Application of the stochastic model for precipitation generation in the complex orographic region (Bashkortostan Republic, Russian Federation)

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Abstract: - This article presents the use of a stochastic approach to generate monthly precipitation total at the regional scale. The data of 32 meteorological stations located in the territory of the Bashkortostan Republic were used. To generate precipitation monthly data the gamma distribution function has been selected. To check the accuracy of approximation of the theoretical distribution of data the χ^2-test has been used. To perform a regional scale application, estimated gamma distribution parameters (α and β) have been interpolated using the geostatistical kriging to map the spatial variability. The parametric Fisher's and Student's tests have been used to test the suitability of the regional approach to describe observed data, for the available point stations. The stochastic model based on gamma distribution function shows acceptable results of precipitation generation. Differences between observed and synthetic data could be caused by the particular climate conditions and orography features of the region.

Key-Words: - precipitation, precipitation generation, gamma distribution, homogeneity test, interpolation methods

1 Introduction

Global warming has accelerated since the 1980s in the last century and is characterized by increasing surface air temperature. Air temperature changes in some months are different from the average temperature changes [40-43]. Climate research in recent years have been shown the surface air temperature has been increased on 0,6 ° C for 100 years due to the global warming.

The Intergovernmental Panel on Climate Change was found a statistically significant trend of surface temperature increasing at all Russian Federation territory in the past 25 years [38,42].

Warmer air contains a lot of moisture, hence the rate of evaporation and cloudiness are increased, which may lead to the hydrological cycle acceleration, and increasing of precipitation.
According references, the increasing of average annual precipitation is expected to 2015 year. The reason of this tendency is increasing of precipitation amount in the winter period. The precipitation amount will be more than currently by 4-6% [42].

The climatological time-series generation is an essential task in order to describe the behavior of the data, obtain estimates and predictions given the observed values or simulate series under the same statistical pattern or under future climate scenarios. In rainfall series, usually characterized by a large variability and a significative empirical autocorrelation function, there is a need of finding suitable models that correctly capture the data behavior. Most works dealing with such a problem have considered the modeling of annual or monthly totals, where the normal distribution in the first case, and several asymmetrical distributions (as the lognormal, or incomplete gamma distributions) in the second case, have been suggested[47].

Gamma distribution efficiency for analyzing the amount of precipitation has been demonstrated for the model and observational data many authors [45-47].

The Gamma distribution with shape parameter α and scale parameter β is often assumed to be suitable for distributions of precipitation events [11-13].This distribution has been proven to be effective for the analysis of precipitation data in previous studies [14-17]. Gamma distribution efficiency for analyzing the amount of precipitation has been demonstrated for the model and observational data by many authors [16,18,19, 20].

Gamma distribution can be applied to the series, in which there is at least 4 non-zero value. To check the accuracy of approximation of the theoretical distribution of data there are various statistical tests such as χ_2- test.

Maps of precipitation have a wide range of applications and many different interpolation methods have been used to derive maps from data collected as part of monitoring networks. Conventional methods such as arithmetic mean, Thiessen polygons [21], simple trend surface analysis [22] and Delauney triangulations [23] have been used to estimate precipitation from ground-based point data. The inverse distance weighting (IDW) is another simple technique which has been used by Bosch and Davis [24] for spatial prediction of rainfall in the southern region of the USA. Precipitation, however, shows a significant spatial variation [25,26] suggesting that interpolation techniques which explicitly incorporate this spatial variability into the estimation process should be employed.

A well-known geostatistical method, i.e., kriging provides unbiased estimates with minimum variance taking into account the spatial relationship between the data points. Several authors [25,27,28,29] have shown that the geostatistical prediction technique (kriging) provides better estimates of rainfall than conventional methods. A major advantage of kriging over simpler techniques, such as IDW, besides providing a measure of estimation uncertainty (kriging variance), is that it can make use of correlated dense secondary variables to improve the prediction of sparsely sampled primary variable.

Consequently, the purpose of this research is to verify to suitability to use a gamma distribution stochastic generator for monthly precipitation for a particular region.

2. Material and Methods

2.1. The studied area

The 32 meteorological stations are located in the Bashkortostan Republic, Russian Federation. The Bashkortostan territory is bounded by the Perm Territory and the Sverdlovsk Region on the north, by the Chelyabinsk Region on the east, by the Orenburg Region on the south-west, south and south-east, by the Republic of Tatarstan on the west and by the Udmurtian Republic on the north-west (Fig. 1). The territory is characterised by a continental type of climate, with a warm summer and a snowy and cold long winter.

