Conceptual framework of on-the-fly web map generalization process

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Abstract Today, great numbers of users want access to spatial data on the web specific to their needs. This may be possible by applying the suitable generalization process which consists to simplify the objects of the map and may transform the topological relations between them into another ones in real-time. This process called on-the-fly generalization maps. Many approaches were proposed for improving this process, but those do not suffice to guide a powerful and efficient process. In this paper, we will propose a conceptual framework to transform the topological relationships during the on-the-fly web map generalization and treat the problem associated to modeling of streets and rivers as Linear objects because, in the reality (real word), they have some widths or areas. By considering a road or a river as a line or as an area, topological relationships can be different. We use mainly two types of object; ribbon and regions.

Keywords: On-the-fly maps generalization, Agent, Genetic Algorithm, topological relationships, Ribbons, Visual acuity, Scale.

1. Introduction

The web mapping has known great growth in parallel to the rapid development of the internet. To provide on-the-fly web mapping to the user, the process of on-the-fly map generalization must rely on fast, effective, and powerful methods. A principal challenge of such on-the-fly maps generalization is to offer the user a spatial data in real-time and in high quality , it must allow also to solve spatial conflicts that may appear between objects especially due to lack of space on display screens [16]. To optimize the on-the- fly maps generalization: we have to formulate an efficient generalization process. The generalization process is an important research area in this last decade and continues in the future. It is a set of operations, inspired by traditional cartographic generalization. Its main role is to simplify geographic data when they are very detailed, in order to satisfy user needs in cartographic applications. The principal objective of this process is creating an elegantly map from a vector geographic database very detailed [22]. All these aspects of generalization process are treated in our previous work [1].

In this work, we focus on the variation the topological relationships during the on-the-fly map generalization because topological relations may vary according to scale. Suppose a decision-maker who wants to create a new motorway running along a lake with the help of a computer, taking this consideration into account, any reasoning system will generate difficulties because the spatial relations hold differently: any conceptual framework dealing with spatial relationships must be robust against scales.

Another problem comes from mathematical modeling of streets and rivers and their visualization in the map. Often, they are considered as linear objects even if they have some widths or areas. By considering a road as a line or as an area, topological relationships can be different. In order to solve this problem, the concept of ribbon will be developed. Depending on the scale, or more exactly on visual acuity and granularity of interest, a ribbon will be a longish rectangle (area), a line or will disappear. In other words, ribbons can be seen as an extension of polylines. Moreover, in order not to be stuck to cartography, the concept of granularity of interest will be introduced. Also, we use the region feature to represent the areal objects as buildings.

The main contribution of this paper is the description of a conceptual framework of topological relationships in on-the-fly generalization map based on our previous work presented in [1] using the ribbon concept to represent the linear objects in the map.

This paper is organized as follow:

First, we present some definitions of ribbon, generalization process and visual acuity applied to geographic objects…etc. Then, we define the on–the-fly web map generalization, we present the approaches used for modeling this process and the state of the art. Also, we present the definition of topological relationships and the state of the art. Then, we describe the conceptual framework of topological relationships in on-the-fly generalization based on our previous work presented in [1]. Then, we present experimental examples. Finally, we present the conclusion and a certain future work.

2. Definitions

2.1 Ribbon

We claim that ribbons may elegantly model rivers, roads and streets (so-called linear objects): a ribbon can be loosely defined as a line or polyline with a width. Mathematically speaking, a ribbon is defined as longish rectangle [20]. The ribbon has a skeleton which is its axis. See Figure 1 for an example.

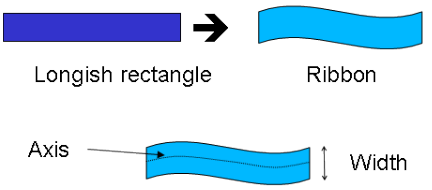


Figure 1 Definition of Ribbon

Let us note Width(R) and Skeleton(R) respectively the width and the Skelton of a ribbon. Remember that the ribbon can contain holes which can be useful for modeling islands in rivers.

In the sequel of this paper, to simplify the presentation, a ribbon will be represented by a longish rectangle. For instance a motorway (Figure 2) can be described by several ribbons corresponding to several driving lanes, emergency lanes and one median.

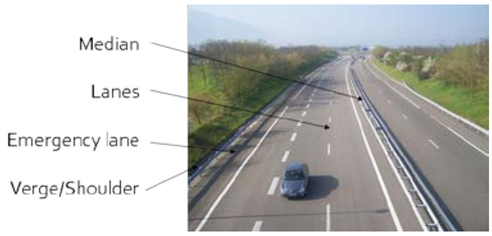


Figure 2 Ribbon model applied to a motorway.

2.2 Region

This feature may represent areal objects, as buildings. We can define a region as loose polygonal type. See Figure 3 for example, each region has an interior, boundary, and exterior. Using these primitive, nine topological relationships can be formed by two regions, called 9-intersection model.

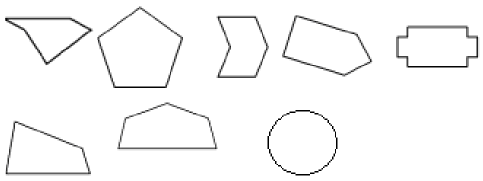


Figure 3 Example of Regions

2.3 Generalization Process

In most cases the required representation scale do, however, not exist in geographical database, thus a derivation from existing representation of required representation is necessary. This process of adaptation and reduction of the representation content to a requested scale is called as downscaling process.

During the downscaling, the topological relationships can vary as the changes of objects geometry. We treat in this context, two principal objects; Ribbons and regions. We can use the process as it is described in [20]:

Step 0: original geographic features only modeled as areas and/or ribbons,

Step 1: as scale diminishes, small areas and ribbons will be generalized and possibly can coalesce,

Step 2: as scale continues to diminish, areas mutate to points and ribbons into lines (its Skeleton),

Step 3: as scale continues to diminish, points and lines can disappear.

