Analyzing the Performance of a GPS Device

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Abstract: The insucess of proceeded baselines of considerable length, when the ionospheric and tropospheric delays are not properly modeled, is a serious problem. In order to minimize such problem, some models have been proposed to minimize the biases. For example, the combination of L_1 and L_2 carrier-phase can vanish 98% of the first-order ionospheric biases. Generally, the LGO device, the equipment under evaluation, uses this solution to the majority of the baselines considered in our work. But it is not enough, the tropospheric bias still needs to be minimized or vanished. The objective of this study, is to verify and quantify the improvements, by the evaluation of the rate of successful processed baselines when an improved tropospheric bias mitigation strategy is used in opposition to a tropospheric bias mitigation approach. LGO equipment uses as a priori tropospheric model the simplified Hopfield model. The main aim of the investigations presented in this work was to determine the increase, or not, in the rate of baselines successfully produced by adopting an advanced tropospheric bias mitigation strategy as opposed to a simpler one. In the first case, LGO uses an improved strategy with a zenith tropospheric scale factor per station. We built some models by general least squares (GLM) to evaluate the performance of the equipment. We are aware that 1D and 2D present different behaviors, we analyzed both cases individually with each strategy. In this article, we present such analysis for 1D case.

Key–Words: Baselines, Bias, General Least Squares, Performance, GPS equipment

1 Introduction

The objective of this research is to evaluate GPS static relative positioning [16], regarding accuracy, as the equivalent of a network real time kinematic (RTK) and to address the practicality of using either a continuously operating reference station (CORS) or a passive control point for providing accurate positioning control. The precision of an observed 3D relative position between two GNSS antennas, and how it depends on the distance between these antennas and on the duration of the observing session, was studied.

This goal was achieved by comparing the outputs from the Leica Geo Office v5 (LGO) software and the Ordnance Survey (OS) active stations coordinates, assumed as true [10, 11]. The methodology followed was using observation files from OS active stations to simulate different scenarios for the baseline length, in order to answer the question of how long should be the observing session for the LGO to process those baselines within a pre-establish threshold of accuracy.

A brief introduction of GPS, the navigation system used in the present work, can be found in [13, 4, 2] where some details about observation modeling of systematic biases and errors affecting GPS measurements are described. Other significant references can be found easily. For example, [3, 7, 8] are significant references where a description and discussion about the navigation system are given.

This work investigates the performance of commercial software LGO when processing baselines in static mode. This article is a continuation of a preliminary approach presented in[14, 13]. The parameter to be tested is the time of observation needed to achieve a given accuracy (1D and 2D) for a set of ranges of baseline lengths. A statistical approach by general liner models (GLM) is applied. Four different scenarios were created.

Summarizing, the present work is comprised of introduction and final remarks sections, a section with the statistical methodology and two sections containing the description of data and results details respectively.

2 GLM method

In the classical linear model, a vector X with p explanatory variables $X = (X_1, X_2, \ldots, X_p)$ can explain the variability of the variable of interest Y (response variable), where $Y = Z\beta + \epsilon$. Z is a specification matrix with size $n \times p$ (usually $Z = X$, considering an unitary vector in first column), β a parameter vector and ϵ a vector of random errors ϵ_i, independent and identical distributed to a reduced Gaussian.

The data are in the form (y_i, x_i), $i = 1, \ldots, n$, as result of observation of (Y, X) n times. The response variable Y has expected value $E[Y|Z] = \mu$.

GLM is an extension of classical model where the response variable, following an exponential family distribution [15], do not need to be Gaussian. Another extension from the classical model is that the function which relates the expected value and the explanatory variables can be any differentiable function. Y_i has expected value $E[Y_i|x_i] = \mu_i = b'(\theta_i)$, $i = 1, \ldots, n$.

It is also defined a differentiable and monotone link function g which relates the random component with the systematic component of response variable. The expected value μ_i is related with the linear predictor $\eta_i = z_i^T \beta_i$ using the relation

$$\mu_i = h(\eta_i) = h(z_i^T \beta_i), \qquad \eta_i = g(\mu_i) \qquad (1)$$

where h is a differentiable function; $g = h^{-1}$ is the link function; β is a vector of parameter with size p (the same size of the number of explanatory variables); Z is a specification vector with size p.