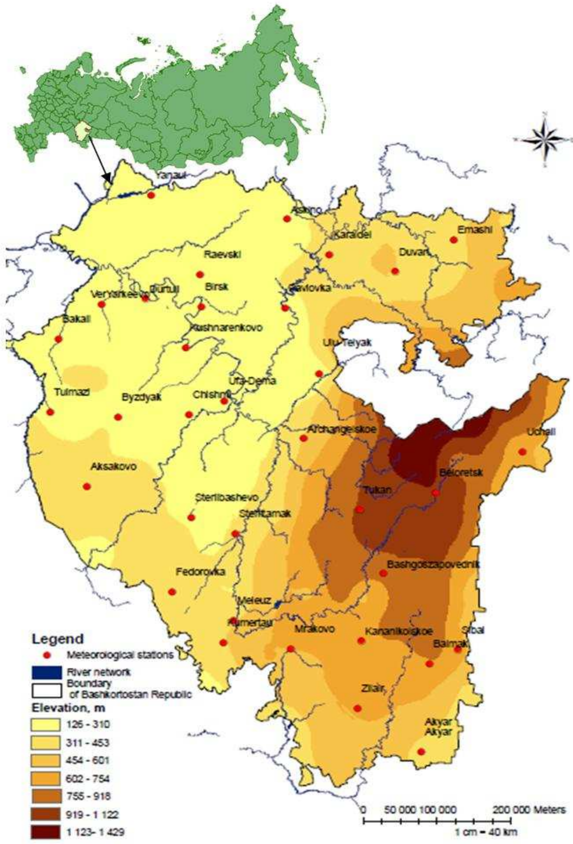

Fig.1: The studied area and stations location.

The Republic of Bashkortostan is a complex orography region. Plains and upland areas are located in the western part whereas the Ural Mountains relieves are located along the eastern part.

The relieves of the territory affect the spatial distribution of precipitation. The maximum amount of precipitation is indeed observed in the mountainous area and the minimum in the west of the low land territory. The minimum amount of precipitation is observed from February to April and the maximum from July to September (Fig.2 a, b)

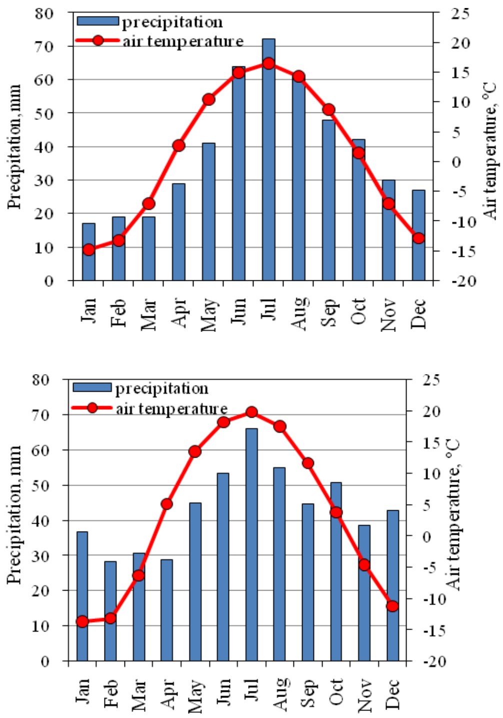

Fig.2: The mean monthly precipitation and mean air temperature for meteorological station located in different orographic conditions: mountain (Uchali station, a) and lowland (Sterlitamak stations, b).

2.2 Data and Methodology

Monthly precipitation data have been collected through the Bashkir Territorial Administration of Hydrometeorology and Environmental Monitoring Agency (BashUGMS). The data base includes monthly records in a total of 32 meteorological stations in Bashkortostan Republic, from 1936 to 2009. Station name, elevation and mean annual precipitation for the 41 meteorological stations are shown in the Table 1.

Fig.3 shows a flowchart of the precipitation generator algorithm. The algorithm proceeds as follows. Firstly, the probability distribution function is selected. In particular, the gamma distribution has been used it well preserves the important statistical characteristics, i.e., mean, standard deviation, as compared to the usually used distributions, such as the exponential distribution [15-17,30].

