

The results are presented at the figs. 1 – 4 and in the Table 1.

The figs. 1 and 2 illustrate traffic flow density distribution for the different time moments which are shown on the figs, for the case of street lights traffic controlling. The green lights duration was taken $t_g = 50$ s, and the inflow density was $\rho_0 = 0.18$ (fig. 1) and $\rho_0 = 0.3$ (fig. 2). The time moments on those figures correspond to the following street lights cycles: fig. 1a – first cycle, end of the green period; fig. 1b – first cycle, yellow period; fig. 1c – first cycle, end of the red period; fig. 1d – second cycle, green period; fig. 1e – second cycle, green period; fig. 1f – second cycle, end of the red period. Fig. 2a corresponds to the first cycle, yellow period; fig. 2b – first cycle, end of the red period; fig. 2c – second cycle, end of the green period; fig. 2d – fifth cycle, end of the green period; fig. 2e – fifth cycle, end of the red period; fig. 2f – sixth cycle, end of the green period. It can be seen from the figures that at $\rho_0 = 0.18$ traffic jam does not occur (fig. 1), and at $\rho_0 = 0.3$ the traffic jam arises and propagates counter-flow decreasing vehicles velocity essentially.

The dependence of critical inflow density ρ_0^* under which the traffic jam does not occur, on the duration of the green period of the street lights t_g was investigated, and the results are placed in the Table 1. All the other parameters were fixed as shown above. The dependence of ρ_0^* on t_g can be described by the following formula:

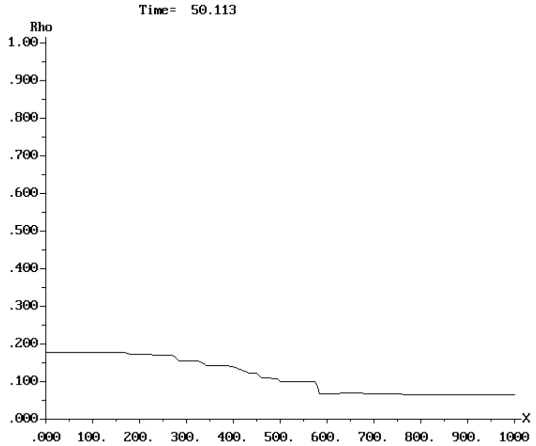
$$\rho_0^* = a \ln\left(b t_g\right) \tag{3}$$

where a, b are parameters depending on many factors including the red lights duration t_r. For the parameters used, we have got $a = 0{,}054,\ \mathrm{b} = 0{,}87$. The table 1 contains also the difference of the critical inflow density obtained by traffic flow simulation and the value obtained using the formula (4.3). The mean square deviate is equal to 0.011629, and the maximal difference is 0,021566.

The figs. 3, 4 illustrate the profiles of $\rho(x)$ for different time moments in the case of "laying policemen". The fig. 3 correspond to the inflow density $\rho_0 = 0.1$, and the fig. 4 – to $\rho_0 = 0.3$.

The case $\rho_0 = 0.1$ corresponds to the free traffic passage via the zone regulated by two "laying policemen", and the case $\rho_0 = 0.3$ corresponds to origination of the traffic jam and its propagation counter-flow. It can be seen from both figs. 3 and 4 that two locations with increased density take place. In case $\rho_0 < 0.2$ this does not prevent the traffic flow from the free passage. If the inflow density is higher than 0.2, then the traffic jam originates in front of the first "laying policeman"; it propagates towards the inflow portion and finally the density at the inflow increases thus decreasing the inflow velocity, and the traffic turns to be very slow (fig. 4).