2.4 Visual acuity applied to geographic objects

In the GIS, "*Cartographic representation is linked to visual acuity*" [20]. Thresholds must be defined. In classical cartography, the limit ranges from 1 mm to 0.1 mm. If one takes a road and a certain scale and if the transformation gives a width more than 1 mm, this road is an area, between 1 mm and 0.1mm, then a line and if less that 0.1mm the road disappears. The same reasoning is valid for cities or small countries such as Andorra, Liechtenstein, Monaco, etc. In these cases, the "holes" in Italy or in France disappear cartographically.

With the defined thresholds as; ε_i = 0.1 mm , ε_{lp} = 1 mm , we can formally get (in which *2Dmap* is a function transforming a geographic object to some scale possibly with generalization , in the 2-dimension):

With the defined thresholds ε_i ,ε_{lp} , we can formally get (in which *2Dmap* is a function transforming a geographic object to some scale possibly with generalization , in the 2-dimension):

a/ Disappearance of a geographic object (O) at scale σ:

$$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O_\sigma = 2Dmap(O,\sigma) \wedge$$
$$Area(O_\sigma) < (\varepsilon_{lp})^2 \Rightarrow O_\sigma = \phi.$$

b/ transformation of an area into a point (for instance the centroid of the concerned object, for instance taken as the center of the minimum bounding rectangle):

$$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O_\sigma = 2Dmap(O,\sigma) \wedge$$
$$(\varepsilon_i)^2 > Area(O_\sigma) > (\varepsilon_{lp})^2 \Rightarrow O_\sigma = Centroid(O).$$

c/ Transformation of a ribbon *R* into a line (for instance its skeleton):

$$\forall R \in \text{Ribbon}, \forall \sigma \in \text{Scale} \wedge R_\sigma = 2Dmap(R,\sigma) \wedge$$
$$\varepsilon_i > Width(R_\sigma) > \varepsilon_{lp} \Rightarrow R = Skel(R).$$

3. On-the-fly web map generalization

3.1 Definition

The on-the-fly web map generalization is defined as the creation in real-time and according to the user's request, of a cartographic product appropriate to its scale and purpose, from a largest-scale database. The main characteristics of on-the-fly web mapping are:

- Required maps must be generated in real-time [16].
- Generation of a temporary and reduced scale dataset for visualization purposes from the database [12] in order to use the computer's memory efficiently [4].
- A real-time map generation process has to take into account users' preferences and contexts.
- A real-time map generation process must adapt maps' contents to display space and resolution of display media as well as to the contextual use of these maps [16].
- The scale and theme of the map are not predefined [4].
- There is no way to verify the quality of the final map that will be sent to the user [16].

The main problems linked to on-the-fly map generalization are the time of delivering the cartographic data and its quality. The generalization process time is a crucial factor to provide a user cartographic data. The waiting time must be compatible with Newell's cognitive band, which is less than 10 seconds [14]. Also, in order to produce maps suited to a user's requests, on-the-fly map generalization must be flexible enough to take into account the level of detail, the kind of the map [1]…etc.

To optimize this process, we must find a solution that puts the constraints in a state of maximum satisfaction. The constraints divided into two components; internal constraints and relational constraints. Thus, we must find within a reasonable time, a compromise that satisfied these constraints in better for solving the most important spatial conflicts, as overlapping of two objects. Thus, the generalization process can be modeled as an optimization problem, where different constraints have to be satisfied simultaneously as faithfully as possible [17].

3.2 Approaches of the on-the-fly map generalization

There are three fundamental approaches to providing on-the- fly map generalization:

3.2.1 Generalization-oriented approaches

This approach is based on map generalization which is known to be a complex and time consuming process. In order to accelerate this process, certain authors propose methods based upon pre-computed attributes [16]. Cartographic generalization operators have to be applied to spatial objects on-the-fly. The generalization operators are principally the selection, the simplification, the displacement and smoothing. The generalization oriented approach is very flexible [19]. However, it is not widely used because of the time it takes to provide requested maps. Furthermore, due to its complexity, generalization process cannot be carried out by simply applying generalization algorithms sequentially without taking into account the objects' spatial neighbourhood.

3.2.2 Representation-oriented approaches

Currently, it is the ideal solution to allow users to get data at the desired level of abstraction compared by the previous approach. This approach proposes to store several pre-defined representations of a given object (usually at different scales) within the same database [19]. The simplest representations are usually obtained from the manual or semi-automatic generalizations of the most detailed representations. However, in terms of personalization, multi-representations are extremely limited because all scales need to be predefined. Other important problems related to multiple representations are the difficulty to create necessary map scales [19]. All these problems limit the effective use of multi-representations for on-the-fly map generalization.

3.2.3 Hybrid approaches

Hybrid approaches take advantage of the flexibility of generalization oriented approaches and the suitability of representation-oriented approaches to generate maps in real-time by combining their use [16]. Several authors proposed an approach based on this hybrid approach, such as [16], [19] and [4]. Its advantage is that it reduces the effort needed for generalization process and improves the quality of the result because smaller the difference between the initial map scale and the desired one, easier the generalization process. However, to be truly efficient, this method must rely on a database that includes several scales, leading to the typical problems associated with multiple representations [19]. To improve this third approach, it is necessary to develop new methods that minimize, as much as possible, the problems associated with automatic generalization and multiple representations and resolve the spatial conflicts.

3.3 State of art for on-the-fly web map generalization

Several methods and concepts were proposed to model and implement the generalization process but a framework for their combination into a comprehensive generalization process is missing [2].

Many works model the spatial objects by agents such as the works of ([3], [12], [15], [16], [17], [18] and [19]). The strategy presented in [3] offers a good method for automated the generalization process; nevertheless, it is not flexible enough since it does not permit the agent to choose the best action to perform according to a given situation. An important work that was suggested by [16], it present an approach based on the implementation of a multi-agent system for the generation of maps on-the-fly and the resolution of spatial conflicts. This approach is based on the use of multiple representation and cartographic generalization.