The probability density function for gamma distribution is given by

$$f(x) = \frac{(x/\beta)^{\alpha-1} exp[-x/\beta]}{\beta\Gamma(\alpha)} \tag{1}$$

where Γ(α) indicates the gamma function evaluated at α. α is the shape parameter and β is the scale parameter of the gamma distribution, estimated by the Maximum Likelihood Estimation Method.

The gamma function is defined by

$$\Gamma(\alpha) = \int_0^{\alpha} x^{\alpha-1} e^{-x} dx \tag{2}$$

for α>0

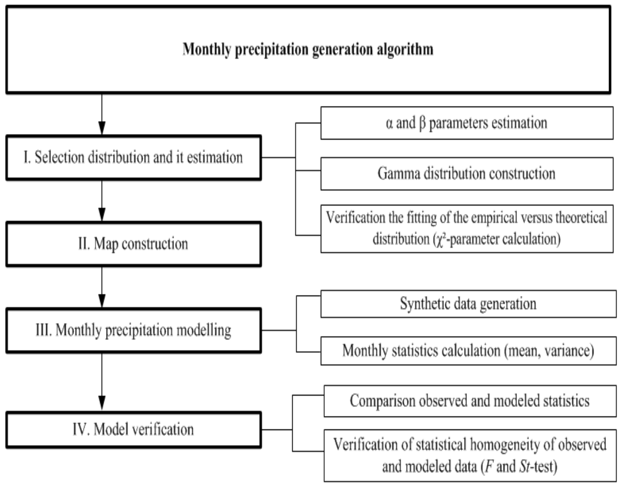

Fig.3: Flowchart of the monthly precipitation generation algorithm

№	Station	Latitude	Longitude	Records of data	Annual precipitation, mm	Elevation, m
1	Aksakovo	54.033	54.183	1936-2009	535,9	348
2	Akyar	51.866	58.183	1936-2009	340,5	341
3	Archangelskoe	54.406	56.781	1936-2009	601,6	142
4	Askino	56.083	56.583	1936-2009	566,5	207
5	Baimak	52.591	58.311	1949-2009	317,7	488
6	Bakali	55.183	53.795	1936-2009	424,7	125
7	Bashgoszapovednik	53.330	57.770	1952-2009	539,2	494
8	Beloretsk	53.933	58.333	1945-2009	468,4	568
9	Birsk	55.416	55.533	1935-2009	527,0	186
10	Byzdyak	54.566	54.516	1962-2009	417,5	190
11	Chishmi	54.589	55.380	1936-2009	437,3	118
12	Duvan	55.695	57.899	1936-2009	493,9	338
13	Emashi	55.941	58.604	1949-2009	476,9	233
14	Fedorovka	54.687	56.136	1980-2009	584,2	341
15	Kananikolskoe	52.780	57.482	1945-2009	553,2	532
16	Karaidel	55.816	57.083	1946-2009	579,9	156
17	Kumertau	52.756	55.797	1960-2009	570,0	349
18	Kushnarenkovo	55.106	55.342	1949-2009	467,7	99
19	Meleuz	52.958	55.966	1936-2009	437,4	180
20	Mrakovo	52.717	56.623	1936-2009	522,0	238
21	Pavlovka	55.420	56.561	1960-2009	823,1	282
22	Raevski	54.083	54.916	1936-2009	413,2	120
23	Sibai	52.719	58.666	2000-2009	364,6	360
24	Sterlibashevo	53.466	55.266	1952-2004	490,1	277
25	Sterlitamak	53.616	55.983	1936-2009	520,2	191
26	Tuimazi	54.583	53.733	1936-2009	418,7	135
27	Tukan	53.833	57.516	1945-2009	643,8	550
28	Ufa-Dema	54.750	56.020	1937-2009	533,5	118
29	Ulu-Telyak	54.913	56.976	1941-2009	672,2	119
30	VerYarkeevo	55.430	54.320	1971-2009	477,1	108
31	Yanaul	56.266	54.928	1936-2009	443,9	102
32	Zilair	52.216	57.448	1936-2009	559,8	521

Table 1: Gauge station

To check the fitting of the empirical versus theoretical gamma distribution the goodness-of-fit between cumulative distribution function values (F_i)

and probability distribution function values (P_i) is calculated based on the quantity [31]:

$$\chi^2 = \frac{\sum_{i=1}^{n}(F_i - P_i)^2}{P_i} \quad (3)$$

where χ^2 is a value of a random variable whose sampling distribution is approximated very closely by the chi-squared distribution with $v=n-1$ degrees of freedom (the χ^2 test).

