A Statistical Approach for Calibrating a Microsimulation Model for Freeways

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Abstract: - In this paper the calibration of a traffic microsimulation model based on speed-density relationships is presented. Hypothesis test was applied in the calibration process to measure the closeness between empirical data and simulation outputs and determine whether the difference between (observed and simulated) speed-density relationships was statistically significant. Statistical regressions between the variables of traffic flow were developed by using traffic data observed at the A22 Brenner Freeway, Italy. Similar relationships were obtained for a test freeway segment in uncongested conditions of traffic flow by using the Aimsun microscopic simulator; thus on field conditions were reproduced varying some selected parameters until a good match between measurement and simulation was achieved.

Key-Words: - freeway, traffic, microsimulation, speed-density relationship, calibration, Aimsun

1 Introduction

Simulation may perform sampling experiments on the model of the system rather than the real system itself [1][17][23]. Over time, the evolution of the system model is then able to imitate correctly the evolution of the modeled system, and conclusions can be drawn about the system behavior by analyzing samples of the variables of interest and using statistical analysis techniques. With reference to traffic simulation, a model should accurately represent the system behavior with the result that the model can be used as a substitute of the real system for experimental purposes. Commercial traffic microsimulation models began to emerge in the 1990s and now represent, together with agent-based simulation (see e.g. [2][24]), the latest generation of traffic models available for developing and evaluating of a broad range of road traffic management and control systems, and useful to predict future driving conditions [6].

According to car following, lane changing and gap acceptance rules, traffic microsimulators model the movements of individual vehicles traveling around road networks; as a consequence, they try to reproduce each individual driving behavior making the modeling process more complex from the model calibration stage.

Microsimulation models must be carefully calibrated on empirical observations before a model can estimate traffic performances [15]. According to [1], calibration of a traffic microsimulation model is an iterative process consisting in changing and adjusting numerous model parameters and comparing model outputs with a set of empirical data until a good match between the two data sets is achieved. The guidelines for applying traffic microsimulation software published in the Traffic Analysis Toolbox Vol. III [8] highlight that every microsimulation software comes with a set of user-adjustable parameters which allow to calibrate the model; thus, users can set the calibration setup parameters to obtain the best match possible between model and field measurements. Moreover, in order to reproduce each individual driving behavior, that is the mechanism of the decision made by an individual driver when he/she changes lane or negotiates the intersection waiting for an acceptable gap in the major road traffic stream, each traffic microsimulation model is provided with several sub-models each having several parameters to be set.
The calibration process should focus on adjusting parameters with strong effect on model outputs. However, direct measurement of these parameters is often complex, since they represent features difficult to observe or collect on field. Currently the lack of data explaining individual driving behavior has imposed the use of aggregate data in the process of model calibration; in these cases, the result often limits the behavioral power [15].

The selection of calibration parameters should be put in relation to the purpose of the calibration problem. Since model calibration is an iterative process, achievement of calibration objectives requires to focus on calibration parameters more appropriate to the problem to be solved and having strong influence on the performance measures that will be used to assess calibration. In some studies, calibration focuses on the driver behavior parameters only [16][18], whereas in other studies the driving behavior is just one part of a broader problem, including the calibration of a route choice model and/or an o-d matrix [7].

The 2014 Traffic Analysis Handbook [27] suggests that the parameters to be adjusted should be divided into global and local parameters: global parameters regard all elements of the simulated network and they should be changed and adjusted prior to local parameters; on the contrary, local parameters affect individual links or points of the network. Moreover, it must be said that there is no certainty yet on the most appropriate number of parameters that must be calibrated before the model can be used as a prediction tool. For a few number of parameters, the calibration process can be developed through a manual procedure. When manual calibration is proposed, a reasonable number of adjustable parameters (dependent on the network type and traffic conditions) should be used to appropriately calibrate the model within the required degree of accuracy [27]; thus, some parameters are calibrated often through multiple retries [25]. For large parameter subsets, calibration process normally uses automated algorithms, which should allow a closer approach to the optimal solution. Despite more parameters give more degrees of freedom to better fit the calibrated model to the specific location [27], an automated procedure makes harder to follow changes in the value of each parameter [22].

Proper calibration requires an assessment of the degree of closeness of the simulation outputs to the on-field measured data. Since microsimulation models are by definition a simplified representation of reality, improper calibration may be influenced by the simplification of which some technical features of the above models are affected: for instance, updating of transport system, randomness, traffic generation, driver/vehicle characteristics, vehicle interactions, etc.

