Error Analysis in Unbiased FIR Filtering with Time-Stamped Discretely Delayed and Missing Data

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**Abstract:* This investigation is motivated by communication effects such as delays and information loss in control loops on data travelling along wireless channels. The state-space model with k-step-lags in observations is transformed to have no latency and expanded on a finite horizon of N most recent data points. An analysis of error is provided for a novel unbiased finite impulse response (UFIR) filtering algorithm developed for discrete-time state-space models with time-stamped discretely delayed and missing data. A comparative analysis with the standard Kalman filter (KF) and robust H_∞ filter is also provided. It is shown that the UFIR filter is more robust than the KF and H_∞ filter under the communication delays and missing data. The results are obtained by simulation and verified experimentally.

Key–Words: Delayed data, missing data, unbiased FIR filter, Kalman filter, H_∞ filter.

1 Introduction

In the last two decades, linear estimation has received much attention due to many possible applications of science and engineering. Advances of sensors technology have enabled binding the wireless systems with the estimation problem in monitoring and control system such as environmental monitoring, navigation and control of moving vehicle [1], etc. A common feature of such systems is the ability to cause significant communication delays and data loss due to a variety of physical reasons [2]. Latency, limited bandwidth, intermittence, failures in measurements, and accidental loss of some collected data are some reasons of this problem [3, 4].

In applications such as vehicle tracking, data are required to arrive at the destination in real time. If it does not happen, errors occur in control systems. Overall, it had been inferred [2, 5] that the time delay and missing data are often the main causes of instability and poor performance of systems. Mostly two kinds of approaches were developed to deal with signals experienced delays and missing data: 1) Kalman filter (KF), which minimizes the mean square error (MSE), and 2) H_∞ filter, which minimizes estimation errors for maximized error matrices. For the KF, noise is required to be white Gaussian and information about the initial values should be available, which is not always the case in practice. In other words, the KF is optimal for linear systems when it matches the system perfectly [1, 6, 7, 8, 9, 10]. In turn, the H_∞ filter is designed to have a robust performance to minimize errors with less information required than for the noise statistics. Applied to uncertain models, the H_∞ filtering estimate is provided by bounding the error matrices for admissible parameter perturbations and delays [5, 11]. With intensive external impacts, noise is neglected and the MSE is minimized under the worst performed case [4].

Another way to achieve better robustness is to provide estimation over most recent data [12] using finite impulse response (FIR) filters [13]. During decades, the FIR approach has been developed by many authors [14, 15, 16, 17, 18, 19, 20, 21]. Among the available solutions, the iterative unbiased FIR (UFIR) algorithm [22, 23], which ignores the noise statistics and initial values, is considered as most robust. This filter is blind on given horizons and bounded-input bounded-output (BIBO) stable. However, we do not find developments of this approach for observations with delayed and missing data.

In this paper, we develop the UFIR filter for measurements with discretely delayed and missing data and provide an analysis of error produced for this filter in a comparison with the KF and H_∞ filter.

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2 Problem Statement

To illustrate the problem, one may consider three typical scenarios of a process observed with delays and missing data at a receiver: 1) data may arrive with no delay, 2) delay may occur with missing data, and 3) missing data may occur with no delay. These scenarios can be represented in discrete-time state-space with equations

$$
\begin{aligned}
x_n &= Fx_{n-1} + w_n\,, &(1)\\
\tilde{y}_n &= HFx_{n-1}\,, &(2)\\
y_n &= \kappa_n H x_{n-k_n} + (1-\kappa_n)\tilde{y}_n + v_n\,, &(3)
\end{aligned}
$$

where n in the discrete time index corresponding to time t_n and sampling time $T = t_n - t_{n-1}$, $x_n \in \mathbb{R}^K$ is the state vector, $y_n \in \mathbb{R}^M$ is the observation vector, $F \in \mathbb{R}^{K\times K}$ is not singular, $H \in \mathbb{R}^{M\times K}$, κ_n is the missing data factor, and $k_n \geqslant 0$ is a discrete delay-step-lag. When data arrive successfully, a data sensor generates $\kappa_n = 1$ and $\kappa_n = 0$ otherwise. The initial state x_{n-1} is supposed to be known and the noise vectors, $w_n \in \mathbb{R}^K$ and $v_n \in \mathbb{R}^M$, are white Gaussian with known covariances, $Q = E\{w_n w_n^T\}$ and $R = E\{v_n v_n^T\}$, and the property $E\{w_n v_r^T\} = 0$ for all n and r .