When the difference between cumulative distribution function values and probability distribution function values is small, the χ^2- value will be small, indicating a good fit. A good fit leads to the acceptance of H_0, whereas a poor fit leads to its rejection. For a level of significance equal to 0.05 the critical value $\chi^2_{0.05}$ is calculated, and then if $\chi^2 < \chi^2_{0.05}$ the H_0 hypotheses is accepted, otherwise it is rejected.

Secondly, the maps of α and β parameters are constructed. For spatial representation of the distribution parameters the geostatistical kriging is used. This method produces visually appealing maps from irregularly spaced data [32]. The theory of ordinary kriging is based on the same geostatistical model we have been using all along with tow important restrictions:

- the mean m (s) is assumed to be constant;
- the semivariogram γ(h) is assumed to be known.

The basic equation used in ordinary kriging is as follows:

$$F\,(x, y) = \sum_{i=1}^{n} w_i \cdot f_i \quad (4)$$

where n is the number of scatter points in the set, f_i are the values of the scatter points, and w_i are weights assigned to each scatter point. This equation is essentially the same as the equation used for inverse distance weighted interpolation except that rather than using weights based on an arbitrary function of distance, the weights used in kriging are based on the model variogram.

Further, using the interpolated values of the gamma distribution parameters, generation of the precipitation amounts for each month and each station is carried out and the statistics (mean and variance) are calculated.

To determine the model suitability at the regional scale the *t*-test and *F*-test criteria are calculated. It assesses the significant differences between averages and variance of modeled and observed data samples. This hypothesis tests have widely been used in different countries, such as: Russia, Italy, USA, China and etc. to detect distinguish between samples [33-36].

When setting a datum point for an *n*-sample series, the subsequence x_1 of the n_1 samples was obtained before the datum point with an average of X_{cp1} and a variance of σ_1^2, and the subsequence x_2 of the n_2 samples was obtained after the datum point with an average of X_{cp2} and variance of σ_2^2. The *t*-statistic is given as [31]:

$$St = \frac{X_{cp1} - X_{cp2}}{\sqrt{n_1\sigma_1^2 + n_2\sigma_2^2}} \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{n_1 + n_2}} \quad (5)$$

The *F*-statistic is given as:

$$F = \frac{\sigma_1{}^2}{\sigma_2{}^2}, \quad (6)$$

The null hypothesis that assumes no differences will be rejected if $|t| > t_{\alpha/2}$, ($|F| > F_{\alpha/2}$) given a significance level α. A typical confidence level of 95% was applied.

3. Results

3.1 Fitting performances of the gamma distribution function

The first step is to choose the type of theoretical distributions that best describe the empirical distribution. According to cited references, the gamma distribution is a particularly suitable distribution for monthly precipitation data description in a number of climate conditions [19]. As a preliminary study, this density probability distribution has been considered as relevant for the case study and has been applied at the point stations in the whole region.

Model shape and scale parameters α and β parameters were estimated using the maximum likelihood (ML) method.

The gamma cumulative distribution and empirical distribution have been plotted for each month and for each station (Fig.4). On a visual inspection it is clear that the gamma distribution curves well fit the empirical curves and could be used to model monthly precipitations.

Besides graphical comparisons, the theoretical and empirical distributions were compared by the Chi-Squared goodness-of-fit test at the 5% significance level. Table 2 shows χ^2 and χ^2_{crit} values calculated for Ufa meteorological station (as an example). For the 32 precipitation stations in the study region, χ^2 was calculated for each station and for each month. It was found that all 384 sample data (32 stations ×12 months) passed the χ^2 test. This indicates that gamma distribution is an appropriate distribution to represent the synthetic precipitation data.