Microsimulation tools require data collection or analytical determination of data for calibration and validation purposes [3]. It is noteworthy that each test used in the calibration process can also be used in the validation of the simulation model only if a new data set is used. This allows for checking whether a valid model is obtained, or only a representation of the particular set of input data is provided. In order to gain a valid model, indeed, two data sets are necessary: the first data set should be used for calibrating the model parameters, whereas the second independent data set (i.e. not previously used in the calibration) should be used for running the calibrated model so that the resulting model outputs can closely match the existing conditions; thus, validation represents the process of checking to what extent the model replicates reality [10][25][26].

In this article the calibration method based on the development of speed-density functions in the microsimulation calibration process is presented. Speed-density functions for empirical and simulated data were developed so that traffic patterns were implemented. In order to obtain speed-density relationships for a test freeway segment in uncongested traffic conditions, reference was made to traffic data surveyed at A22 Freeway, Italy, whereas microscopic traffic simulation was performed by using Aimsun software. On field conditions were reproduced varying some calibration parameters until a good matching between empirical and simulated data was achieved. Hypothesis test was applied in the calibration process to measure the closeness between empirical data and simulation outputs and determine whether the difference between (observed and simulated) speed-density relationships was statistically significant.

The organization of the paper is as follows: section 2 presents calibration methods for traffic microsimulation models and discusses issues on calibration approaches, whereas section 3 describes the path followed to develop the speed-flow-density relationships based on field observations at the A22 Freeway, Italy. Section 4 introduces the test freeway segment selected as case study and the parameters to be calibrated within the model; the calibration method and its implementation is described in section 5. At last the results are presented and discussed in section 6.
2 Calibration of microsimulation traffic models

Several calibration methods for microsimulation traffic models have been developed up to now in order to improve the calibration process.

Calibration methods use a single measure [18] or more than one measure through calibration of different sub-processes; each sub-process can use a different traffic measure for performing the calibration of a separate group of parameters. On this regard, Dowling et al. [7] proposed a three step calibration method: first, driving behavior parameters were calibrated by comparing capacities, then route choice parameters were calibrated by comparing flows; at last, calibration was performed by comparing travel times and queue lengths. Furthermore, Hourdakis et al. [16] proposed another procedure to be followed to improve the efficiency of the calibration effort: in the first step, observed and simulated flows were compared for calibrating global parameters (e.g. maximum acceleration and other vehicle characteristics), whereas in the second step, observed and simulated speeds were compared for calibrating local parameters; the third step was suggested as an optional calibration stage for comparing any measure selected by the analyst.

According to [8], calibration of the model to traffic capacity is one of the steps in the microsimulation model calibration, within which the set of model parameters better matching on-field measurements of capacity is searched for. A global calibration phase and a fine-tuning phase compose the capacity calibration step. In order to derive the appropriate network-wide values of the capacity parameters best reproducing on-field conditions, global calibration is first performed. Then link-specific capacity parameters are changed and adjusted to best match the on-field measurements of capacity at each bottleneck. In order to estimate a numerical value for capacity, queue discharge flow rate can be used. Defining capacity as a single numerical value results in loss of information, since a distribution of capacity values gives more information than a single numerical value. Brilon et al. [4] introduced the stochastic nature of capacity, whereas Menneni et al. [22] noted that if the calibration process is based on a single numerical value, the search for the match of the means of capacity distribution does not match the other important properties of the distribution; moreover, other traffic parameters characterizing capacity as speed or density, can be neglected. Since the main target in the calibration process should be to maximize the information appropriate for replicating real system performances and improving the efficiency of the calibration effort, the 2014 Traffic Analysis Handbook [27] has also suggested that capacity calibration is completed by route choice calibration and system performance calibration.

Generalized relationships among speed, density, and flow provide information on the capacity, but also information regarding free-flow and congested regions, not deducible from a single numerical value or a distribution of capacities. A calibration procedure could replicate the whole range of traffic behavior (not just peak periods), if it could be based on the generalized relationships among speed, density and flow, from which information about the free-flow, congested, and queue discharge regions can be derived. In order to enhance the overall model performance, consistently to model calibration purposes, just a part of the speed-density graph (or the speed-flow graph or flow-density graph) instead of the whole graph could be used [22]. Since a large amount of data is required for fitting empirical/simulated data, further information can be derived from speed-flow-density relationships; hence, a higher number of parameters can be submitted to the calibration process, resulting in a better fine-tuned simulation model. The calibration of speed-flow, speed-density, or flow-density relationships should be one step in microsimulation calibration and, as stated above, should be followed by route-choice calibration and system performance calibration. Despite the potential benefits to calibration process, technical literature still presents few studies related to the use of the fundamental relationships of traffic flow in the microsimulation calibration process. However, some studies have already introduced the concept of replicating field speed-flow relationships and the use of the corresponding simulated functions to demonstrate closeness of field and simulated data [30], whereas some other researches have already highlighted the ability of a simulation model to replicate speed-flow graphs from real-world freeways [10].