In the same context and for reducing the spatial conflicts in the map, a good method was proposed in [15], this method is based on the genetic algorithm. Also, the technique presented in [17] is very important, it uses the least squares adjustment theory to solve the generalization problems, but it can’t implement certain operations of generalization, such as elimination, aggregation or typification…etc.

Then, to improve the process of on-the-fly map generalization, another approach was proposed in [19] which based on a new concept called SGO (Self-generalizing object). Foerster et al. propose an approach based on user profiles, which formally captures the user requirements (preferences) towards the base map and deploys those profiles in a web-based architecture to generate on-demand maps[8].

All these methods and approaches were presented to define a good generalization process but the majority of them do not treat the transformation of topological relationships between the spatial objects when downscaling.

In this paper, we will mainly define a framework based on our previous work presented in [1] combined with an efficient sub-module of topological relationships which compute the topological relations between objects and propose the best transformation of them into other relations according to certain rules. In our previous work presented in [1], we use the multi agent system equipped with genetic algorithm in order to generate data on arbitrary scales thanks to the on-the-fly map generalization process.

4. Topological relationships

4.1 Definition

Topology is defined as the mathematical study of the properties that are preserved through deformations, twistings, and stretchings of objects. Topology is foremost a branch of mathematics, but some concepts are of importance in cartographic generalization, such as topological relationships [21]. Topological relationships describe relationships between all objects in space, the points, lines and areas for all possible kinds of deformation. Several researchers have defined topological relationships in the context of geographic information [7], [9] and [10].

4.2 State of art for topological relationships

From a historical point of view, different topological models were proposed. First, Allen (1991) proposed a model organizing pieces of a linear model which can also be used for temporal reasoning. Then, Max Egenhofer (1990) with his colleagues proposed the first topological model for two-dimensional objects, and then Lee and Hsu (1990, 1992) define the relations between rectangles. Let us examine them rapidly.

4.2.1 Allen Model

The objective of the Allen model is to represent the relations between two segments as illustrated in Figure 4.

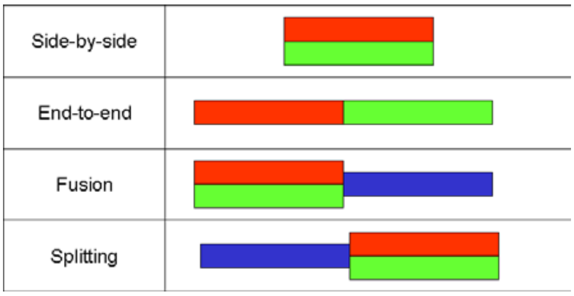


Figure 4 The Allen topological relations.

4.2.2 Egenhofer topological relationship

To define a model of topological relationships, Egenhofer and Herring (1990) proposed a spatial data model based on topological algebra. The algebra topological model is based on geometric primitives called cells that are defined for different spatial dimensions 0-D, 1-D, and 2-D. A variety of topological properties between two cells can be expressed in terms of the 9-intersection model [5]. The 9-intersection model between two cells A and B is based on the combination of six topological primitives that are interiors, boundaries, and exteriors of A $(A^{\circ}, \partial A, A^{-})$ and B $(B^{\circ}, \partial B, B^{-})$.

These six topological primitives can be combined to form nine possible combinations representing the topological relationships between these two cells. These 9-intersections are represented as one 3×3 matrix [9]:

$$R(A,B) = \begin{pmatrix} A^{\circ} \cap B^{\circ} & A^{\circ} \cap \partial B & A^{\circ} \cap B^{-} \\ \partial A \cap B^{\circ} & \partial A \cap \partial B & \partial A \cap B^{-} \\ A^{-} \cap B^{\circ} & A^{-} \cap \partial B & A^{-} \cap B^{-} \end{pmatrix}$$

The value represented in the matrix will be only a symbol indicating whether the intersection is null (ϕ) or not null ($\neg\phi$). When, the value of the intersection is not important, it is represented by (-). Based on these nine possible intersections, one can construct 512 theoretical relationships. However, they are not all available. The detection of possible relations made using negative conditions which prevent the association between pairs of primitives (non-existing topological relations). Therefore, the result implies eight possible topological relations between two regions in $\Re^2$. These eight relations are explicitly represented in Figure 5 (note that sometimes, the MEET relation is called TOUCHES in some papers):

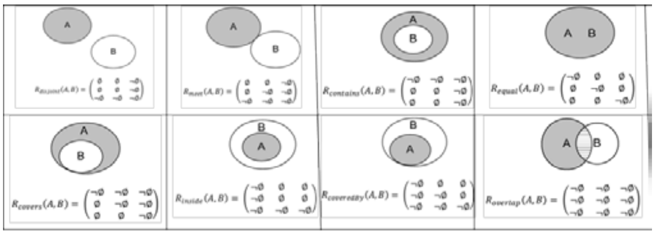


Figure 5 The eight topological relations between two regions A and B.

4.2.3 Lee and Hsu Model

In this model, Lee and Hsu (1990, 1992) study the rectangle relations; they proposed a table representing all spatial relations between two rectangles. They found a total of 169 types (See figure 6) in which they number: 48 disjoint, 40 joint, 50 partial overlaps, 16 contains and 16 belongs (= inside). Due to the semantics of ribbons, a lot of them can be discarded. We shall not examine all of them, but the more interesting ribbon relations, namely disjointing, meeting, merging and crossing.

Figure 6 The 169 types of spatial relations

All the models presented above define topological relationships between objects but they do not treat the transformation of topological relationships between the spatial objects, when downscaling. In the work [12] present an implementation of a topological vario-scale data structure. The purpose of this structure is to store the data only once, with no redundancy of the geometry, and derive different representations of this same data on-the-fly according to the level of detail needed. In this paper, we will discuss the variation of topological relationships during the generalization process.