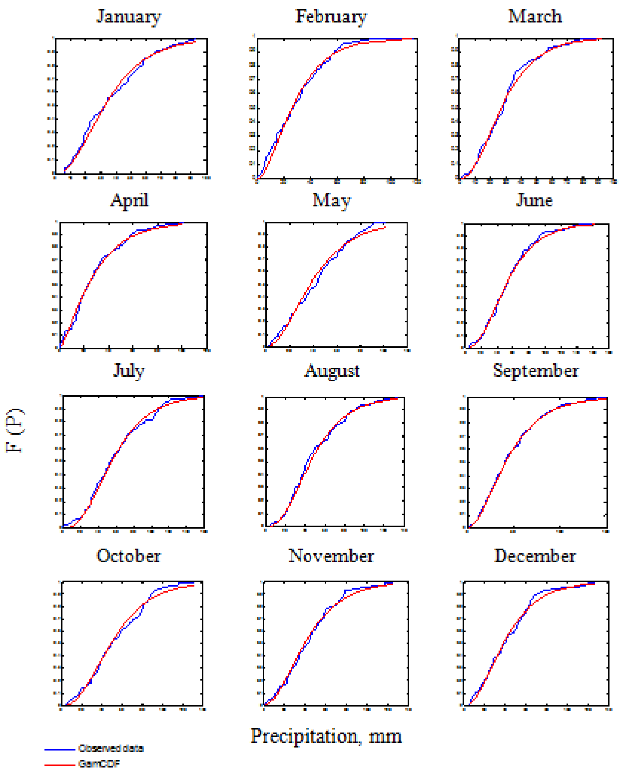

Fig.4: Gamma cumulative distribution function and empirical distribution function (Ufa meteorological station) for different data sets

Months	χ^2	χ^2_{crit}
Jan	0.15	28.9
Feb	0.22	29.7
Mar	0.31	31.4
Apr	0.35	33.9
May	0.24	38.9
Jun	0.15	40.6
Jul	0.14	38.9
Aug	0.09	39.8
Sep	0.31	39.8
Oct	0.72	33.1
Nov	0.19	35.6
Dec	0.15	29.7

Table 2: χ^2 and χ^2_{crit} values for each months for Ufa meteorological station

3.2. Observed and modeled data comparison

For regional scale applications, the gamma function shape and scale parameters have been interpolated over the study area. The geostatistical kriging interpolation has been used to generate monthly maps (January to December) for α and β spatial variation. These estimated values are used with the SURFER software to draw the contour maps of these parameters spatial distribution. As an example, the contour maps of the gamma distribution parameters obtained for January are shown in Fig.5.

To test the capability of the approach to represent, at the regional scale, the monthly rainfall occurrences process, the shape and scale parameters, for each of the monitored sites, have been derived from the interpolated kriging maps. Monthly precipitation time series have been modeled than, at each monitored site using the regional parameters, and comparison, for each gauged station, have been provided between modeled and observed rainfall time series.

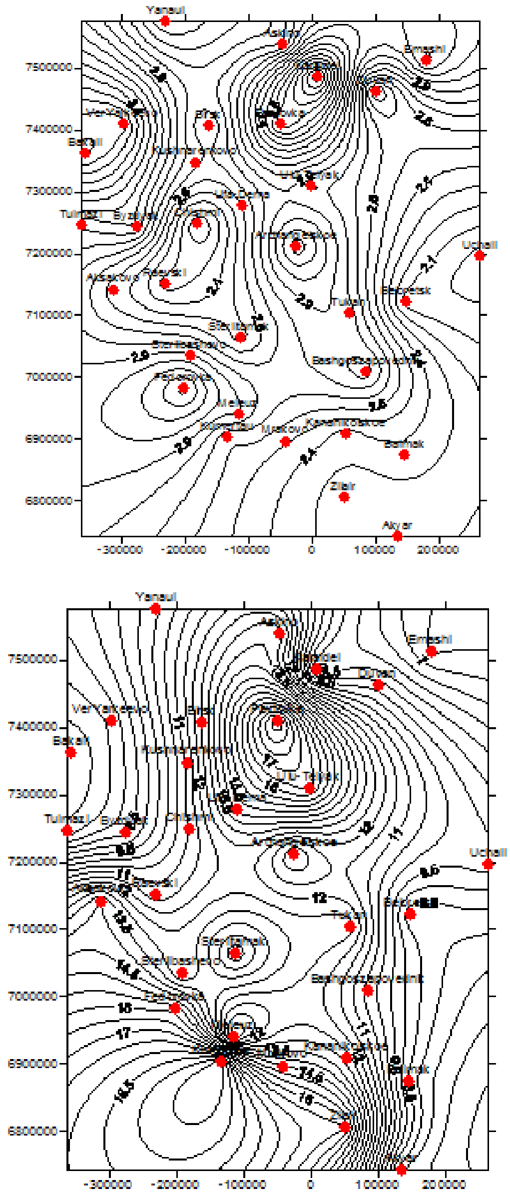

Fig5: Spatial distribution of the gamma distribution parameter (α parameters in the upper panel, β parameters in the lower panel)

To verify the model results, modelled and observed precipitation time series have been compared for monthly main statistics, such as the mean and the variance, and, in terms of distribution, using the t-test and the Fisher test.