The problem can be discussed as follows. Given model (1)–(3) with delayed and missing data, how can we apply the standard estimators? The answer can be found if to transform (1)–(3) to have no latency [24]. We do it by using the backward-in-time solutions and represent x_{n-k_n} via x_n as

$$
x_{n-k_n} = F^{-k_n}\left(x_n - \sum_{i=0}^{k_n-1} F^i w_{n-i}\right). \quad (4)
$$

The observation vector can be switching from y_n to $\tilde{y}_n$ if κ_n take the value of 1 or 0, respectively. For $\kappa_n = 1$, one can employ (4) and go from (3) to

$$
y_n = \bar{H}_n x_n + \bar{v}_n\,, \quad (5)
$$

where $\bar{v}_n = v_n - H\sum_{i=0}^{k_n-1} F^{-k_n+i} w_{n-i}$, $\bar{H}_n = HF^{-k_n}$, and the covariance of $\bar{v}_n$ is given by

$$
\bar{R}_n = E\{\bar{v}_n \bar{v}_n^T\} = R + \tilde{R}_n\,, \quad (6)
$$

where $\tilde{R}_n = \bar{H}_n \sum_{i=0}^{k_n-1} F^i Q F^{i^T} \bar{H}_n^T$. For $\bar{B}_n = \left[\, F^{-1}\; F^{-2}\; \ldots\; F^{-k_n} \,\right]$ and $w_{p_n,n} = \left[\, w_{p_n}^T\; w_{p_n+1}^T\; \ldots\; w_n^T \,\right]^T$, where $p_n = n - k_n + 1$ and $\bar{B}_n = 0$ and $w_{p_n,n} = 0$ when $k_n = 0$, we finally obtain

$$
\begin{aligned}
\bar{v}_n &= v_n - H\bar{B}_n w_{p_n,n}\,, &(7)\\
\bar{R}_n &= R + H\bar{B}_n \bar{Q}_n \bar{B}_n^T H^T\,, &(8)
\end{aligned}
$$

where $\bar{Q}_n = \mathrm{diag}[\, Q\; Q\; \ldots\; Q\,]$ has k_n diagonal components. Provided (5), any linear estimator can be applied , such as the KF and H_∞ filter.

3 UFIR Filter Design

UFIR filtering can be applied to data with $k_n \geqslant 0$ if to extend model (1) and (5) on a horizon $[m, n]$ of N points, from $m = n - N + 1$ to n. Referring to [23], the extended model becomes

$$
\begin{aligned}
x_{m,n} &= A_N x_m + B_N w_{m,n}\,, &(9)\\
y_{m,n} &= C_{m,n} x_m + G_{m,n} w_{m,n} + v_{m,n}\,, &(10)
\end{aligned}
$$

where the extended state vector $x_{m,n}$ and the extended observation vector $y_{m,n}$ and the extended matrices are $x_{m,n} = \left[\, x_m^T\, x_{m+1}^T\, \ldots\, x_n^T \,\right]^T$, $y_{m,n} = \left[\, y_m^T\, y_{m+1}^T\, \ldots\, y_n^T \,\right]^T$,

$$
A_N = \left[\, I\; F^T\; \ldots\; F^{N-1^T} \right]^T, \quad (11)
$$

$$
B_N = \begin{bmatrix} I & 0 & \ldots & 0 & 0 \\ F & I & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F^{N-2} & F^{N-3} & \ldots & I & 0 \\ F^{N-1} & F^{N-2} & \ldots & F & I \end{bmatrix}, \quad (12)
$$

$$
C_{m,n} = \begin{bmatrix} \bar{H}_m \\ \bar{H}_{m+1}F \\ \bar{H}_{m+1}F^2 \\ \vdots \\ \bar{H}_n F^{n-1} \end{bmatrix}, \quad (13)
$$