5. Conceptual framework

5.1 The bases of our framework

This work is based on the approach proposed in our previous work presented in [1]. We combine genetic agent, map generalization process and multiple representations approach for improving the delivery time of map and resolving spatial conflicts to increase the quality of result map. This approach aims exclusively to improve the map generalization process. The spatial objects are modelled as agent. Each agent is equipped with genetic patrimony. Thus, genetic agent has some knowledge of its internal state, and some sensory information concerning environmental context, which permit it to decide what action (or action sequence) executed in order to achieve its goals. In this context, the role of genetic agent is to identify the best sequence of generalization operators with good parameters that allow perform the best map generalization process. In order to implement this process, each agent is able to identify and assess its internal constraints, and applies generalisation operators to its self in order to satisfy as well as possible these constraints. They are divided into two kinds [1]:

- Internal constraints: The constraints relate an isolated object. In this context, we take into account for region agent; constraints of size, granularity, squareness, preservation of shape,…etc. we mainly based on the visual acuity associated to each object.
- Relational constraints: They are the constraints which involve more than one spatial object, the constraint that prevents symbols from overlapping each others (e.g the symbol of a road should not overlap with that of a house), or the constraint that requires aligned buildings to remain aligned. In this work, we defined a new object called ribbon in order to represent the linear object as roads and streets to

elegantly the map and we study the possible transformation of topological relationships between the objects.

5.2 Genetic agent Module

As mentioned in our prevised work [1], the main role of a genetic agent is to generalize its self, in order to adapt it to the level of detail requested by the user. Thus, the genetic agent is responsible for the satisfaction of its constraints. It must be collaborate with the other agent to avoid a constraints violation. It applies the best solution composed of a sequence of generalization operators, which is generated by its optimizer [1]. The architecture of genetic agent is composed of two main modules (see figure 7).

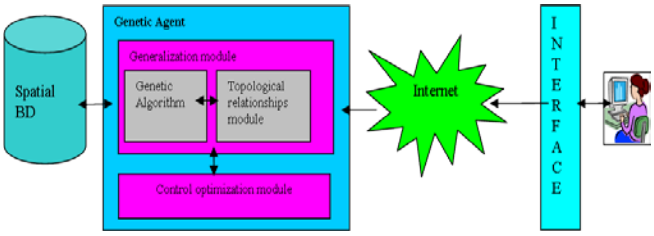


Figure 7 Architecture of the system

5.2.1 Map Generalization module

This module carries out the generalization process; it applies the solution found by the optimizer. The optimizer execute GA algorithm to define the best chromosome. It follows the classical steps of a genetic algorithm are selection, crossover and mutation. The solution is represented by sequence of algorithms and their good parameters. If the best solution is found, the algorithm stopped and the modified solution replaces the original solution in the next generation [1].

We will define also a sub-module of topological relations during downscaling. First, we will compute and store all the topological relations between spatial objects. Then, the mathematical assertions will be applied according to certain conditions. The main objective of this module consists to maintain the consistency of map under the geometric transformation, when downscaling. Thus, this sub-module performs the transformations of topological relations into other relations according to certain mathematical assertions, for more detail see section 6.

5.2.2 Optimization Control module

The control optimization module could achieve a satisfactory balance between discovery time of best solution and quality of the results. Thus, this module controls the time of generalization process to not exceeds the maximum limits and receive the message from neighboring agents which contain relevant information, such as the number of conflict agents, the distance between the neighboring objects …etc [1]. We add this model to control the time because the genetic method is typically time consuming.

We add also an interface between the user and the map generalization module; it allows user to transmit its requests. The request transmitted carries important information for the researched data, such as identification of the zone, the kind of map and its level of detail [1]. The level of detail is arbitrary and not predefined.

In this approach, genetic agents negotiate with each others, via messages which are used as input in genetic algorithm, these data allow the agent to solve various conflicts at once and prevent new conflicts from appearing in order to carry out the best generalization process [1].

6. Topological relationship sub-module

The objective of this module is to treats the transformations of topological relations according to certain metric conditions. But before defining this transformation of topological relationships for ribbons and regions, let us present some mathematical background.

6.1 Basic theory

In this section, we give certain definitions of the intersection which they will be used to formal the mathematical description for each topological relationship between two ribbons or between ribbons and regions:

Def 01 # the intersection**:**

If R^1 and R^2 are two ribbons , to define the intersection of $R^1 \cap R^2$, we have three cases :

- Point P (*x,y*).
- Line L (*y* = *a x* + *b*).
- Area A.

In other terms, this is an exclusive "belonging to" defined as follows $(P \oplus L \oplus A)$. Therefore, we can formulate it as:

$$R^1 \cap R^2 = \{x / x \in (P \oplus L \oplus A)\}$$

Def 02 # complement of the intersection:

Let be two ribbons R^1 and R^2. The **relative complement** of intersection $R^1 \cap R^2$ can be set of Points belongs to R^1 *or* R^2, but not to $R^1 \cap R^2$. Therefore, we can formally define the relative complement of intersection between ribbons as:

$$CMP(R^1 \cap R^2) = \{x / x \in (R^1 \oplus R^2) \quad et\ x \notin (R^1 \cap R^2)\}$$

6.2 Ribbons-Ribbons relations

In this section, we will classify the topological relations between ribbons according certain characteristics, then a mathematical description will be given for each types. Thus, two ribbons can be disjoint or intersect. The disjunction is defined by a distance separate the two ribbons, but the intersection of them can be Point (0D), Line (1D) or area (2D) according the case. In the following, we can formally get the mathematical description for each topological relationship, when we use thresholds and metric measurements; as area, distance etc.

6.2.1 Disjoint relations

For disjoint relation between two ribbons *Disj(* R^1*,*R^2*)*, the first condition is the inexistence of an intersection between them. Figure 8 shows vive cases:

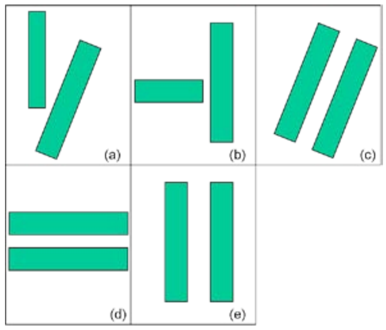


Figure 8. Disjoint relations between two ribbons.