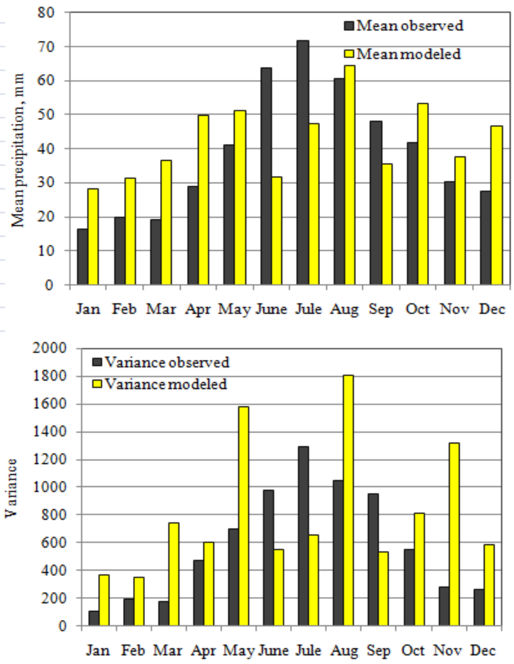

Fig.6: Histogram of observed versus modeled monthly precipitation statistics of Beloretsk station located in the mountainous (mean values in the upper figure , variance values in the lower figure)

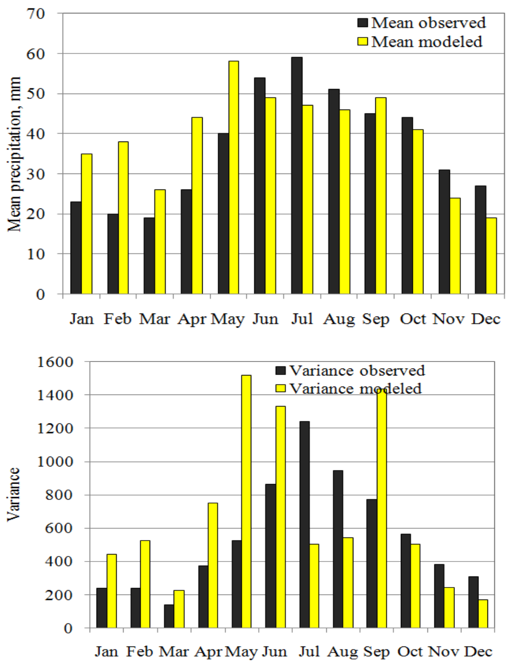

Fig7: Histogram of observed versus modeled monthly precipitation statistics of Chishmi station located in the lowland (mean values in the upper figure , variance values in the lower figure).

As an example, in the following (Fig 6 and Fig. 7) the observed and modeled monthly main statistics are represented for two different gauged stations, respectively located in the mountainous and lowland. The observed and modeled mean monthly precipitation for another station are presented in the Fig.8.

As shown in Fig.6 and Fig.7, the synthetic precipitation generated by gamma distribution model on successive months overestimates the mean monthly precipitation in winter and spring months and underestimates the mean monthly precipitation in summer months on both gauged stations.