One of the most recent and more comprehensive efforts to calibrate traffic data for freeway is reported by [22]. In this study calibration was based on scatter plots of speed-flow pairs. These diagrams were very useful for calibration because they contained information on a broad range of traffic situations. More specifically they showed how the traffic flow behaves around capacity. This is the reason why the speed-flow diagrams are used for comparing simulation results with real-world data.
The calibration procedure was applied using a genetic algorithm to systematically modify the parameters of the behavior model and to fit the speed-flow diagrams from measurements and simulation [22]. For information on the process of identifying the optimal parameters for an optimization algorithm see also [5].

Menneni et al [22] developed an objective function based on minimizing the dissimilarity between (observed and simulated) speed-flow graphs; the same authors have measured the dissimilarity of two graphs by measuring the area not covered by the other graph. Since speed and flow measurements are represented as point sets, discretization to convert point information to area was necessary. Moreover, considering that the information derived from the field and simulation was often represented by an incomplete speed-flow graph, the comparison was only made over the space occupied by the field graph.

Differently from [22], in this paper the measure of the closeness between observed and simulated data was achieved through a statistical approach including hypothesis test and confidence intervals (see next section 4).

3 The fundamental diagram of traffic flow for the A22 Brenner Freeway, Italy

Traffic data have been surveyed extensively at observation sections on the A22 Brenner Freeway, Italy; hence, the relationships between the fundamental variables of traffic flow (i.e. the speed-flow-density relationships), for a traffic flow only made of cars, were modeled [19]. Data were collected over different locations and multiple days and then used to create a complete graph between the pairs of traffic flow variables. These speed-flow, speed-density, or flow-density relationships were developed for the roadway, the right lane and the passing lane, after treatment and processing of traffic data surveyed at specific observation sections (San Michele, Rovereto and Adige sections) on the A22 Freeway [19].

For the same framework, a criterion for predicting the reliability of freeway traffic flow by observing speed stochastic processes in uninterrupted flow conditions has been already proposed [20].

First the relationship between speed and density was created. This choice was derived from the following: considering the real traffic flow phenomenon, the speed-density relationship is a monotonically decreasing function, which implies a mathematical relation simpler than the flow-density and speed-flow relationships; moreover, the relationship $V=V(D)$ explains the interaction between vehicles in a traffic stream, where users perceive, through the spacing among consecutive vehicles, the density and then adapt their speed. The speed-density models proposed by literature were considered [9][13][14][21][29], but the single-regime models were selected because they were deemed more appropriate in interpreting the observed phenomenon.

Among the single-regime models, May's model [21] was chosen; it appeared as the best in interpreting the data surveyed on field and the traffic flow phenomena at the observed sections, especially the maximum density values under congested traffic conditions. According to May's model [21], the speed-density relationship was expressed as follows:

$$V = V_{ff} \cdot \exp \left[ -0.5 \cdot \left( \frac{D}{D_c} \right)^2 \right]$$  \hspace{1cm} (1)

where $V_{ff}$ and $D_c$ are the free flow speed and the critical density (that is the density $D$ to which capacity $C$ has been reached), respectively. Equation (1) can be expressed into linear form through the following logarithmic transformation:

$$\ln(V) = \ln(V_{ff}) - \frac{1}{2} \cdot \frac{D^2}{D_c^2}$$

or

$$V_1 = a + b \cdot D_1$$  \hspace{1cm} (2)

where $V_1$ is $\ln(V)$, $a$ is $\ln(V_{ff})$, $b$ is $\frac{1}{2} \cdot \frac{1}{D_c^2}$ and $D_1$ is $D^2$. Starting from the above equation, by means of the fundamental relation between flow $Q$, density $D$ and speed $V$, $Q = D \cdot V$, $Q$ was obtained as follows:

$$Q = V \cdot \left[ \ln \left( \frac{D}{D_c} \right) \cdot 0.5 \cdot \frac{D^2}{D_c^2} \right]$$  \hspace{1cm} (3)

$$Q = V_{ff} \cdot D \cdot \exp \left[ -0.5 \cdot \left( \frac{D}{D_c} \right)^2 \right]$$  \hspace{1cm} (4)

Equations (3) and (4) allowed to develop the speed-flow function, $V=V(Q)$, and the flow-density function, $Q = Q(D)$. For the complete specification
of the relationships shown before the parameters $V_{FF}$ and $D_c$ were estimated. Thus, traffic flow models were calibrated for the roadway, the right lane and the passing lane at the observation sections by using the values of $Q$ [veh/h] and $V$ [km/h], and calculating the density $D$ [veh/km/lane] from $D=Q/V$; then, for each speed value $V$, corresponding to each lane and the roadway, the natural logarithm $\ln V$ was calculated to derive from each of the pairs $(D, V)$ the corresponding pair $(D^2, \ln V)$.