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1, \sigma)) \wedge (R^2_\sigma = 2Dmap(R^2, \sigma)) \wedge Inters(R^1, R^2) = \phi \wedge (Dist(R^1, R^2) > \varepsilon_{Ds}) \Rightarrow Disj(R^1_\sigma, R^2_\sigma).$$

6.2.2 Meeting relations

Two ribbons R^1 and R^2 are linked by a meeting relation Meet(R^1, R^2) when:

The intersection of two ribbons is $P(x, y) \vee L(y = ax + b)$, such as P is Point (0D) and L is Line (1D). (See Figure 9)

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1, \sigma)) \wedge (R^2_\sigma = 2Dmap(R^2, \sigma)) \wedge (Inters(R^1, R^2) = \{P \vee L\}) \wedge (Dist(R^1, R^2) = 0) \Rightarrow Meet(R^1_\sigma, R^2_\sigma).$$

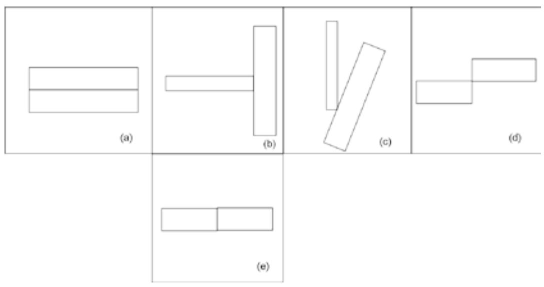


Figure 9. Several cases for meeting from (b to (d). Except (a) corresponding to a side-by-side and (e) to end-to end.

6.2.3 Merging relations

Two ribbons R^1 and R^2 are linked by a

Merging relation Merge(R^1, R^2), if the intersection of these ribbons is an area. We obtain six cases, (See Figure 10):

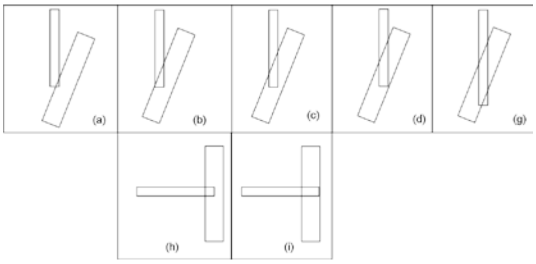


Figure 10. Example of merging.

Formally, we can state:

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1,\sigma)) \wedge (R^2_\sigma = 2Dmap(R^2,\sigma)) \wedge Inters(R^1,R^2) \neq \phi \wedge (Area(R^1 \cap R^2) > \varepsilon_{Mr}) \wedge (Area(CMP(R^1 \cap R^2)) = 0) \Rightarrow Merge(R^1_\sigma, R^2_\sigma).$$

6.2.4 Crossing relations

This topological relationship is very important because 80% of spatial objects are polyline-type [31]. Common examples include road-road crossings and river-road crossings. For instance, see the Fig 11.

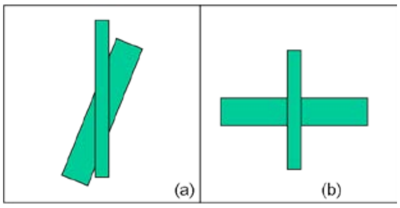


Figure 11. Example of crossing.

This relation based on the area of the intersection between two Ribbons R1 and R2. For instance, a threshold ε_{Cr} can be given.

So, we have:

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1,\sigma)) \wedge (R^2_\sigma = 2Dmap(R^2,\sigma)) \wedge Inters(R^1,R^2) \neq \phi \wedge (Area(R^1 \cap R^2) > \varepsilon_{Cr}) \wedge Area(CMP(R^1 \cap R^2)) > 0 \Rightarrow Cross(R^1_\sigma, R^2_\sigma).$$

6.3 Transformation of Topological relationships

The generalization of spatial data implied the generalization of topological relations according to certain accurate rules. The objective of this section is to formulate the list of these rules between objects; regions and ribbons. The regions represent the building and ribbons represent streets, roads or rivers. Then, sliver polygons will be taken into account in order to relax those relations, including the case of tessellations.

6.3.1 Transformation of topological Region-Region relations

In this section, the Egenhofer's relations are treated mainly. After the generalization, the object geometries are adapted to the perceptual limits imposed by the new (smaller) scale. In this context, the disjoint relations transformed into meet relation. Also overlap relations transformed into cover or meet according to certain metric conditions. We use the thresholds for distance, width and areas for modeling the conditions of the assertions. We will present in this context an example of the transformation of relation disjoint into meet, contain to cover and overlap to cover, when downscaling.

1) Transformation Disjoint-to-Meet

The relation "Disjoint" mutates to relation "Meet" (Figure 12). This transformation can be applied according to this assertion:

$$\forall O^1, O^2 \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O^1_\sigma = 2Dmap(O^1,\sigma) \wedge O^2_\sigma = 2Dmap(O^2,\sigma) \wedge Disjoint(O^1,O^2) \wedge Dist(O^1,O^2) < \varepsilon_1 \Rightarrow Meet(O^1_\sigma, O^2_\sigma).$$

It noted that 2Dmap is a function transforming a geographic object to some scale possibly with generalization, but a smaller object can disappear or be eliminated if its area is too small to be well visible. So in this case, the initial relation does not hold anymore.

$$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge (O_\sigma = 2Dmap(O,\sigma)) \wedge (Area(O_\sigma) < (\varepsilon_{lp})^2) \Rightarrow O_\sigma = \phi.$$

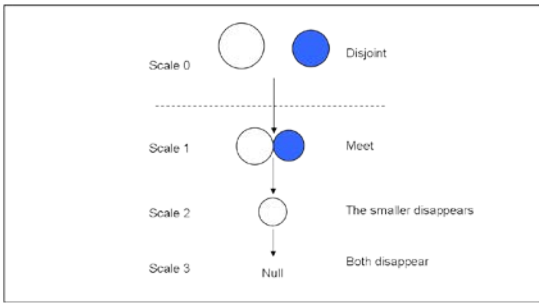


Figure 12 The mutation Disjoint-to-Meet.

