

Development and Investigation of the Moebius Strip in Tensioned Membrane Structures

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Abstract: - Tensioned membrane structure in the form of Moebius strip can be considered as a sustainable development for the green environment and technology, it also can be used to support the effectiveness used of energy and the structure. Sustainable development of tensioned membrane green structure in the form of Moebius strip has been studied. Shape-finding of tensioned membrane surface bordered by Moebius strip is investigated. Moebius strip has the mathematical property of being non-orientable and with only one side and only one boundary component. In this research, the possibility of adopting the form of Moebius strip as surface shape for tensioned membrane structure has been scrutinised. The combination of shape and internal forces for the purpose of stiffness and strength is an important feature of tensioned membrane surface. For this purpose, shape-finding needs to be carried out. Nonlinear analysis method is used for shape-finding analysis in this research. Development and pattern of prestress in the resulting tensioned membrane surface is also studied. The development of Moebius strip problem has been able to overcome. Shape-finding has been found to converge for Moebius strip. It provides an alternative choice for engineer to consider the Moebius strip, radius over width = 0.7, 1.2, 1.7, 2.2 and 2.7 applied in tensioned membrane green structure.

Key-Words: - Shape-finding, Moebius strip, tensioned membrane, non-orientable, nonlinear analysis method and prestress.

1 Introduction

Tensioned Membrane Structures (TMS) include a wide variety of systems that are distinguished by their reliance upon tensile only members to support load. TMS have been employed throughout recorded history as in rope bridges and tents. However, large permanent tension structures were generally a 19th century development in bridges and a 20th century development in buildings. Tensioned membrane structure is the best alternative to cover area with low cost. One of the greatest benefit is their translucent. Woven fabric coated with a polymeric resin allows a light transmission value of around 10%. This provides a very comfortable level of illumination compared to the full brightness of outside the structure. [1] have stated large energy consumption of energy heating for residential and commercial use in the world. [2], [3], [4], [5] and [6] have stated energy saving. The energy saving problem can be overcome by application of TMS.

One of the most exciting shapes that captured the focus of the mathematics is Moebius strip surface. Moebius strip surface is a surface with only one side, one boundary component and can be made by twisting a strip by 360° and joining both ends of the

strip. It was discovered by two German mathematicians name August Ferdinand Moebius and Johann Benedict Listing in 1858.

Moebius strip has the mathematical property that is non-orientable surface. [7] have mentioned that Moebius strip surface has great potential be as an architecture form. [8] have carried out the possibility implement Moebius strip surface in a shell structure. [9] and [10] have proposed nonlinear analysis method for shape-finding of tensioned membrane structures in the form of Moebius strip. Applicability of the computational strategies proposed has been verified by shape-finding carried out models of tensioned membrane structures in the one of the form of Moebius strip (with $R/W > 1.3699$) have shown good agreement with mathematically defined surfaces. Shape-finding of Moebius strip TMS models by assuming an initial assumed shape with the opening at the center is not able to yield converged result for value $R/W > 1.3699$. The shape obtained after shape-finding are found to be different from the form of typical Moebius strip surface with opening at the center. The opening at the center of Moebius strip is found

to be non-existent. Shape of Moebius strip (without opening) has been verified through soap film model with $R/W = 1$. Shape-finding has been found to converge for Moebius strip TMS model (with $R/W \leq 1.3699$) with initial assumed shape specified to follow the topology without opening which has been observed in experiment.

In this research, only shape-finding using nonlinear analysis method of Moebius strip TMS with $R/W = 0.7, 1.2, 1.7, 2.2$ and 2.7 has been carried out. Shape-finding is the step to determine the initial equilibrium shape that satisfies the prescribed prestress system and boundary condition.