There are different link functions in GLM. When the random component of response variable has a Poisson distribution, the link function is logarithmic and the model is log-linear. In particular, when the linear predictor $\eta_i = z_i^T \beta_i$ coincides withe the canonical parameter θ_i, $\theta_i = \eta_i$, which implies $\theta_i = z_i^T \beta_i$, the link function is denominated as canonical link function. Sometimes, the link function is unknown being estimated simultaneously with the linear component of the semi-parametric model for electricity spot prices. A detailed description of GLM methodology can be found in several references such as [9, 15].

3 Data

OS active stations were used to investigate the relation between time of observation and length of the baseline. A total of 105 baselines were processed using LGO, separated into six range groups ($R_i,\ i = 1, \ldots, 6$) according with their lengths in kilometers:

- $R_1 = [000 - 100]$ →(5 baselines)
- $R_2 = [100 - 200]$ →(14 baselines)
- $R_3 = [200 - 300]$ →(27 baselines)
- $R_4 = [300 - 400]$ →(29 baselines)
- $R_5 = [400 - 500]$ →(14 baselines)
- $R_6 = [500 - 900]$ → (16 baselines)

All the stations are permanent stations of clear sky visibility and with low multipath conditions. The quality of the data is therefore expectantly high. Day $13/06/2013$ of receiver independent exchange (RINEX) data of GPS week 1744 was downloaded from the data archive of the active GPS network of Ordnance Survey (OS Net) for each of the 106 stations [11]. These RINEX data include phase measurement of the carrier waves L_1 and L_2, P_1, P_2 and C/A pseudo-range code at a 30 seconds interval.

For this experiment, 24 hours of dual-frequency GPS carrier phase observations for each of 105 baselines formed by ABEP, chosen as reference station, and all the other active stations, designated as rover, from OS Network were used. These 105 baselines range in length from $61\ km$ to $898\ Km$ and correspond to all active stations considered 'healthy' on $13/06/2013$. The data for each baseline comprised the same 24-hour session that was further subdivided into periods of time of 1, 2, 3, 4, 6, 8, 12 and 24 hour as follow, where the two first digits represent the beginning of the observation period and the last two the end:

- 1 hour periods: [0001], [0607], [1213], [1819];
- 2 hour periods: [0002], [0608], [1214], [1820];
- 3 hour periods: [0003], [0609], [1215], [1821];
- 4 hour periods: [0004], [0408], [0812], [1216], [1620], [2024];
- 6 hour periods: [0006], [0612], [1218], [1824];
- 8 hour periods: [0008], [0816], [1624];
- 12 hour periods: [0012], [1224];
- 24 hour period: [0024].

The division of time in this way was done in order to evaluate the performance of the software for different lengths of observation time.

The criteria followed to select the reference station were primarily based on location. Thus ABEP, on the west coast of England, was chosen, because of its high altitude and location, providing a well distributed range of radial vectors to all the other active stations, either in latitude and longitude. Its 3D positional coordinates were fixed to the official values adopted by OS.

In order to evaluate at what range of baseline lengths the use of precise ephemerides become worthwhile, both results using broadcast and precise ephemerides[1] are presented as well. The corresponding SP3 files were downloaded from the data archive of [6]. These include precise ephemerides at a sampling interval of 15 minutes and the high-rate precise satellite clocks with a sampling of 30 seconds.

Hence, the four different scenarios can be compared as follows:

- Direct comparison of the results obtained using the broadcast ephemerides and the precise ephemerides (BH versus PH and BC versus PC);
- Direct comparison of the results obtained using Hopfield model and computing the troposphere (BH versus BC and PH versus PC).