2) Transformation Contain-to-cover

The transformation of relation "contain" to "cover" was expressed by the following assertion (Figure 13),

$$\forall O^1, O^2 \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge$$
$$O_\sigma^1 = 2Dmap(O^1, \sigma) \wedge O_\sigma^2 = 2Dmap(O^2, \sigma) \wedge$$
$$Disjoint(O^1, O^2) \wedge Dist(O^1, O^2) < \varepsilon_1 \Rightarrow Meet(O_\sigma^1, O_\sigma^2).$$

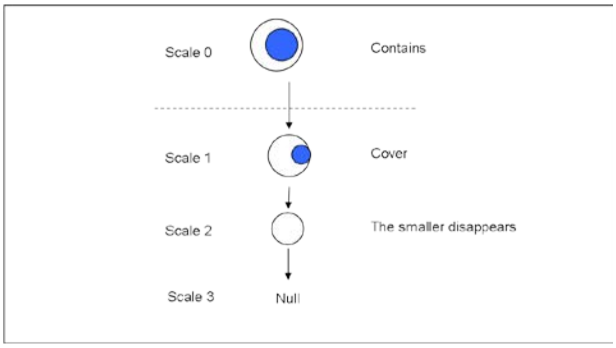


Figure 13 The transformation Contain-to-Cover.

The region can be disappeared, if its area is too small to be well visible:

$$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge (O_\sigma = 2Dmap(O, \sigma)) \wedge$$
$$(Area(O_\sigma) < (\varepsilon_{lp})^2) \Rightarrow O_\sigma = \phi.$$

3) Transformation Overlap-to-Cover

Also, the relation "overlap" may be mutating to relation "cover" (see Figure 14), to formulate this mutation, one use the following assertion:

$$\forall O^1, O^2 \in \text{GeObject}, (\forall \sigma \in \text{Scale}) \wedge$$
$$(O_\sigma^1 = 2Dmap(O^1, \sigma)) \wedge (O_\sigma^2 = 2Dmap(O^2, \sigma)) \wedge$$
$$(Overlap(O^1, O^2)) \wedge (Area(O^1 \cap O^2) > Area(\neg(O^1 \cap O^2)))$$
$$\Rightarrow Cover(O_\sigma^1, O_\sigma^2).$$

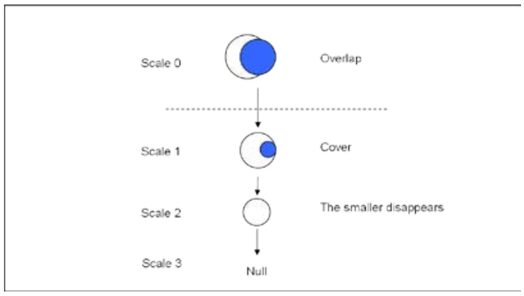


Figure 14 The transformation Overlap-to-Cover.

6.3.2 Transformation of topological Ribbon-Ribbon relations

This topological relationship is very important because 80% of spatial objects are polyline-type [11]; common examples include road-road crossings or river-road crossings.

1) Mutation of crossing relations

The figure 15 shows the case of crossing two ribbons. This relation varies corresponding to the following steps when downscaling:

i. First ribbons continue to cross (ribbon-ribbon intersection),
ii. then the smaller ribbon is mutate to a line (ribbon-line intersection),
iii. one ribbon disappears, so the intersection becomes void,
iv. and both ribbons are mutated to lines (line-line intersection).

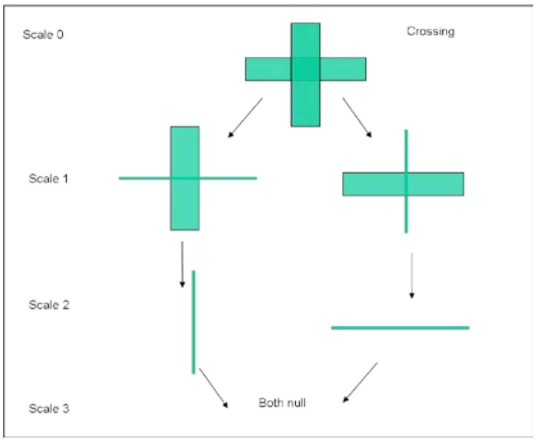


Figure 15 Mutation of crossing relations between two rectangular ribbons

This process can be modeled as follows:

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1,\sigma)) \wedge (R^2_\sigma = 2Dmap(R^2,\sigma)) \wedge (cross(R^1,R^2)) \wedge (\varepsilon_1 > Width(R_\sigma) > \varepsilon_2) \Rightarrow R_\sigma = Skel(R).$$

When a ribbon becomes very narrow, we apply this assertion:

$$\forall R \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R_\sigma = 2Dmap(R,\sigma)) \wedge (Width(R_\sigma) < \varepsilon_1) \Rightarrow R_\sigma = \phi.$$

2) Transformation of disjoint to merge

This disjoint relation transformed into merging, when downscaling (See Figure 16).

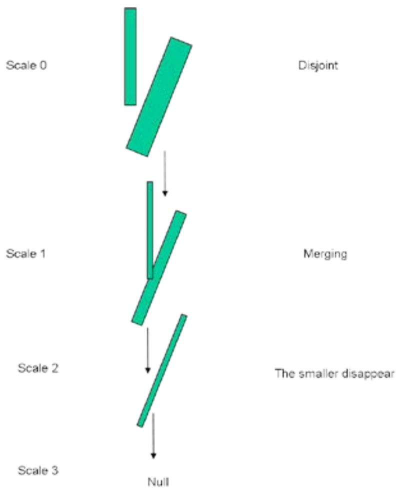


Figure 16 Transformation of disjoint relation between two ribbons.