2 Development of Moebius Strip

For this research, the software [11] has been used for the purpose of model generation. Aspect of modeling of surface of Moebius strip and form as well prestress pattern of the resulting TMS through shape-finding using nonlinear analysis method are studied. Moebius strip as shown in Fig.1 can be represented parametrically by the following set of equations [12]:

$$X = \left(R + S \cos \frac{\tau}{2} \right) \cos \tau$$

$$Y = \left(R + S \cos \frac{\tau}{2} \right) \sin \tau$$

$$Z = S \sin \left(\frac{\tau}{2} \right)$$

for $\{S: -W, W\}$ and $\{\tau: 0, 2\pi\}$ (1)

The problem with this development model is single continuous surface involve only one curve along the band in a Cartesian plane. The problem is impossible to join the band border and to overcome this problem, [9] has proposed by applying discontinuous surface in modeling with small gap across the width of the strip. It was therefore decided to settle for a sufficiently close approximation of this shape, the solution adopted

was to introduce a small gap (1°) across the width of the strip as depicted in Fig. 1. In this way, the surface becomes discontinuous over the gap.

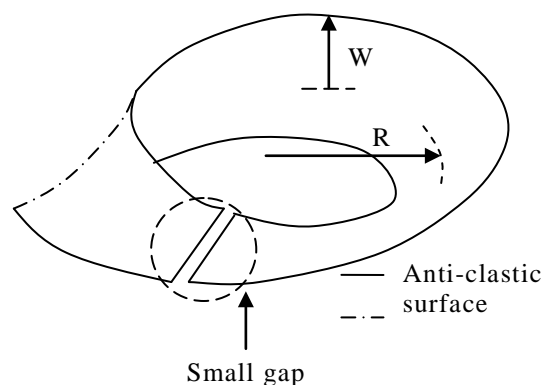


Fig.1: A Moebius strip with midcircle radius R and half-width W

3 Shape-Finding using Nonlinear Analysis Method

A nonlinear finite element analysis program by [9] for the analysis of tensioned membrane structures has been used in this study. The procedure adopted is based on the work by [9]. 3-node plane stress element has been used as element to model the surface of TMS. All x, y and z translation of nodes lying along the boundary edge of the Moebius strip have been restrained. Similarly, all nodes lying on either side of the gap introduced were also restrained from translating in all directions. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero.

Two stages of analysis were involved in the procedures of shape-finding analysis in one cycle proposed by [9]. First stage (denoted as SF1) is analysis which starts with an initial assumed shape in order to obtain an updated shape for initial equilibrium surface. The initial assumed shape can be obtained from any pre-processing software and reference [11] is chosen for this research. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned membrane properties, E with very small values are used. Both warp and fill tensioned membrane stresses are kept constant. In the second stage of (SS1), the actual values of tensioned membrane properties are used. Resulting warp and fill tensioned membrane stresses

are checked at the end of the analysis against prescribed tensioned membrane stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be < 0.01 . The resultant shape at the end of iterative step n (SS_n) is considered to be in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step. As a first shape for the start of shape-finding procedure adopted in this research, initial assumed shape is needed. For the generation of such initial assumed shape, knowledge of the requirement of anti-clastic nature of TMS is used. The incorporation of anti-clastic feature into the model will help to produce a better initial assumed shape.

Different combination parameters radius, R and haft-width, W for Moebius strip have been studied. The determination of R and W has been carried out as follow: $W = 10$ and $R = 7, 12, 17, 22$ and 27 . Moebius strip $R/W = 0.7, 1.2, 1.7, 2.2$ and 2.7 has been studied. The opening of the opening at the center becomes larger when R has been increased.

Two different variables $R/W = 0.7$ and $R/W = 1.2$ which is $R/W \leq 1.3699$ for Moebius strip have been chosen for the first discussion. Total number of nodes and elements used in the model are 343 and 576, respectively. Fig. 2 shows the initial assumed shape for Moebius strip $R/W = 0.7$. The surface models $R/W = 0.7$ has been found intersect with each other after shape-finding as shown in Fig. 3. The convergence of $R/W = 0.7$ not able to achieve due to intersection of surface.