$$
G_{m,n} = \begin{bmatrix} \bar{H}_m & 0 & \ldots & 0 \\ \bar{H}_{m+1}F & \bar{H}_{m+1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{H}_n F^{N-1} & \bar{H}_n F^{N-2} & \ldots & \bar{H}_n \end{bmatrix}, \quad (14)
$$

$$
v_{m,n} = \begin{bmatrix} v_m - C\bar{B}_n w_{p_m,m} \\ v_{m+1} - C\bar{B}_n w_{p_{m+1},m+1} \\ \vdots \\ v_{n-1} - C\bar{B}_n w_{p_{n-1},n-1} \\ v_n - C\bar{B}_n w_{p_n,n} \end{bmatrix}. \quad (15)
$$

Below, we develop the UFIR filter for $k_n \geqslant 0$ in the batch and fast iterative forms.

3.1 Batch UFIR Filter

Most generally, the FIR estimate $\hat{x}_n \triangleq \hat{x}_{n|n}$ can be obtained at n in a batch form over data available on $[m, n]$ as [14]

$$\hat{x}_n = \mathcal{H}_{m,n} y_{m,n} \,, \tag{16}$$

where $\mathcal{H}_{m,n}$ is the *filter gain*. The estimate $\hat{x}_n$ will be unbiased if it satisfies the unbiasedness condition $E\{x_n\} = E\{\hat{x}_n\}$, where $E\{z\}$ means averaging of z. To find $\mathcal{H}_{m,n}$ obeying this condition, represent x_n on the horizon $[m, n]$ by the last row vector in (9) as $x_n = F^{N-1} x_m + B_N^{(N)} w_{m,n}$, where $B_N^{(N)} = [\, F^{N-1} \;\; F^{N-2} \;\; \ldots \; F \;\; I \,]$ is the last row vector in (12). Substituting $\hat{x}_n$ with (16), in which $y_{m,n}$ is given by (10), and averaging with $E\{w_{m,n}\} = 0$ and $E\{v_{m,n}\} = 0$ for known k_n-step-lag yields the unbiasedness constraint

$$I = \mathcal{H}_{m,n} \mathcal{C}_{m,n} \,, \tag{17}$$

where

$$\mathcal{C}_{m,n} = \begin{bmatrix} HF^{-N+1-k_m} \\ \vdots \\ HF^{-1-k_{n-1}} \\ HF^{-k_n} \end{bmatrix} , \tag{18}$$

which gives the UFIR filter gain $\mathcal{H}_{m,n} = (\mathcal{C}_{m,n}^T \mathcal{C}_{m,n})^{-1} \mathcal{C}_{m,n}^T$ [13] and the batch UFIR estimate (16) becomes

$$\hat{x}_n = G_n \mathcal{C}_{m,n}^T y_{m,n} \,, \tag{19}$$

where $y_{m,n}$ is a vector of real data and $G_n = (\mathcal{C}_{m,n}^T \mathcal{C}_{m,n})^{-1}$ is the *generalized noise power gain* (GNPG) [23].

Let us define errors in the UFIR filtering estimate $\hat{x}_n$ by $\epsilon_n = x_n - \hat{x}_n$, where $x_n = F^{N-1} x_m + B_N^{(N)} w_{m,n}$ is the last row vector in (9) and $B_N^{(N)} = [\, F^{N-1} \;\; F^{N-2} \;\; \ldots \; F \;\; I \,]$ is the last row vector in (12). The error covariance matrix

$$P_n = E\{\epsilon_n \epsilon_n^T\} \tag{20}$$

can then be represented as, if we employ $\hat{x}_n = \mathcal{H}_{m,n} y_{m,n}$ with $y_{m,n}$ given by (10),

$$\begin{aligned} P_n \;=\; & [B_N^{(N)} - \mathcal{H}_{m,n} G_{m,n}] \bar{Q}_N \\ & \times [B_{m,n}^{(N)} - H_{m,n} G_{m,n}]^T \\ & + H_{m,n} \bar{R}_N H_{m,n}^T \,, \end{aligned} \tag{21}$$

where $\bar{Q}_N = \mathrm{diag}[\, Q \; Q \; \ldots \; Q \,]$ and $\bar{R}_N = \mathrm{diag}[\, R \; R \; \ldots \; R \,]$ are square matrices with N diagonal elements.