At starting points 1D, 2D and 3D accuracy criteria were established for each baseline, as only successful processed baselines are of interest for this research. The chosen values were set to 1D and 2D accuracies to be better than $3\ cm$ and 3D better than $4.5\ cm$. These are realistic values, as the OS active stations have 1D accuracy of about $2\ cm$ in magnitude and close to $1\ cm$ in 2D. Therefore, assuming the $3\ cm$ as 1D and 2D threshold seems to be reasonable due the fact that this tolerance allows for the absorption of errors inherent to the coordinates of the stations. Despite how perfectly the baseline was calculated an error of up to $4\ cm$ in height and $2\ cm$ in plan could arise due to the uncertainty associated with the coordinates.

The published coordinates of each of these stations (in Cartesian format on the header of the corresponding RINEX file) are assumed as true and used to compute the errors (1D, 2D and 3D) in the solutions processed by LGO.

[1]In astronomy and celestial navigation, an ephemeris (plural: ephemerides; from Latin ephemeris, ”diary”, from Greek: ephemeris, ”diary, journal”) gives the positions of naturally occurring astronomical objects as well as artificial satellites in the sky at a given time or times. Historically, positions were given as printed tables of values, given at regular intervals of date and time. Modern ephemerides are often computed electronically from mathematical models of the motion of astronomical objects and the Earth. Even though the calculation of these tables was one of the first applications of mechanical computers, printed ephemerides are still produced, as they are useful when computational devices are not available. (Cited from https://en.wikipedia.org/wiki/Ephemeris.)

One of the major challenges in processing high-accurate long baselines is the presence of un-modelled ionospheric and tropospheric delays. There are effective mitigation strategies for ionospheric biases, such as the ionosphere-free linear combination of L1 and L2 carrier-phase, which can remove about 98

4 Numerical Results

In [13] was studied the relation for single baselines between lengths ranges and between the different ranges and the observation time required to obtain high-accurate positioning, using commercial software LGO using analysis of variance techniques [12]. The results are considered valid for the applied software and under the conditions of the experiments.

In the present work the same four different strategies were established and evaluated through the processing of a total of 11760 baselines. The data processing and testing used several options concerning the best thresholds for accuracy. A brief analysis for different amplitudes of time interval of exposure, considering the four strategies is reproduced partially in the present paper

In order to access the relation between time of observation and success, the data can be grouped by class of lengths averaging the observing sessions with the same length. The results obtained by GLM modeling were promising. Analyzing the percentage of successful baselines we can establish that while for the 2D case, in all strategies and for each class, the success increases with session length, in 1D that trend is not clear. Applying the Hopfield model [5] for baselines in class R_3 and higher do not appear to produce better results with longer sessions. By adopting the strategy of computing the troposphere the benefits of longer sessions are only significant to baselines up to class R_3 and, even yet, 12 hours sessions are required (74% of success with BC, 65% with PC).

In 1D, BC may be better than PC, regardless of baseline length. One of the reasons for this behavior may be when using the computed option due to the LGO attempt to absorb errors in Broadcast Ephemerides as troposphere. We can arise some questions:

- LGO computed option constraints designed for use with BC?
- BC may be better because of interpolation method used in LGO?

In 2D, PC behaves better than BC even for the shortest range, although the difference is not expressive.

Typically, sessions of 1 hour are enough to produce good results in 2D for baselines belonging to R_1 class while for classes R_2 to R_4 4 hours are needed (75% and 86% of success with BC and PC respectively to R_4). For baselines of R_5 class that time should be 12 hours.

In 1D, for R_3 class, minimum of 12 hours sessions are needed (74% of success with BC) or 24 hours (81% of success with BC). For longer baselines ($\geq 300\ km$) the percentage of success is always less than 50%, regardless the strategy and length of the observing session.

CQ indicators provided by LGO post-processing are often overlay optimistic in all components, when compared with the correspondent 1D, 2D and 3D errors found throughout this research, and should therefore be used with caution.