Basing on the scatter plot $(D^2, \ln V)$ corresponding to each observation section and according to equation (2), a least squares estimation was performed; then, the model calibration parameters ($V_{FF}$ and $D_c$) were estimated for all observation sections [19]. Thus, the relationships between the fundamental variables of traffic flow were specified for all observation sections by using equations (1), (3) and (4); estimations of capacity $C$ and speed $V_c$, corresponding to $C$, were then provided. For all cases, $R^2$ values corresponding to speed-flow and flow-density relationships were found to be higher than 0.7.

In order to calculate the speed-flow-density relationships for the roadway, the right lane and the passing lane for the A22 Freeway (Italy), the determinations of $V_{FF}$ and $D_c$ obtained at each of the three observation sections were averaged.

Using the $V_{FF}$ and $D_c$ values, the speed-flow-density relationships for the A22 Freeway were obtained (see Fig. 1). Table 1 shows the averaged values of the parameters ($V_{FF}$, $D_c$, $C$ and $V_c$) for relationships between the fundamental variables of traffic flow for the A22 Freeway.

Table 1 The parameters of speed-flow-density relationships for the A22 Freeway, Italy

<table>
<thead>
<tr>
<th>lane/lanes of travel</th>
<th>$V_{FF}$</th>
<th>$D_c$</th>
<th>$C$</th>
<th>$V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>right lane</td>
<td>106.95</td>
<td>23.65</td>
<td>1534</td>
<td>64.86</td>
</tr>
<tr>
<td>passing lane</td>
<td>130.28</td>
<td>25.09</td>
<td>1983</td>
<td>79.02</td>
</tr>
<tr>
<td>roadway</td>
<td>117.45</td>
<td>48.56</td>
<td>3459</td>
<td>71.23</td>
</tr>
</tbody>
</table>

A case study was then selected; in the following sections, indeed, empirical data considered for calibrating the microsimulation model are those corresponding to S. Michele observation section (southbound), chosen as case study; for this observation section, Table 2 shows the speed-flow-density relationships.

Fig. 1 The speed-flow-density relationships for the A22 Brenner Freeway, Italy [19]

### 4 Calibration Parameters

Microsimulation analysis involved application of Aimsun micro-simulator to replicate traffic flow on the transportation facility under examination.

As for any other currently existing microscopic traffic simulator, Aimsun is based on the family of car-following, lane changing and gap acceptance models to model the vehicle’s behavior [1].
Aimsun, indeed, uses input information (e.g. traffic volume, facility type, vehicle-driver characteristics, etc.) to move traffic using simple acceleration, gap acceptance, and lane change rules on a split second (time step) basis. Aimsun performs the car-following model in an evolved form compared to the empirical model proposed by [11][12]; in Aimsun the model parameters are determined by the influence of local parameters, depending on the type of driver, the road characteristics, the influence of vehicles driving in the adjacent lanes, etc. This model includes two components: i) acceleration, that is the intention of a vehicle to achieve a certain desired speed; ii) deceleration, reproducing the limitations imposed by the preceding vehicle, when trying to drive at the desired speed. The car-following model proposed by Gipps [11] considers only the vehicle and its leader, whereas its implementation in Aimsun also includes the influence of the adjacent lanes. Briefly when a vehicle is driving along a section, the influence that a certain number of vehicles driving slower in the adjacent lane may have on the vehicle, is considered. Thus the model determines a new maximum desired speed of a vehicle in the section, which will be used in the car-following model, considering the mean speed of vehicles driving downstream of the vehicle in the adjacent slower lane and allowing a maximum difference of speed [1].

Lane changing model in Aimsun can also be considered an evolution of the lane changing model proposed by [12]. Thus, the lane change is modeled as a decision process which examines both the desirability of a lane change and the benefits of a lane change resulting from the attainment of the desired speed, when the leading vehicle is slower, and the feasibility conditions for a lane change depending on the position of the vehicle in the road network. Car following and lane-changing model parameters for freeways are widely discussed by [1].