Green lights durati on, s	**Critical density determined numerically**	**Critical density calculated using formula (4.3)**	**Table 4.1.** **Divergen ce**
40	0,18	0,191679	0,011679
50	0,19	0,203729	0,013729
60	0,21	0,213574	0,003574
70	0,22	0,221899	0,001899
80	0,23	0,229109	-0,00089
90	0,23	0,23547	0,00547
100	0,23	0,241159	0,011159
110	0,25	0,246306	-0,00369
120	0,25	0,251004	0,001004
130	0,26	0,255327	-0,00467
140	0,26	0,259329	-0,00067
150	0,27	0,263054	-0,00695
160	0,27	0,266539	-0,00346
170	0,28	0,269813	-0,01019
180	0,28	0,2729	-0,0071
190	0,29	0,275819	-0,01418
200	0,29	0,278589	-0,01141
210	0,3	0,281224	-0,01878
220	0,3	0,283736	-0,01626
230	0,3	0,286136	-0,01386
240	0,31	0,288434	-0,02157
250	0,31	0,290639	-0,01936
260	0,31	0,292757	0,01724
270	0,31	0,294795	-0,01521
280	0,31	0,296758	-0,01324
290	0,31	0,298653	-0,01135
300	0,31	0,300484	-0,00952



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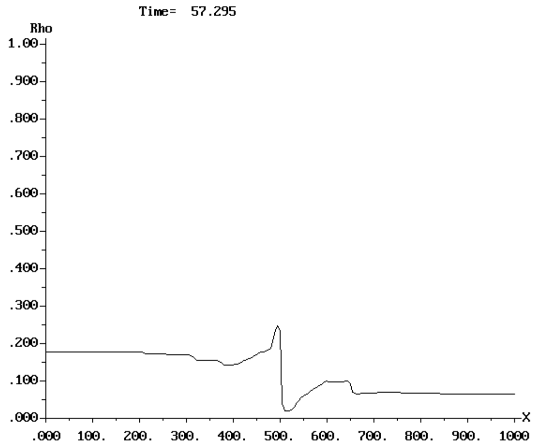

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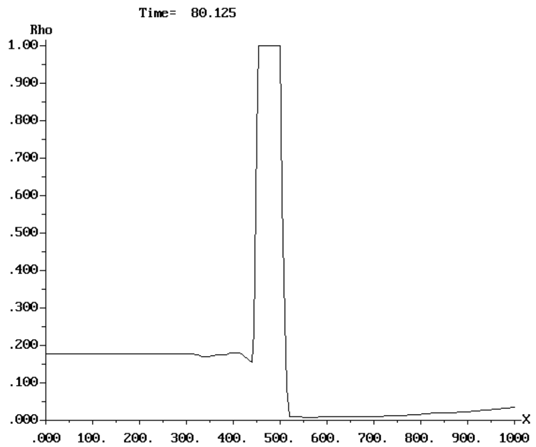

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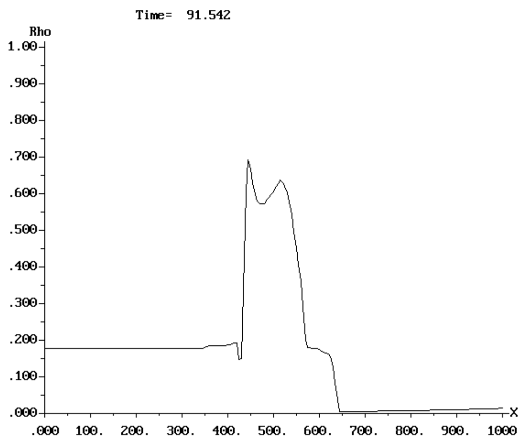

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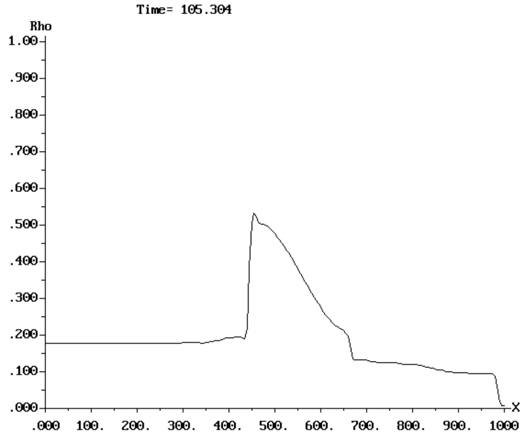