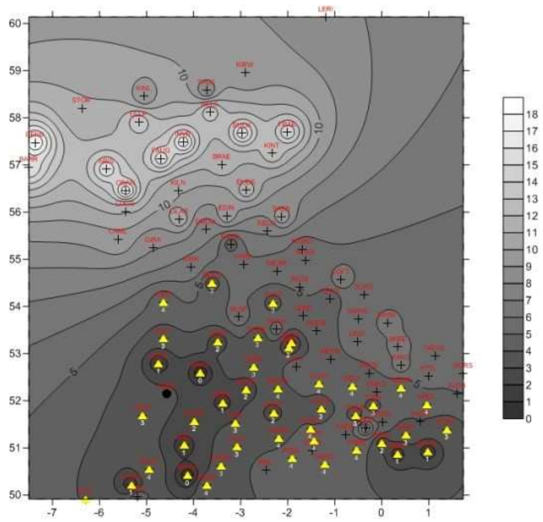

Figure 1: BHH24h1D.

5 Detailed analysis: 1D case

The main aim of the investigations presented in this work was to evaluate the improvements, or not, of the rate of baselines successfully produced by adopting an advanced tropospheric bias mitigation strategy as opposed to a sample tropospheric bias mitigation approach. In both cases LGO uses as a priori tropospheric model the simplified Hopfield model, improved in the first case with a zenith tropospheric scale factor per station. Being aware that 1D and 2D present different behaviours, both cases should be analysed individually with each strategy. In this section we present an analysis for 1D.

In the figures below the yellow triangles represent successful baselines in 3D (1D and 2D simultaneously) and red triangles represent successful baselines only in 1D. The colour scale bar is scaled in centimetres. An analysis of figure 5 shows that, considering the 24-hour session, Hopfield is only acceptable up to a certain distance (Wales and Southern England) where δZHD (δP), assumed by Hopfield is consistent with real δZHD (Difference between Zenith Hydrostatic Delays) and real δZWD (Difference between Zenith Wet Delays) is small. On the other hand, the computed option is acceptable over greater distances (Wales, Southern England, North England and Southern Scotland), as can be seen in figure 6, as it accounts for real δZWD and for differences between real δZHD (δP) and assumed δZHD (δP).

Does this pattern follow for sub-sets of the 24-hour session?

Taking, as an example, the four 6-hour observing sessions, evolution during the day can be assessed by examining Figs 9 to 16. In principle, while using the same processing strategy, the research results pro-

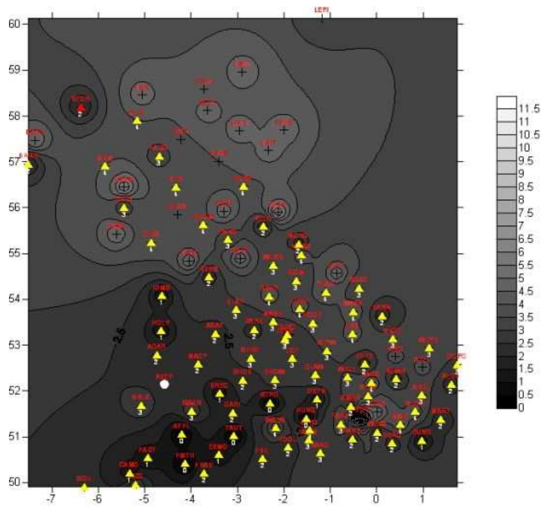

Figure 2: BCH1D 24 hours.

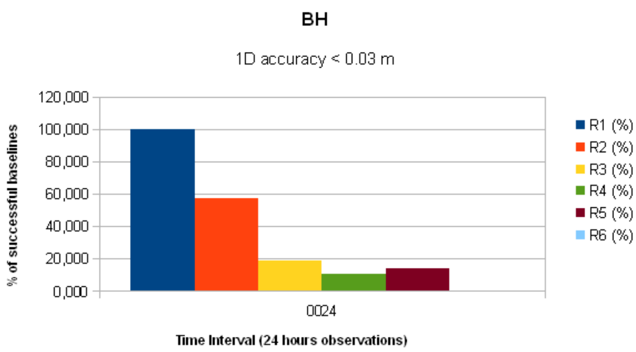

Figure 3: BH1D 24 hours.