It is worthwhile to note that the Aimsun output processor comes with a set of user-adjustable parameters for calibrating the model to local conditions through minimizing the difference between the empirical and the simulated values of the variables of interest. In order to find the set of values for the model parameters best reproducing local traffic conditions at the A22 Brenner Freeway, Italy, the default values for the model parameters were used in trial simulation runs for testing possible coding errors.

Since the default parameter values did not allow to reproduce properly the current features of traffic flow, as it was noted by comparing empirical and simulated data, the fine tuning process required the iterative changing of some parameters and simulation replications until the best match between the simulated pairs of speed and density and the corresponding pairs on field observed was obtained. The use of Aimsun default parameter values, indeed, gave unrealistic simulation results compared to empirical data; thus some parameters were changed, basing on engineering knowledge and best practices.

Among the adjusted parameters, the following ones were included: i) the minimum headway, representing the time in seconds between the leader and the follower vehicle; ii) the reaction time, or the time in seconds it takes a driver to react to speed changes in the preceding vehicle; iii) the minimum distance between vehicles or the distance, in meters, that a vehicle keeps between itself and the preceding vehicle when stopped.

Having explored different combinations of values for the above mentioned parameters, the minimum headway parameter equal to 1.70 s was set instead of the default value of 2.10 s, whereas a value of 0.8 s was set for the reaction time parameter instead of the default value of 0.7 s; for the minimum distance between vehicles a value of 1 m was selected instead of the default value of 1.10 m. All the default/used parameter values used in the calibration process, together with their combinations as explored in this research, are shown in Table 3.

The calibration process also included adjustments for the desired speeds, namely the maximum speed that a certain type of vehicle can travel at any point in the network.

Table 2 The speed-flow-density relationships for the S. Michele observation section [19]

<table>
<thead>
<tr>
<th>relationship</th>
<th>roadway</th>
<th>speed-density</th>
<th>flow-density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$V = 118.20 \cdot \exp \left[-0.5 \frac{D}{48.35}\right]$</td>
<td>$Q = 105.00 \cdot D \cdot \exp \left[-0.5 \frac{D}{24.36}\right]$</td>
</tr>
<tr>
<td>right lane</td>
<td></td>
<td>$V = 105.00 \cdot \exp \left[-0.5 \frac{D}{24.36}\right]$</td>
<td>$Q = 105.00 \cdot D \cdot \exp \left[-0.5 \frac{D}{24.36}\right]$</td>
</tr>
<tr>
<td>passing lane</td>
<td></td>
<td>$V = 131.50 \cdot \exp \left[-0.5 \frac{D}{24.67}\right]$</td>
<td>$Q = 105.00 \cdot D \cdot \exp \left[-0.5 \frac{D}{24.36}\right]$</td>
</tr>
</tbody>
</table>
Table 3 Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
<th>Used</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum headway [s]</td>
<td>2.10</td>
<td>1.70</td>
<td>1.70</td>
</tr>
<tr>
<td>minimum distance between vehicles [m]</td>
<td>1.10</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>reaction time [s]</td>
<td>0.70</td>
<td>0.80</td>
<td>0.70</td>
</tr>
</tbody>
</table>

On this regard, Aimsun introduces a car vehicle type having as default values a mean desired speed of 110 km/h and a deviation of 10 km/h; the desired speed for the car vehicle type is sampled from a truncated Normal distribution (110, 10).

Consistently to the observed data on the A22 Freeway and according to what introduced by [28], the desired speed values on right lane were assumed lower than the values in the passing lane.

Trial runs allowed to observe that the desired speed was sensitive to flow rate, decreasing as flow rate values significantly increased. Thus, adjustments for the mean desired speeds (used here as a proxy variable) were differentiated by traffic demand values, for each lane and the roadway as shown in Table 4.

Table 4 Adjustments for the desired speed

<table>
<thead>
<tr>
<th>flow rate [pcu/h]</th>
<th>desired speed, mean [km/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>right lane</td>
</tr>
<tr>
<td>&lt;1500</td>
<td>110</td>
</tr>
<tr>
<td>2000</td>
<td>100</td>
</tr>
<tr>
<td>2500</td>
<td>95</td>
</tr>
<tr>
<td>&gt;3000</td>
<td>90</td>
</tr>
</tbody>
</table>

A short (2–km long) segment on the S. Michele observation section (southbound), having the cross section of A22 Freeway and a grade of 0.09%, was used in the simulation process; this length was chosen so that all vehicles entering the network were able to reach the end of the link. No traffic entered and exited in the middle was considered.

Thus, for the freeway segment under examination, 10 simulation replications were performed for 7 different values of traffic flow, increasing from 500 to 3500 veh/h with step 500 during a time interval of 4 hours; the values of traffic variables generated during the first half-hour of warm up were excluded, since they were considered corresponding to a motion condition not fully operational, and therefore they were considered unreliable.