This process can be modeled as follows:

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1,\sigma)) \wedge (R^2_\sigma = 2Dmap(R^2,\sigma)) \wedge (Dist(R^1,R^2) < \varepsilon_{Dj}) \Rightarrow Merg(R^1_\sigma, R^2_\sigma).$$

When a Ribbon becomes very narrow, we apply this assertion:

$$\forall R \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R_\sigma = 2Dmap(R,\sigma)) \wedge (Width(R_\sigma) < \varepsilon_{lp}) \Rightarrow R_\sigma = \phi.$$

6.3.3 Transformation of topological Ribbon-Region relations

In this section, we study the relations which can hold between ribbon and region. To describe these relations, we based on the basic relations who may be classified into six types, namely disjoint, touches, and cross, covered-by, contained-by and on-boundary, as shown in Figure 17:

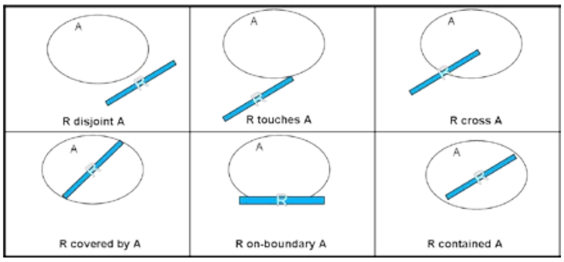


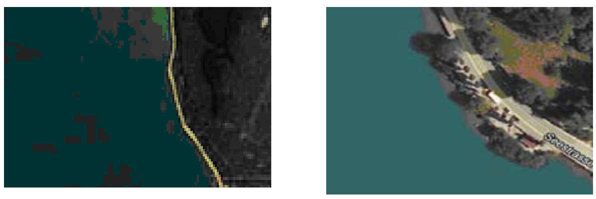
Figure 17 Basic relations between Region and Ribbon

Therefore, the spatial relation varies according to scale. In this context, one says that a road runs along a sea; but in reality, in some place, the road does not run really along the water of the sea due to beaches, buildings, etc. At one scale, the road TOUCHes the sea, but at another scale at some places, this is a DISJOINT relation (see Figure 18). Let consider two geographic objects O^1 and O^2 and $O_{\sigma 1}$ and $O_{\sigma 2}$ their cartographic representations, for instance the following assertion holds:

$$\forall O^1, O^2 \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O^1_\sigma = 2Dmap(O^1,\sigma) \wedge O^2_\sigma = 2Dmap(O^2,\sigma) \wedge Disjoint(O^1,O^2) \wedge Dist(O^1,O^2) < \varepsilon_1 \Rightarrow Meet(O^1_\sigma, O^2_\sigma).$$

Similar assertions could be written when CONTAINS, OVERLAP relationships. In addition, two objects in the real world with a TOUCH relation can coalesce into a single one.

As a consequence, in reasoning what is true at one scale, can be wrong at another scale. So, any automatic generalization system must be robust enough to deal with this issue.



a) (b)

Figure 18 According to scale, the road TOUCHes or not the sea.

6.3.4 Generalized irregular tessellations when downscaling

By irregular tessellation (or tessellation), one means the total coverage of an area by sub-areas. For instance the conterminous States in the USA form a tessellation to cover the whole country. Generally speaking administrative subdivisions form tessellations, sometimes as hierarchical tessellations. Let us consider a domain D and several polygons Pi; they form a tessellation iff (See Figure 19b):

- For any point p_k, if p_k belongs to D then there exists P_j, so that p_k belongs to P_j
- For any p_k belonging to P_j, then p_k belongs to D.

A tessellation can be also described by Egenhofer relations applied to P_i and D, but in practical cases, due to measurement errors, this definition must be relaxed in order to include sliver polygons (Figure 19a). Those errors are often very small, sometimes a few centimeters at scale 1. In other words, one has a tessellation from an administrative point of view, but not from a mathematical point of view.

When downscaling, those errors will be rapidly less than the threshold ε_{lp} so that the initial slivered or irregular tessellation will become a good-standing tessellation.

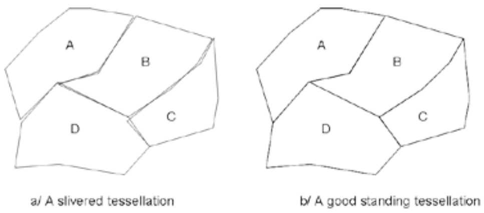


Figure 19 A tessellation with sliver polygons and a good standing tessellation

The situation becomes complex when a road or river traverses the tessellation, because we have to study all topological relationships between tessellation and ribbons which represent the road or river.

6.4 Frechet distance

This topological relationships module computes the different relations between objects and proposes for genetic agent the best transformation of these relationships. We need to calculate the distance between objects (ribbons and regions) and the area of each polygonal object. We use the Frechet distance:

Considering two objects A and B, what is the distance between them? An interesting definition is given by the Frechet distance which corresponds to the minimum leash between a dog and its owner, the dog walking on a line, and the owner in the other line as they walk without backtracking along their respective curves from one endpoint to the other. The definition is symmetric with respect to the two curves (See Figure 20). By noting a, a point of A, and b of B, the Frechet Distance F is given as follows in which dist is the Euclidean conventional distance:

$$F = \underset{a \in A}{Max}(\underset{b \in B}{Min}(dist(a,b)))$$

But in our case, one must consider two distances, let us say, the minimum and the maximum of the leash, so giving:

$$d_1 = \underset{a \in A}{Min}(\underset{b \in B}{Min}(dist(a,b)))$$

And

$$d_2 = \underset{a \in A}{Max}(\underset{b \in B}{Min}(dist(a,b))).$$

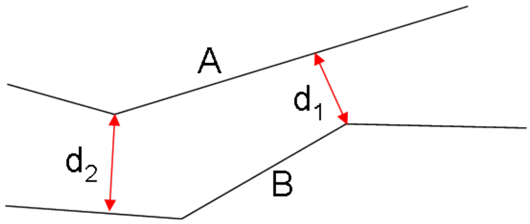


Figure 20 The Distance between two polylines

The thresholds used in the mathematical assertions are defined from this distance. Then, the distance between two regions A and B is defined also as the Frechet distance between both boundaries.