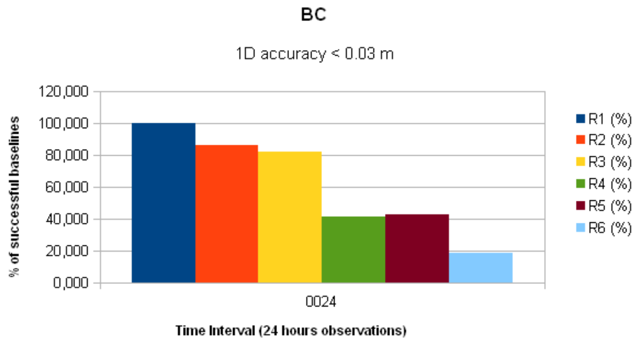

Figure 4: BC1D 24 hours.

vided from the estimated positions vary, depending on the period of the observing session.

As can be seen in Figs 9 and 10, period [0006] does not show much difference between both strategies, only in Wales and Southern England, corresponding to classes R_2 and R_3. The most surprising results concern the period [0612] of BC (Figs 12 and 18). While for BH there is no substantial difference with the previous period, for BC the improvement of success for baselines belonging to class R_4 and higher is remarkable. As a matter of fact, although being a sub-set of [0024], this session presents the best performance of the entire day.

Are water vapour concentrations in the atmosphere the reason for this performance?

Overall, for session [1218] Hopfield results are slightly better (Figs 13 and 14), especially for the longest baselines (Figs 17 and 18). In session [1824] results of both strategies were improved, in particular class R_2 for BH, when compared to the previous session, and classes R_4 to R_6 in BC, corresponding to Wales, Southern and North England, and Southern Scotland.

All in all:

- For BH the period [1824] has similar effect to [0024];
- Identical considerations can be taken for BC, except for Southeast England;
- For BC the period [0612] presents significantly better results than [0024], as it covers the North of Scotland as well.
- For BC the period [1218] does not show relevant improvements compared to the same period for BH and presents a poor performance for classes R_3 to R_6.

The reason why period [0612] appears better than [0024] is because [0024] is a combination of LGO

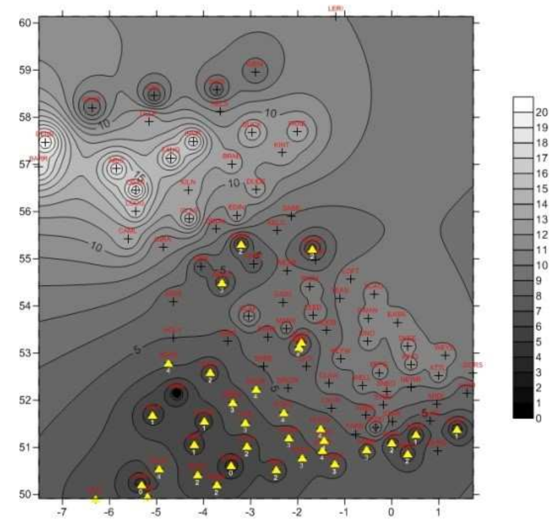

Figure 5: BHH0006h1D 6 hours.

computation being successful in [0612] and [1824] but not successful in [0006] and [1218]. In general, a vertical error was found in all 1D successful baselines.

6 Conclusions

Summarizing, it was developed a GLM model [15] to relate the exposure time, number of success and base lines distance. The remaining details of such approach will be found in a future extended version of this article, where the detailed numerical results about the time exposure duration and which part of day has major influence in the success rate will be discriminated and analyzed. To improve this statistical approach it is ongoing a statistical multivariate analysis [1] using all relevant available data. It was also presented a detailed and empirical analysis evaluating the improvements, or not, of the rate of baselines successfully produced by adopting an advanced tropospheric bias mitigation strategy as opposed to a sample tropospheric bias mitigation approach.