Only cars were simulated, choosing them among cars proposed by Aimsun. For traffic generation, Aimsun micro-simulator proposes different headway models which may be selected by users as interval distributions; the exponential distribution is the default distribution and it was chosen to model time intervals between two consecutive arrivals of vehicles.

Detectors were placed at exactly the same locations as detectors in the field along the test freeway segment built in Aimsun. Speed-density diagrams were developed as shown in Fig. 2, where the plots of empirical and simulated data for S. Michele section (southbound roadway, right lane and passing lane) are represented; moreover, for every graph shown in Fig. 2 the corresponding speed-density relationship of Table 2 is also given. The \( InV-D^2 \) regression lines for observed and simulated data for S. Michele section (southbound) are introduced in the next section, together with the issues on implementing the methodology for calibrating the traffic microsimulation model.

As exploratory survey aimed at improving the interpretation of the empirical data, a two-regime linear model was used for the right lane only at S. Michele section, southbound (see Fig. 3); for the section here examined, indeed, May’s model did not emulate well the empirical values of high speed and low density. Fig. 3 shows simulated data obtained with Aimsun, using the calibration parameters in Table 3; adjustments for the mean desired speeds were also differentiated by traffic demand values as follows:

- for 500 [pcu/h], the values 140 km/h was assumed;
- for 1000 and 1500 [pcu/h], the values 130 km/h and 120 km/h were assumed respectively;
- for 2000 and 2500 [pcu/h], the value 90 km/h was assumed, whereas the value 80 km/h was assumed for 3000 [pcu/h].

Observing the graphs in Fig. 3, it is clear that two-regime model has improved the empirical data fitting, but, on the contrary, it did not match, just as well, the simulation outputs. For this reason and benefit of homogeneity, May’s model was used again for the right lane, the passing lane, and the roadway.

According to [1] the typical situation in which only aggregated values are available (i.e. flow and speed counts at detection stations aggregated to the hour), it can be useful to use joint measures that provide an overall view.
In this situation the GEH index, widely used in the case of microscopic simulation models, was calculated as an indicative criterion for acceptance (or otherwise rejection) of the model [1]. The GEH statistic calculates the index for each counting station as follows:

\[
GEH_i = \sqrt{\frac{2(x_i - y_i)^2}{x_i + y_i}}
\]  

(5)

where:

\(x_i\) = the \(i\)th simulated speed;

\(y_i\) = the \(i\)th observed speed.

For comparison purposes, each observed speed value was calculated from the speed-density equations in Table 2, as specified for the roadway, the right lane and the passing lane, by using the simulated values of density. The index is usually interpreted in the following terms (see e.g. [1]): if the deviation of the simulated values with respect to the measurement is smaller than 5% in at least 85% of the cases, then the model is accepted. The fact that for the three case in Fig. 2 (i.e the roadway, the right lane and the passing lane), each \(GEH_i\) resulted less than 5 (and equal to 1) would lead to the conclusion that the model could be accepted as significantly close to the reality.

4 Hypothesis Test Formulation

A statistical approach based on observed and simulated speed-density relationships was applied in the calibration process to measure the closeness between empirical data and simulation outputs. The comparison established between the lnV-D^2 linear regressions for all simulated (speed/density) values and the corresponding linear regressions for the empirical data allowed to evaluate the quality of the calibration of traffic microsimulation model. Thus, a statistical approach including hypothesis testing using \(t\)-test and confidence intervals was used as described briefly below.

Suppose we observe, for \(i = 1,\ldots,n\), the measured variable \(Y_i (\ln V_i)\) corresponding to certain values of the input variables \(x_i (D_i^2)\) and we want to use them with the objective of estimating the regression parameters (\(\alpha\) and \(\beta\)) in a simple linear regression model. If \(A\) and \(B\) are the estimators that we are searching for, then \((A + Bx_i)\) is the estimator of the response variable corresponding to the input variable \(x_i\).

In order to get the distribution of the estimators \(A\) and \(B\), additional assumptions necessarily have to be made. As starting point the estimators \(A\) and \(B\) are usually assumed to be independent, normally distributed with zero mean and constant variance \(\sigma^2\). Consequently, if for \(i = 1, 2, \ldots, n\), the measured variable \(Y_i\) is the response given to the input variable
xi, we will assume that \( Y_1, Y_2, \ldots, Y_n \) are independent and \( Y_i \sim N(\alpha + \beta x_i, \sigma^2) \).