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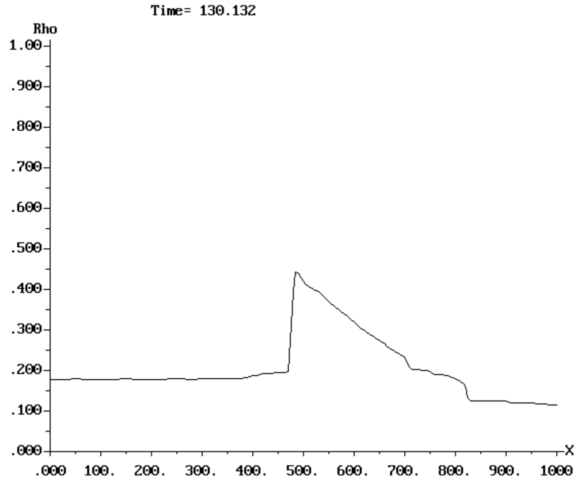
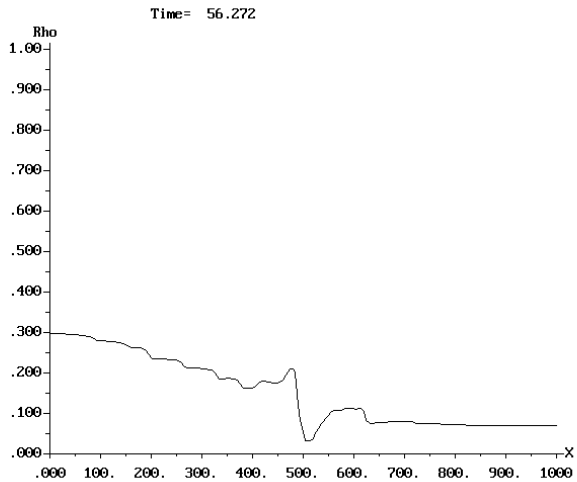


f

Fig. 1. Traffic flow density in the zone traffic regulation by "lights". Time in seconds correspond to second cycle, initial traffic density $\rho_0 = 0,18$: a - the end of "green period", b - "yellow period", c - "yellow period", d - "green period", e – end of "green period", f - end of the "red period",



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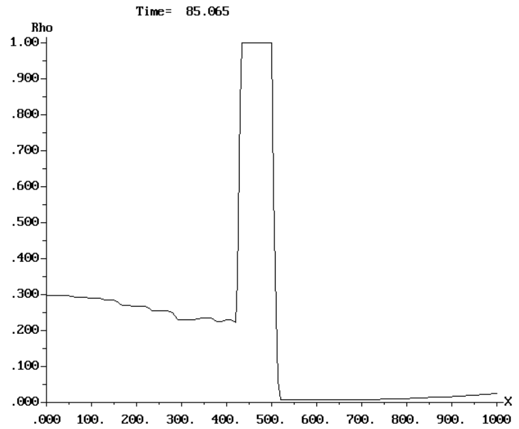

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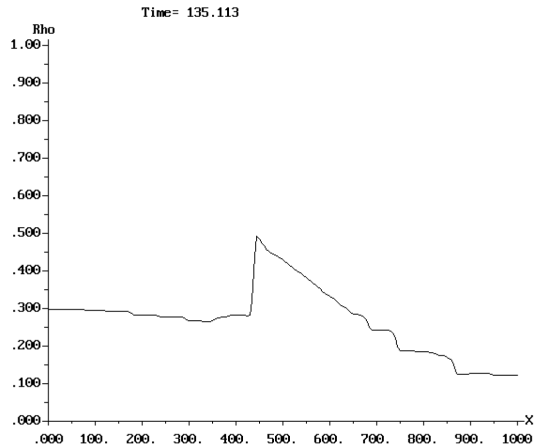