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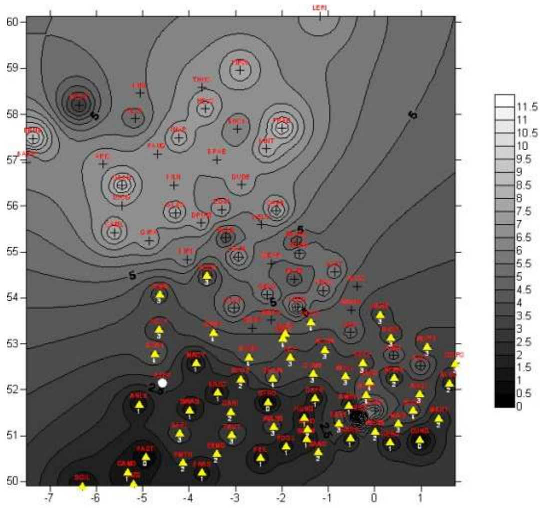

Figure 6: BCH00061D 6 hours.

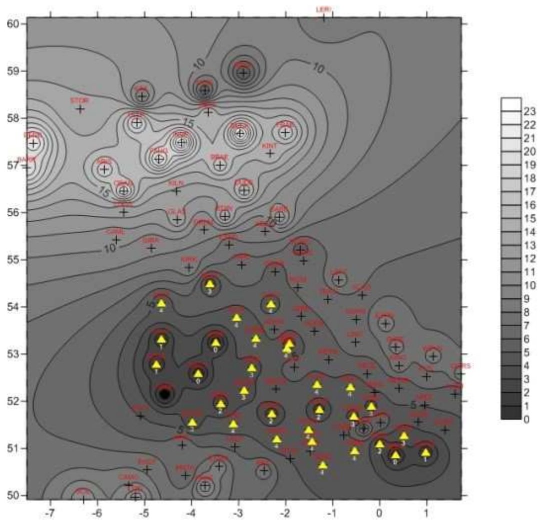

Figure 7: BHH06121D 6 hours.

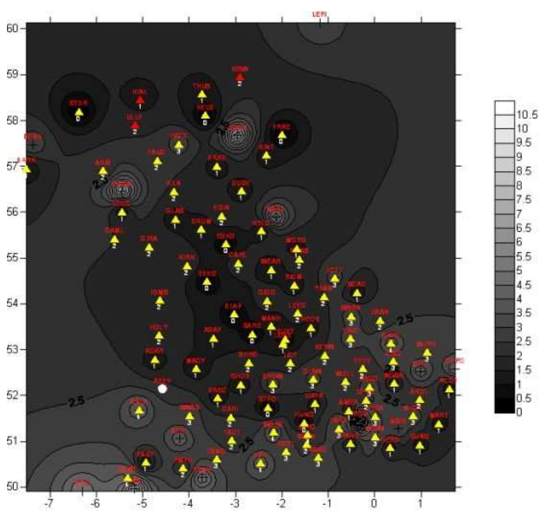

Figure 8: BCH06121D 6 hours.

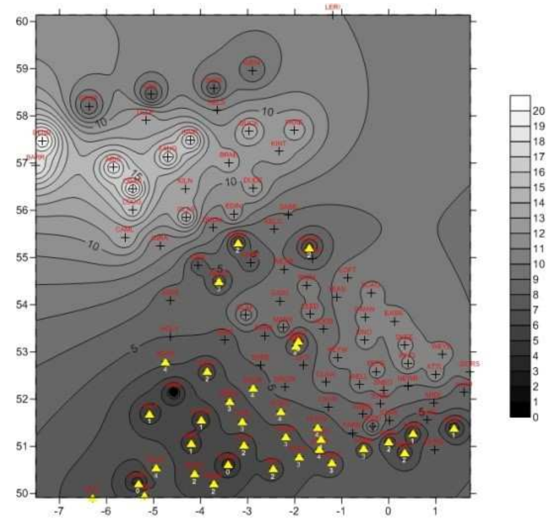

Figure 9: BHH0006h1D 6 hours.