Starting from the above proposition, a statistical test and confidence intervals for the regression parameter \( \beta \) were constructed. As it is well known the hypothesis to be tested is that \( \beta = 0 \) (the response does not depends on the input variable, i.e. there is no correlation between the two variables).

It can be demonstrated that the statistic for the test here considered has a \( t \) distribution with \( n-2 \) degrees of freedom:

\[
\left( \frac{(n-2)S_{xx}}{SS_R} \right)^{1/2} B \sim t_{n-2}
\]

where \( S_{xx} = \sum x_i^2 - n \bar{x}^2 \) and \( SS_R \) is the sum of squared residuals. So, to test \( H_0 : \beta = 0 \) against \( H_1 : \beta \neq 0 \), at the \( \gamma \) significance level, we have to:

- reject \( H_0 \) if \( \left( \frac{(n-2)S_{xx}}{SS_R} \right)^{1/2} |B| > t_{\gamma, n-2} \)
- accept \( H_0 \) otherwise.

Thus an interval containing \( \beta \), at the \( 1-\gamma \) confidence level, is the following:

\[
\left( B - t_{\gamma, n-2} \sqrt{\frac{SS_R}{(n-2)S_{xx}}}, B + t_{\gamma, n-2} \sqrt{\frac{SS_R}{(n-2)S_{xx}}} \right)
\]

The determination of the confidence intervals and statistical tests for the regression parameter \( \alpha \) was obtained as for \( \beta \). So, the confidence interval at the \( 1-\gamma \) level is given by:

\[
A \pm t_{\gamma, n-2} \sqrt{\frac{SS_R \sum x_i^2}{n(n-2)S_{xx}}}
\]

Table 5 shows the coefficients estimates and goodness-of-fit for \( \ln V - D^2 \) regression lines (observed and simulated) for S. Michele section (southbound), for the roadway, the right lane and the passing lane; on each set of data, statistical inference on the regression parameters (intercept and slope) was performed by means of a \( t \)-test at the significance level of 5%. GEH index was calculated again for each pair \((V_{\text{obs}}, V_{\text{sim}})\) obtained from the regressions in Table 5; only undersaturated conditions \((D < D_c)\) were considered. In all the cases we obtained \( \text{GEH} = 100\% \). A \( \chi^2 \) test was also performed considering the percentage of occurrence of a class of speed both for the field case and for the simulated one in Table 5. In all the cases (i.e. the roadway, the right-lane and the passing lane) the test showed that the two populations (observed and simulated) did not differ significantly at the 0.05 level:

- roadway (50 degree-of-freedom)
  \[ \chi^2 = 11.48 < \chi^2_{0.95} = 67.5 \]
- right lane (25 degree-of-freedom)
  \[ \chi^2 = 0.93 < \chi^2_{0.95} = 37.7 \]
- passing lane (25 degree-of-freedom)
  \[ \chi^2 = 16.33 < \chi^2_{0.95} = 37.7 \]

Table 5 Coefficients estimates and goodness-of-fit for S. Michele section – Southbound.

<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>( t )-pr.</th>
<th>t (t pr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>4.7726</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.0002139</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>4.7972</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.00024417</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>4.6540</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.00084291</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>4.6744</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.00088134</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>4.8789</td>
<td>(&lt;.001)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.0007380</td>
<td>(&lt;.001)</td>
</tr>
</tbody>
</table>

Note that: Constant is \( \beta_0 \); \( D^2 \) is \( \beta_1 \).
Comparing the two regression lines (observed and simulated), including confidence areas, a significant overlapping of the regression curves can be seen as shown in Fig. 4.

It is worthwhile to note that the simulated data fell almost entirely within the confidence band of the regression line fitted to the observed data. Thus the microsimulation model was able to reproduce the real phenomenon of traffic flow within a wide enough range of operations, from the free flow conditions until almost to the critical density. At the same time we argue that the methodology has showed that, if only one regime of traffic flow (for example, the congested flow conditions) had been considered, we would not have had any insurance on the ability of the model to reproduce, just as well, the real operations at different regimes of traffic flow. It should be emphasized the exploratory nature of the analysis carried out in this study in which, among all models analyzed, only the single-regime model was considered having the accuracy and consistency to interpret the experimental data which covered the three traffic regions (i.e., free-flow, congested, and queue discharge), and to represent the operating conditions for each lane and the entire roadway.

Nevertheless, in order to improve the calibration process, one can hypothesize to model separately the inside lane and the outside lane and further survey should be conducted to relax the single-regime assumption.