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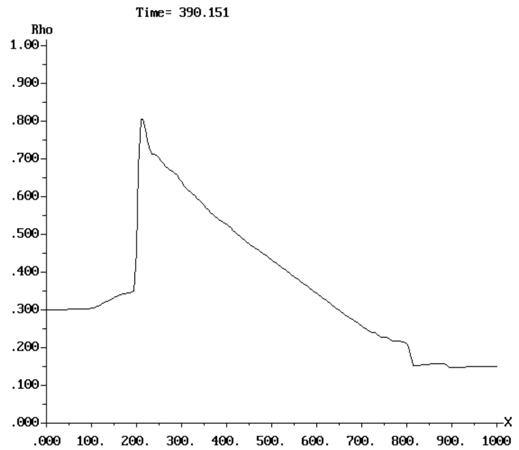

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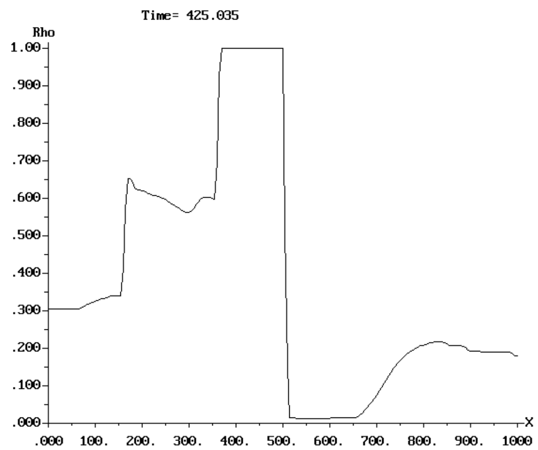

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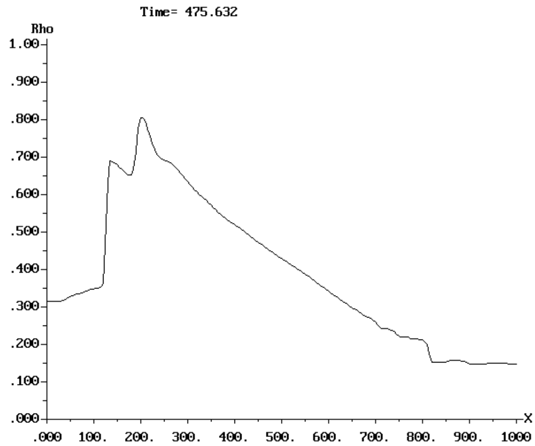

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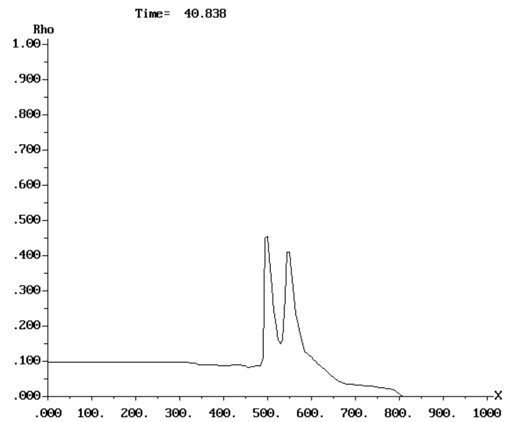
Fig. 2. Traffic flow density in the zone traffic regulation by “lights”. Time in seconds correspond to sixth cycle, initial traffic density $\rho_0 = 0.3$, a - “yellow period”, b - end of the “red period”, c - green period, d - end of the green period, e - end of the red period, f - end of the “green period ”.



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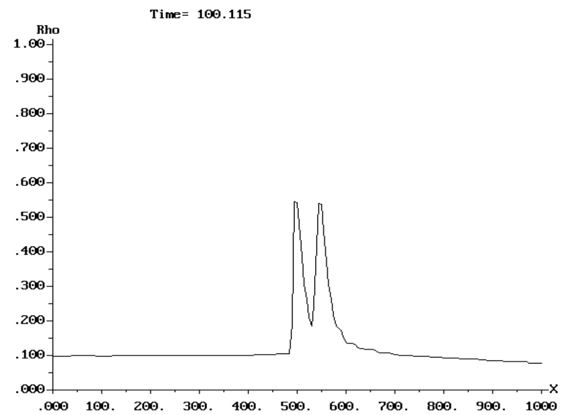

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Fig. 3. Traffic flow density in the zone traffic regulation by a “laying policemen”. Time in seconds is provided on top of the figure; inflow traffic density $\rho_0 = 0.1$.