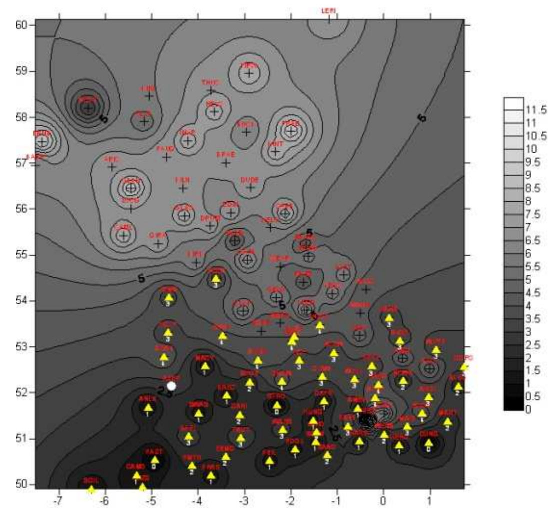

Figure 10: BCH00061D 6 hours.

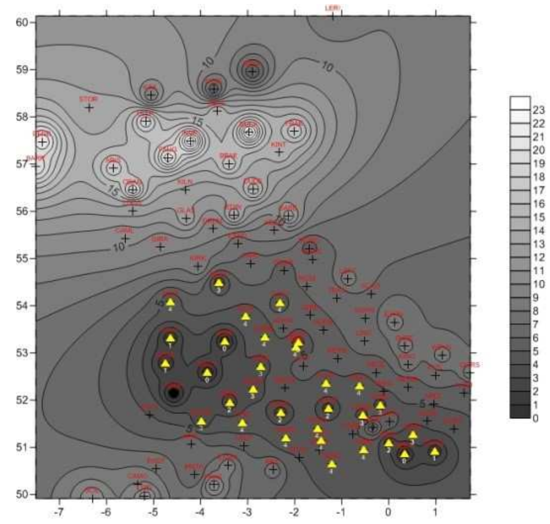

Figure 11: BHH06121D 6 hours.

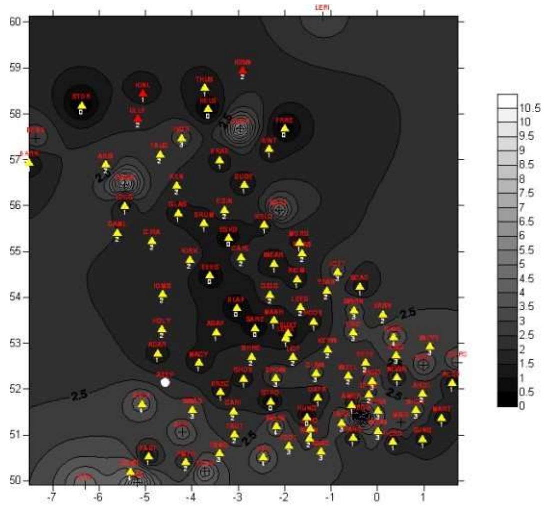

Figure 12: BCH06121D 6 hours.

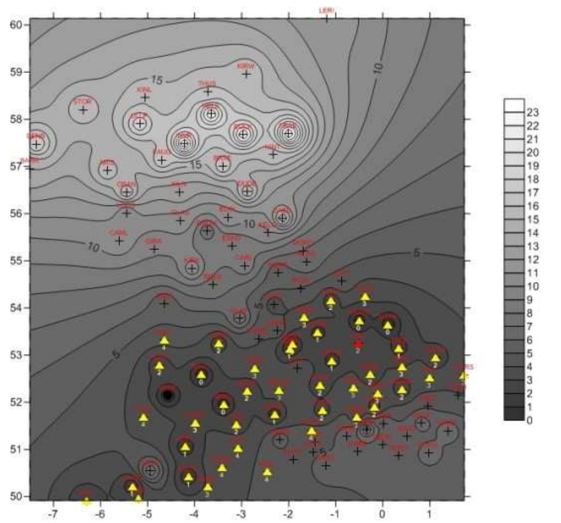

Figure 13: BHH1218h1D 6 hours.