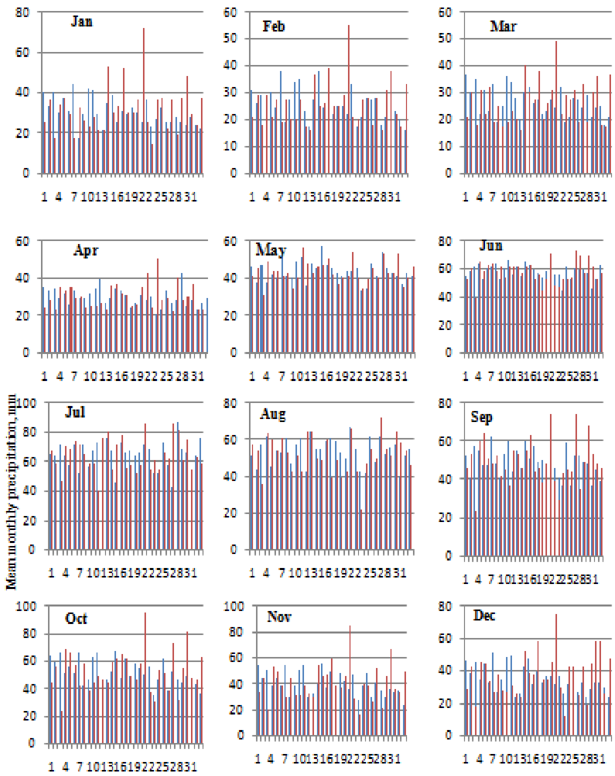

Fig.8: Plots of observed and modeled of mean monthly precipitation

1 - Aksakovo	9 - Birsk	17 - Kumertau	25 - Sterlitamak
2 - Akyar	10 - Byzdyak	18 - Kushnarenkovo	26 - Tuimazi
3 - Archangelskoe	11 - Chishmi	19 - Meleuz	27 - Tukan
4- Askino	12 - Duvan	20 - Mrakovo	28 – Ufa-Dema
5 - Baimak	13 - Emashi	21 - Pavlovka	29 Ulu-Telyak
6 - Bakali	14 - Fedorovka	22 - Raevskii	30 VerYarkeevo
7 - Bashgoszapovadnik	15 - Kananikolskoe	23 - Sibai	31 Uchali
8 - Beloretsk	16 - Karaidel	24 - Sterlibashevo	32 Zilair

To statistically confirm verification results Student's test and Fisher's tests for observed and modeled distribution were calculated. The results of Student's test analysis are shown in the Table 3.

Accept the assumption, that model is suitable for precipitation generation at the regional scale if more than 75% of modeled distributions (9 months from 12) for each station exactly correspond with observed distribution.