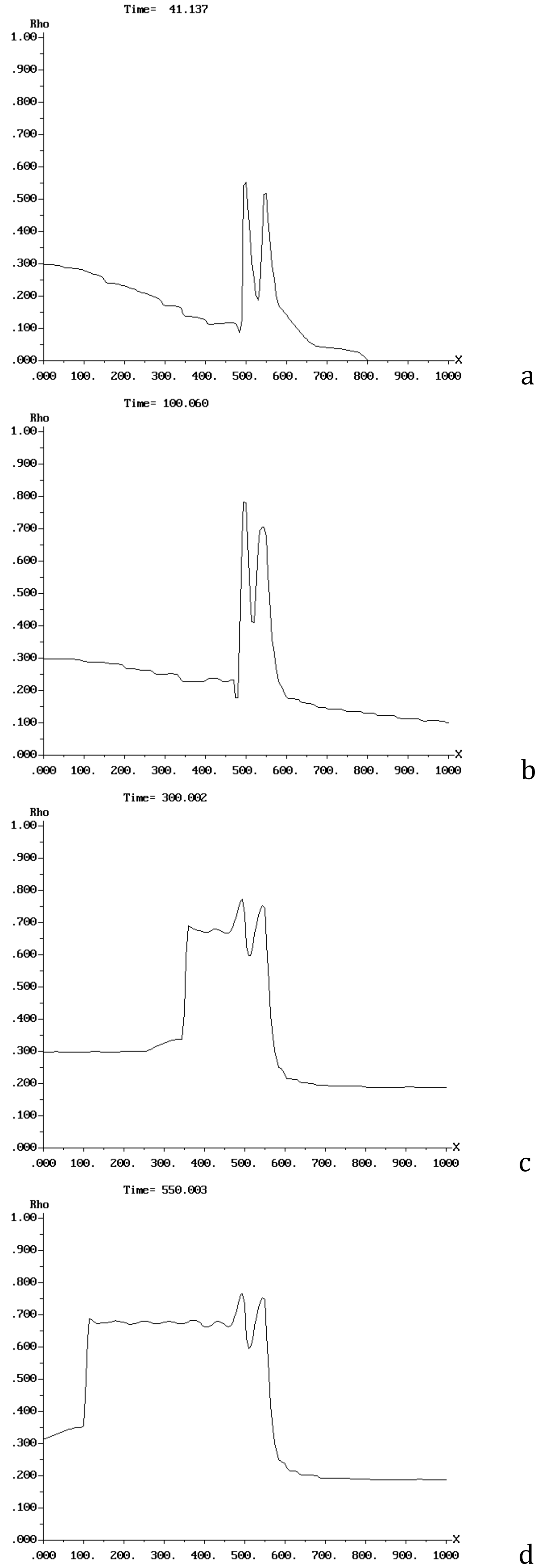

Fig. 4. Traffic flow density in the zone traffic regulation by a "laying policemen". Time in seconds is provided on top of the figure; inflow traffic density $\rho_0 = 0.3$.

The results show (Fig. 4) that for relatively high inflow density artificial hump ("laying policemen") device cannot provide the necessary handling capacity and brings to formation of a growing traffic jam, while for the lower inflow density (Fig. 3) the flow field comes to a steady-state regime. Thus the device successfully serves its function of reducing the traffic speed at a definite place of the road.