### 4 Discussion and Conclusions

In this paper a methodology using speed-density relationships in the microsimulation calibration process is described. Statistical analysis technique of pattern recognition was used to evaluate the match of speed-density relationships from field and simulation. Traffic patterns were implemented developing relationships between the variables of traffic flow for empirical and simulated data: for the former we referred to traffic data observed at A22 Freeway (Italy); for the latter, Aimsun software was applied to a test freeway segment in uncongested traffic conditions for a fleet of cars only. Differently from the methodologies referred by technical literature on this topic, in this paper the measure of the closeness between empirical data and simulation outputs was achieved through a statistical approach which included hypothesis testing and confidence intervals.

Encouraging results were obtained from the comparison of the observed and simulated data:

![Speed-density graphs with plots of field and simulated data for S. Michele section (southbound)](image)

- a) field: \(y = -2.1390 \times 10^{-4}x + 4.7726\) (\(R^2 = 0.9067\))
  sim.: \(y = -2.4417 \times 10^{-4}x + 4.7972\) (\(R^2 = 0.9204\))

- b) field: \(y = -8.4291 \times 10^{-4}x + 4.6540\) (\(R^2 = 0.8762\))
  sim.: \(y = -8.8134 \times 10^{-4}x + 4.6744\) (\(R^2 = 0.9655\))

- c) field: \(y = -8.2173 \times 10^{-4}x + 4.8789\) (\(R^2 = 0.8993\))
  sim.: \(y = -7.3800 \times 10^{-4}x + 4.8819\) (\(R^2 = 0.9395\))

Indeed, a substantial overlapping of the regression curves was obtained and the simulated data fell
almost entirely within the confidence band of the regression line fitted to the empirical data.

Thus we stated that the microsimulation model was able to reproduce the real phenomenon of traffic flow within a wide enough range of operations (from free flow conditions until almost to the critical density). Conversely, the proposed methodology showed that, if only one regime of traffic flow (free flow conditions or, congested flow conditions) had been considered, we would not have had any insurance on the ability of the model to reproduce, just as well, the real operations at different regimes of traffic flow. At last, the deepening of the model calibration as presented in this paper has led the authors to develop some considerations of general order as summarized in the following: i) first, although the results of the calibration process may seem satisfactory, the analyst does not have any guarantee on his/her work: he/she may have changed (or, in the extreme, forced) some parameters, but may have neglected other parameters even more important. However, it must be said that this risk can be contained when information for the calibration process is derived from the speed-flow, speed-density, or flow-density graphs, since a higher number of parameters can be submitted to the calibration process, resulting in a better fine-tuned simulation model. Moreover, the above relationships provide information about the free-flow, congested, and queue discharge regions, which cannot be gained from a single numerical value or a distribution of capacities; ii) second, although microsimulation model gave us data that, on the whole, belong to the population of the observed data, some doubts could relate to what was developed for the right lane. One single model which fits to empirical data both for the right lane and the passing lane, as well as for the entire roadway, does not always represent the best choice. The empirical observations have gradually led to consider that modeling the speed-density relationship (and the associated fundamental diagram) could be improved differentiating by each lane; for example, this can be done with regard to the capability of the model (single regime or multi regime) to fit empirical data reasonably well over the entire range of a traffic variable (i.e. flow, speed or density). The inability of single regime models to perform well over the entire range of density may prompt to think about fitting the data at intervals through multiple equations; iii) third, another question to be deepened relates to the traffic generation. Starting the simulation run, the system is empty; based on the input volumes and an assumed headway distribution, vehicles enter the network from centroids. Although in microsimulation one may choose among different headway models (exponential, uniform, normal, constant, etc.), the default distribution is usually preferred. However, the choice of the distribution should not be so automatic, but it should depend on how much complexity is desired to interpret traffic behavior . Indeed, Poisson distribution for vehicle counts and negative exponential distribution for time headways are only applicable when no interaction between vehicles occurs, thus enabling them to move at random (i.e. traffic flows are light). As traffic becomes heavier so that interaction between vehicles increases, vehicles are restricted in their driving freedom; moreover, the exponential distribution provides nonzero probabilities even for very small values of headways. In order to improve the capability of microsimulation models to replicate the real traffic phenomenon, distributions different from the exponential should have to be used; this is due to the poor agreement between the frequencies of observed headways and the frequencies predicted by the negative exponential distribution (as well as theoretical considerations precluding very short headways). It follows that in microsimulation the use of one of the default headway distributions can produce inappropriate choices in traffic generation, and a user-defined program can be required to feed the network with vehicles not without further computational effort and time.

Acknowledgements
Authors wish to thank Dr. Eng. Walter Pardatscher, CEO for the A22 Brenner Freeway (Italy) for the constant availability shown during the development of this research.

References:


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