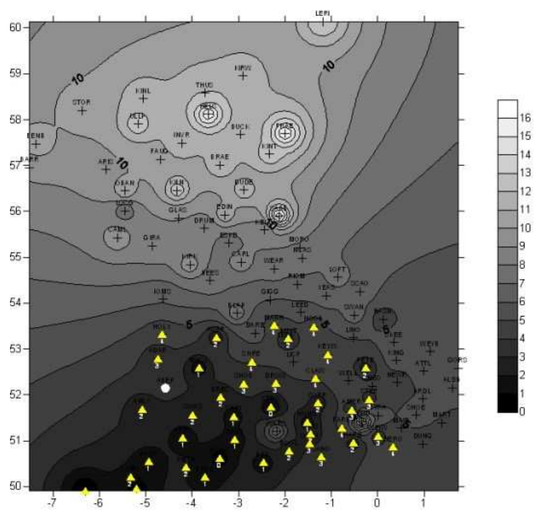

Figure 14: BCH12181D 6 hours.

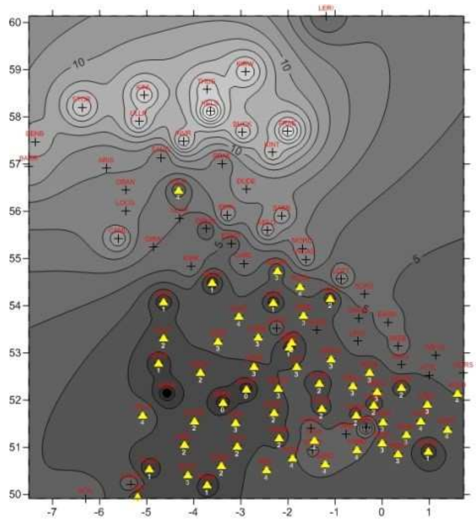

Figure 15: BHH18241D 6 hours.

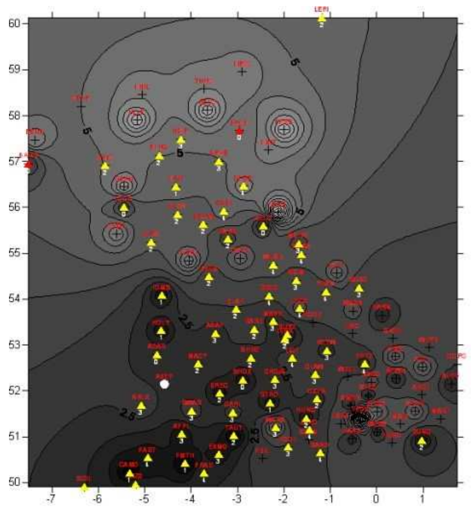

Figure 16: BCH18241D 6 hours.

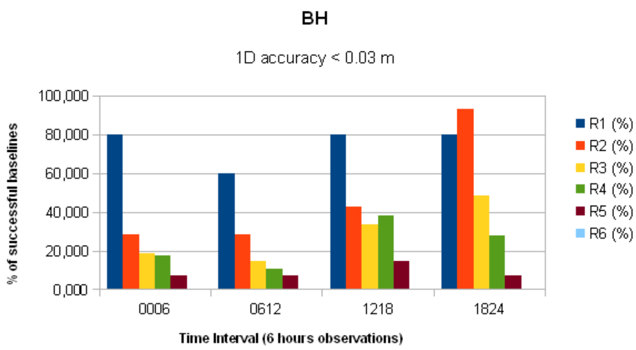

Figure 17: BH1D 6 hours.

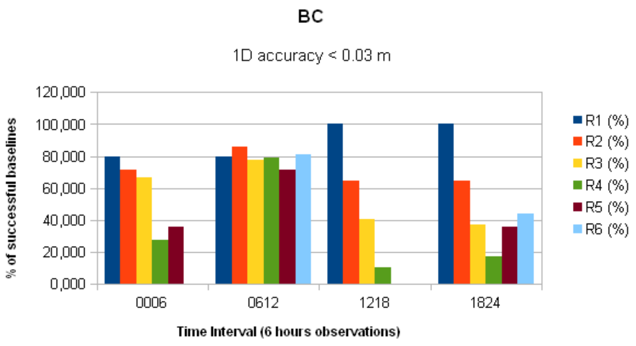

Figure 18: BC1D 6 hours.

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