Another set of Moebius strip $R/W = 0.7$ has been carried out by assuming the initial assumed shape similar to Fig. 3. Total number of nodes and elements used in the model are 154 and 270, respectively. Fig. 4 shows the convergence of the Moebius strip $R/W = 0.7$. The convergence curve in Fig. 5 shows the total warp and fills deviation < 0.01 . Similar case obtained also for the Moebius strip $R/W = 1.2$.

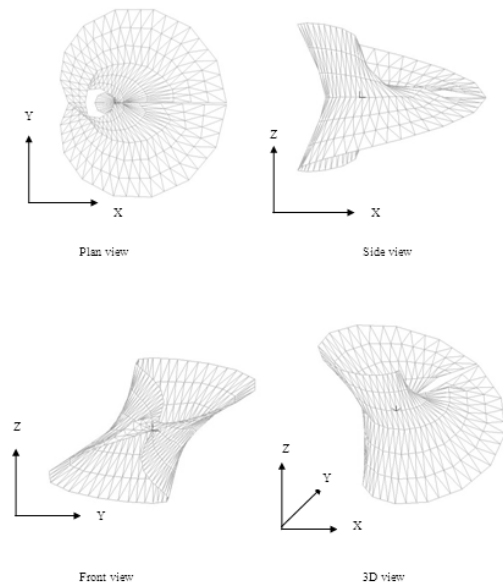


Fig. 2: Initial assumed shape for Moebius strip ($R/W = 0.7$)

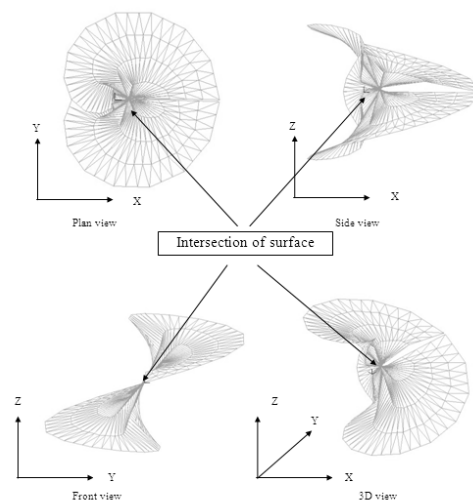


Fig. 3: Models of TMS intersect with each other in the form of Moebius strip after shape-finding ($R/W = 0.7$)

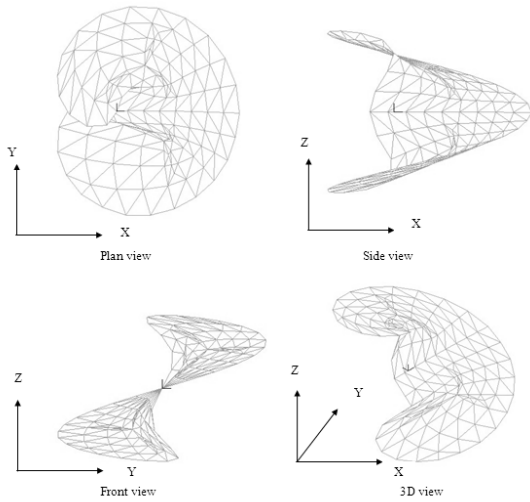


Fig. 4: TMS model in the form Moebius strip (without opening) after shape-finding (R/W = 0.7)

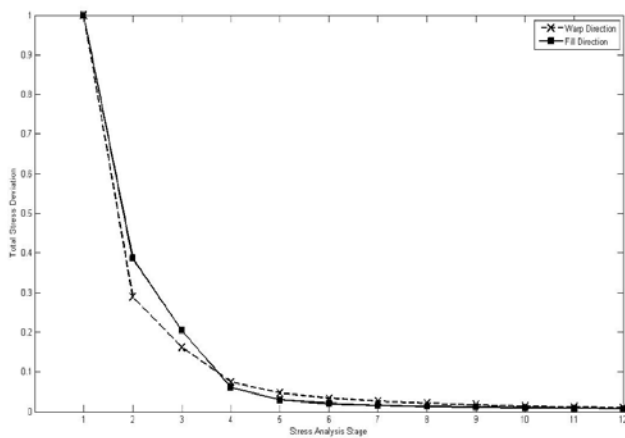


Fig. 5: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Moebius strip TMS Model (R/W = 0.7)