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Aksakovo	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,290	0,634	0,357	0,592	0,501	0,396	0,249	0,511	0,623	0,077	0,369	0,301
Akyar	h	1	1	1	0	0	1	1	1	0	0	0	1
	p	0,000	0,005	0,037	0,172	0,448	0,025	0,006	0,049	0,381	0,680	0,056	0,009
Archangelskoe	h	0	0	1	0	0	0	0	1	1	1	0	0
	p	1,000	0,296	0,005	0,665	0,389	0,212	0,331	0,035	0,000	0,015	0,095	0,646
Askino	h	0	0	0	0	0	1	0	0	0	0	0	0
	p	0,434	0,411	0,283	0,534	0,307	0,001	0,221	0,853	0,879	0,616	0,424	0,406
Baimak	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,453	0,686	0,718	0,902	0,869	0,316	0,577	0,582	0,717	0,376	0,217	0,388
Bakali	h	0	0	0	0	0	0	0	0	0	1	1	1
	p	0,134	0,747	0,281	0,424	0,604	0,487	0,378	0,528	0,365	0,014	0,042	0,013
Bashgoszapovednik	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,394	0,724	0,456	0,358	0,496	0,904	1,000	0,830	0,779	0,897	0,587	0,929
Beloretsk	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,425	0,570	0,978	0,444	0,160	0,546	0,339	0,267	0,459	0,173	0,209	0,705
Birsk	h	0	1	0	1	0	1	0	0	0	1	1	0
	p	0,388	0,024	0,146	0,020	0,054	0,033	0,219	0,336	0,076	0,006	0,019	0,104
Byzdyak	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,968	0,434	0,959	0,985	0,247	0,507	0,753	0,935	0,386	0,515	0,610	0,687
Chishmi	h	0	0	0	1	0	0	0	0	0	0	0	0
	p	0,064	0,213	0,113	0,036	0,062	0,800	0,317	0,069	0,159	0,635	0,782	0,521
Duvan	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,120	0,635	0,505	0,332	0,360	0,655	0,821	0,504	0,277	0,356	0,942	0,625
Emashi	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,726	0,731	0,854	0,573	0,726	0,849	0,736	0,995	0,651	0,210	0,754	0,600
Fedorovka	h	0	0	1	0	0	0	0	0	0	1	0	0
	p	0,910	0,795	0,044	0,519	0,345	0,347	0,344	0,227	0,378	0,044	0,629	0,607
Kananikolskoe	h	1	0	0	0	0	0	0	0	0	0	0	1
	p	0,035	0,176	0,090	0,470	0,922	0,714	0,977	0,880	0,626	0,584	0,762	0,029
Karaidel	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,586	0,534	0,892	0,606	0,847	0,533	0,519	0,422	0,969	0,784	0,658	0,407
Kumertau	h	0	0	0	0	0	0	0	1	1	0	0	0
	p	0,480	0,376	0,766	0,736	0,362	0,235	0,073	0,013	0,004	0,292	0,172	0,158
Kushnarenkovo	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,111	0,908	0,501	0,750	0,822	0,436	0,803	0,073	0,734	0,733	0,505	0,313
Meleuz	h	1	0	0	0	0	0	0	1	1	0	0	0
	p	0,014	0,079	0,083	0,063	0,860	0,911	0,412	0,002	0,026	0,561	0,605	0,362
Mrakovo	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,066	0,087	0,186	0,096	0,572	0,656	0,784	0,150	0,612	0,069	0,164	0,397
Pavlovka	h	0	0	1	1	1	1	0	0	1	0	0	0
	p	0,066	0,079	0,029	0,012	0,010	0,005	0,084	0,220	0,039	0,219	0,192	0,289
Raevskii	h	0	0	1	1	1	0	0	1	1	0	0	0
	p	0,054	0,107	0,001	0,000	0,007	0,549	0,254	0,008	0,012	0,221	0,320	0,051
Sterlibashevo	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,154	0,567	0,755	0,876	0,176	0,587	0,375	0,394	0,250	0,233	0,439	0,305
Sterlitamak	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,088	0,256	0,125	0,209	0,393	0,919	0,479	0,220	0,265	0,695	0,948	0,756
Tumazi	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,343	0,411	0,150	0,135	0,208	0,835	0,491	0,578	0,565	0,108	0,141	0,350
Tukan	h	1	0	1	0	0	0	0	0	0	0	0	0
	p	0,014	0,111	0,023	0,069	0,829	0,537	0,757	0,614	0,792	0,383	0,871	0,234
Uchali	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,903	0,879	0,634	0,478	0,994	0,633	0,586	0,622	0,284	0,208	0,196	0,152
Ufa-Dema	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,276	0,400	0,213	0,194	0,560	0,587	0,389	0,252	0,993	0,797	0,915	0,750
Ulu-Telyak	h	0	0	1	0	0	1	0	0	0	0	0	1
	p	0,662	0,712	0,049	0,385	0,854	0,047	0,232	0,117	0,071	0,533	0,400	0,028
V.Yarkeevo	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,348	0,568	0,403	0,177	0,063	0,833	0,523	1,000	0,498	0,348	0,361	0,608
Yanaul	h	0	0	0	0	0	0	0	0	1	1	0	0
	p	0,053	0,110	0,212	0,136	0,107	0,080	0,264	0,080	0,009	0,035	0,413	0,205
Zilair	h	0	0	0	0	0	0	0	0	0	0	0	0
	p	0,659	0,289	0,737	0,170	0,815	0,196	0,919	0,290	0,312	0,237	0,605	0,606

Table 3: Results of Student’s test analysis

Based on the results of Student's test, the mean monthly precipitation is successfully reproduced by the gamma distribution model and the modeled distribution quality is satisfactory for the five stations. It is mean, that for 84% stations the mean monthly precipitation values lie within the 95% confidence level, which confirms that the modeled values do not differ much from the observed and modeled distributions correspond exactly with observed distributions.

When dealing with Fisher's test, for 71% stations modeled distributions (for 24 stations) correspond with observed.

4. Discussion and conclusion

Monthly precipitation generation at the regional scale has been investigated for Bashkortostan Republic. Precipitation time series have been stochastically generated by using the two-parameter Gamma distribution. It has been found, that the empirical distribution of precipitation corresponds to the theoretical gamma distribution, which is confirmed by the results of goodness-of-fit calculations. The shape and scale gamma distribution parameters have been derived for regional scale model application and using the geostatistical kriging method it has been possible to map their spatial variability. Then, based on estimated parameters the synthetic precipitation data have been generated.

Statistical analyzes of the observed and modeled monthly precipitation time-series has shown that the model provides satisfactory results in terms of average precipitation statistics modeling at the monthly scale. Some difficulties raised in the case of precipitation amounts variance modeling. Model performances have been based on the parametric Fisher's and Student's tests.

Differences between observed and synthetic data could be caused by the particular climate conditions and orography features of the region, which has been to studied to improve the statistical model capability.

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