5. Conclusions

The results of investigations testify that in case of low inflow density of the automobile traffic the "laying policemen" devices allow to maintain traffic control without disturbing the free traffic flow. However, the increase of the inflow density leads to origination of the traffic jam and its propagation counter-flow. The regulation by the traffic lights allows improving the road handling capacity essentially when the duration of the different parts of the traffic lights cycle is properly chosen.

The model developed accounts for the main feature of the automobile traffic flows: their self-organisation. It allows describeing both qualitatively and quantitatively, the conditions for maximal road capacity, the origination and propagation of traffic jams, the influence of different traffic control devices.

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References:

[1] *Lighthill M.J., Whitham J.* On kinetic waves II. A theory of traffic flow on long crowded roads. Proc. of the Royal Society. Ser. A. No. 1178. Vol. 229. London, 1955. 317-345.

[2] *Richards P.L.* Shock waves on the highway. Operations Research. 1956. Vol. 4. 42-51.

[3] *Greenberg H.* An analysis of traffic flow. Operations Research. 1959. Vol. 7. 79-85.

[4] *Prigogine I., Résibois P.* On a generalized Boltzman-like approach for traffic flow. Bull. Cl. Sci., Acad. Roy. Belg., 1962, vol. 48, No. 9, 805-814.

[5] *Whitham J.* Linear and non-linear waves. John Wiley & Sons, New York – London, 1974.

[6] *Soldatov G.P*. On formation of a strong shock wave in a two-directional traffic flow. Applied Math. & Mech., 1970, vol. 34, No. 1, 135-137.

[7] *Bando M., K.Hasebe, A.Nakayama, A.Shibata, Y.Sugiyama*. Dynamical Model of Traffic Congestion and Numerical Simulation // Physical Review. 1995. Ser. E. Vol. 51. P. 1035.

[8] *Hill S.A*. Numerical analysis of a time-headway bus route model // Physica A. 2003. Vol. 328, No. 1-2. P.261-273.

[9] *Karim Md.M., Ohno T*. Air quality planning and empirical model to evaluate SPM concentrations // Journal of Environmental Engineering. 2000. Vol. 64. P. 1116-1124.

[10] *Sukhinova A.B., Trapeznikova M.A., Chetverushkin B.N., Churbanova N.G*. Two dimensional macroscopic model for traffic flows. Mathematical modeling. 2009 vol. 21, #2, pp.118-126.

[11] *Nagatani T*. Bunching transition in a time-headway model of a bus route // Phys. Rev. E. 2001. Vol. 296, № 1-2. P.320-330.

[12] *Regirer, S.A., Smirnov N.N., Chenchik, A.E*. Mathematical model of moving collectives interaction: Public transport and passengers. Automation and Remote Control. 2007, vol. 68, No 7, pp. 1225-1238.

[13] *Kiselev A.B., Kokoreva A.V., Nikitin V.F., Smirnov N.N* Computational modelling of traffic flows // European Conf. on Computational Fluid Dynamics (Egmond aan Zee, The Netherlands, 5-8 Sept. 2006). Proc. on CD-Rom. – 10 p.

[14] *Smirnov N.N., Kiselev A.B., Nikitin V.F., Yumashev M.V*. Mathematical modeling of traffic flows. Moscow University Mechanics Bulletin, 2000, № 4, pp. 39–44.

[15] *Kiselev A.B., Kokoreva A.V., Nikitin V.F., Smirnov N.N*. Mathematical modeling of traffic flow on traffic-controlled roads. Journal of Applied Mathematics and Mechanics 2004, Vol.68, No 5, pp. 1035–1042.

[16] *Kiselev A.B., Nikitin V.F., Smirnov N.N. Yumashev M.V*. Irregular traffic flow on a ring road. Journal of Applied Mathematics and Mechanics 2000, Vol.64, No 4, pp. 627-634.

[17] *Smirnov N.N. Kiselev A.B., Nikitin V.F., Kokoreva A.V.,* Mathematical modeling of traffic flows using continua approach. Two lanes roads: modeling of T-shape crossing and the effect of lane changing on handling capacity. Proceedings of MIPT, 2010, vol.2, N.4, pp.141-151.