Moebius strip R/W = 1.7, 2.2 and 2.7 have been chosen for the following discussion. The surfaces of a Moebius strip also generated from (1). Total number of nodes and elements used in the model are 343 and 576, respectively. Fig. 6 shows the initial assumed shape for Moebius strip (R/W = 1.7). Fig. 7 shows the convergence of the Moebius strip R/W = 1.7. The convergence curve in Fig. 8 shows the total warp and fills deviation < 0.01. The similarity

convergence from the mathematical shape can be clearly seen in Fig. 7. Similar case obtained also for the Moebius strip R/W = 2.2 and 2.7.

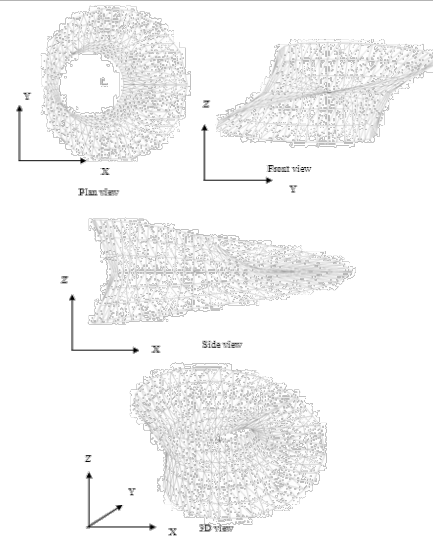


Fig. 6: Initial assumed shape for Moebius strip (R/W = 1.7)

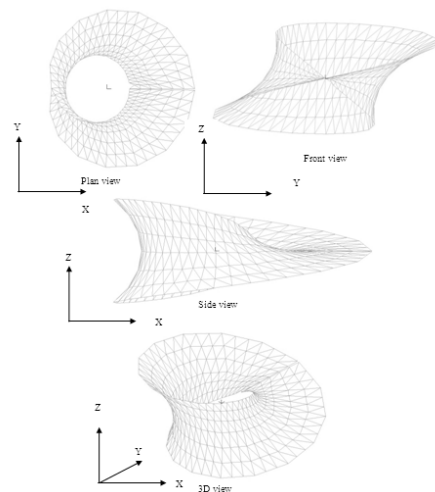


Fig. 7: TMS model in the form Moebius strip after shape-finding (R/W = 1.7)

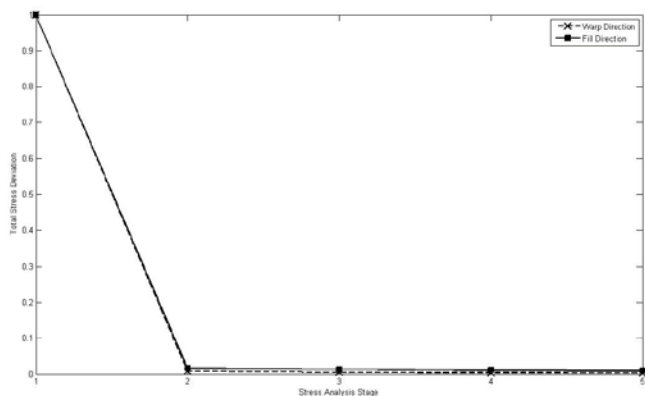


Fig. 8: Variation of total stress deviation in warp and fill direction versus stress analysis stage for Moebius Strip TMS Model (R/W = 1.7)

4 Conclusion

Shape-finding on TMS with surface in the form of Moebius strip R/W= 0.7, 1.2, 1.7, 2.2 and 2.7 has been carried out successfully using the procedure adopted which is based on nonlinear analysis method. The results show that TMS in the form of Moebius strip with proper selection of surface parameter is a structurally viable surface form to be considered. Structural engineer can apply the TMS in the form of Moebius strip to build the large space area.

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