7. Experimental examples

7.1 Example 01

In the figure 21, the Rhone River (is represented by a ribbon) is linked to the sea.

Meet (River, Coast).

When downscaling, the Mediterranean coast is generalized (the coast and the river transformed into polyline) and the topological relation transformed into:

Merge (River, Coast).

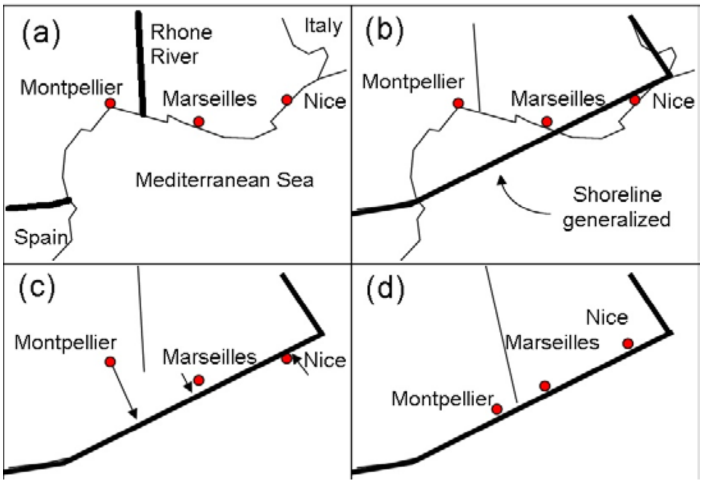


Figure 21 Holding topological constraints for harbors in the Mediterranean Sea. (a) Before generalization. (b) Only the coastline is generalized. (c) Harbors must move. (d) After generalization (The meet relation transformed into merge).

Since certain topological relations must be persistent, regardless the scale of representation, those relations must hold. See for instance in Figure 21 the Mediterranean Coast in the South of France: as the coast is generalized (the coast mutate into a polyline), some harbors will be in the middle of the sea such as Nice, whereas others will be inside the country such as Marseilles and Montpellier; in addition, the confluent of the Rhone river will be badly positioned in the middle of the land. The constraints are as follows:

Covers (France, Nice)

Covers (France, Marseilles)

Covers (France, Montpellier)

Covers (France, Rhone).

Another example of topological constraint when generalizing the Eastern French border is the case of Geneva which must hold outside France (Figure 22): the constraint is as follows:

Meet (France, Geneva).

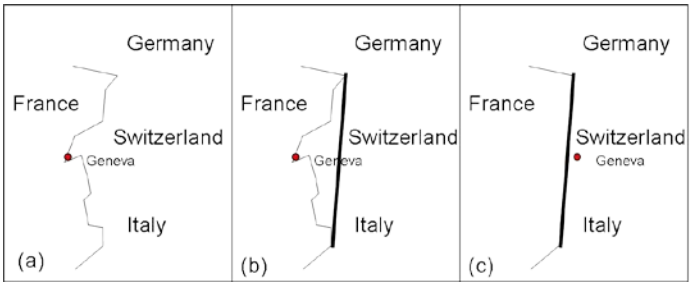


Figure 22 Holding topological constraints for outside border cities.

7.2 Example 02

We want to show in this example, the variation of topological relationships when downscaling, in on-the-fly map generalization. We use a vector data derived from cadastral database. We use the ribbons for representing the streets and roads to increase the quality of the map and to be more real. Because, in the reality the linear objects as roads and streets are not polyline but they have some width. Also, we use the region for representing the areal objects as buildings. Firstly, we apply the simplification operator. Then, we compute the topological relationship between objects and mutate them into another relation according the assertions mentioned in the previous section. The initial results are illustrated in Figure 23.

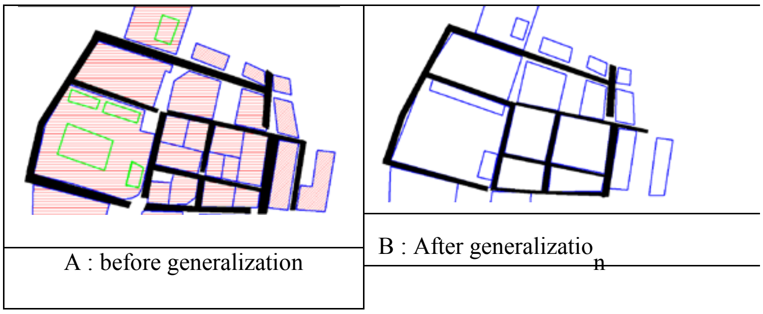


Figure 23 The initial results

The table 1 represents the variation of the topological relationships, when downscaling:

Table 1 The variation of topological relationships

	Scale0	Scale1
Topological relation	05 disjoint	03 meet
	5 contains	2 cover 02 objects aggregated
	10 cross	10 cross

8. Conclusion and futur works

The concept of on-the-fly map generalization process was introduced into the GIS domain since various years. Many propositions were given for modelling this process. In this paper, we treat many aspects in this process. First, we use the concept of ribbons in order that the map will be more real, and we use a known optimization method that is the genetic algorithm, also we treat the variation of topological relationships when downscaling.

The application of the generalization operators may cause topological conflicts. To avoid these conflicts, topological conditions are used to generate the relationships in terms of meeting, overlapping, disjunction, and containment between map objects into others relationships. In this paper, we use these topological conditions to formulate some of mathematical frameworks which composed of a set of assertions for treating the variety of topological relation according the scale. We consider two principal types of objects; regions and ribbons. When downscaling, a spatial objects represented by area, can mutate into a point, or disappear; also a ribbon can mutate into a line, or disappear. These objects have topological relationships between them. So, each topological relation will be also generalized using the assertions given in mathematical framework for each situation.

Our work opens many directions of research:

- The topological relationships module did not apply all transformations of topological relations between objects. In the future, we will try to integrate the other ones.
- The mathematical assertions of the framework considered the geometries of object represented in the 2D domain; we would like to extend our work to deal with geometries of higher dimension, such as the 3D.

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