3.1.1 Iterative UFIR Filtering Algorithm

Provided model (1) and (5), the iterative UFIR filtering algorithm [13] can be applied straightforwardly if to substitute H with $\bar{H}_n = HF^{-k_n}$. A pseudo code of this algorithm for $k_n \geqslant 0$ is listed as Algorithm 1, where MI when $\kappa_n = 0$ is organized by substituting lost data y_n with the predicted observations (lines 4–6), given the initial data on $[0, N-1]$.

Algorithm 1: Iterative UFIR Algorithm for Delayed and Missing Data

Data: y_n, k_n, N, κ_n
Result: $\hat{x}_n$
1 **begin**
2   **for** $n = N-1 : \infty$ **do**
3     $m = n - N + 1, \quad s = m + K - 1;$
4   **if** $\kappa_n = 0$ **then**
5       $y_n = HF\hat{x}_{n-1};$
6   **end if**
7     $\bar{H}_n = HF^{-k_n};$
8     $G_s = (\mathcal{C}_{m,s}^T \mathcal{C}_{m,s})^{-1};$
9     $\tilde{x}_s = G_s \mathcal{C}_{m,s}^T y_{m,s};$
10   **for** $l = s + 1 : n$ **do**
11       $G_l = [\bar{H}_l^T \bar{H}_l + (FG_{l-1}F^T)^{-1}]^{-1};$
12       $K_l^{\mathrm{UF}} = G_l \bar{H}_l^T;$
13       $\tilde{x}_l = F\tilde{x}_{l-1} + K_l^{\mathrm{UF}}(y_l - \bar{H}_l F \tilde{x}_{l-1});$
14   **end for**
15     $\hat{x}_n = \tilde{x}_n;$
16   **end for**
17 **end**
18 † Data y_0, y_1,..., y_{N-1} must be available.

4 Simulations

In this section, we will compare errors produced by the UFIR filter and the KF and H_∞ filter, which algorithms can be found in [28]. We will consider a two-state polynomial model (1)–(3) with $\kappa = 0$, $F = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}$, $H = [\,1 \; 0\,]$, $x_0 = [\,1 \; 1\,]^T$, and zero mean white Gaussian $w_n = [\,0 \; w_{2n}\,]^T$ and v_n with $Q = \sigma_{w2}^2 \begin{bmatrix} \tau^2/2 & \tau/2 \\ \tau/2 & 1 \end{bmatrix}$ and $R = \sigma_v^2$, where $\sigma_{w2} = 0.2/\mathrm{s}$ and $\sigma_v = 2$. Because Q and R are typically not known exactly, substitute them in the algorithms with $\alpha^2 Q$ and $\beta^2 R$, where α and β are positive-valued. We will compare errors via the increments of the bias corrections gains of the UFIR filter ΔK_n^{UF}, KF ΔK_n^{KF}, and H_∞ filter ΔK_n^{∞}.

4.1 Effect of Latency

Distance measurements are provided at 500 discrete points with $\tau = 0.1\,\mathrm{s}$ for different constant k-step-lags that gives $N_{\mathrm{opt}} = 29$ when $k = 0$. Figure 2 sketches the RMSEs $\sqrt{\mathrm{tr}\, P_n}$ of the UFIR filter, KF, and H_∞ filter as functions of k for two scenarios when 1) $\theta_{\mathrm{opt}}(k = 0)$ and $N_{\mathrm{opt}}(k = 0)$ are specified at $k = 0$ and then applied to arbitrary k and 2) $\theta_{\mathrm{opt}}(k)$ and $N_{\mathrm{opt}}(k)$ are specified and applied to each k individually. The KF is self-tuned to k and we use it as a benchmark.

In Fig. 1a, we sketch the results for $\alpha = \beta = 1$ and Fig. 1b for $\alpha = 0.5$ and $\beta = 2$, which lead to the following inferences. Like in the k-step predictors, errors grow in all filters with an increase in k-step-lag. Provided $\theta_{\mathrm{opt}}(k)$, and $N_{\mathrm{opt}}(k)$ for $\alpha = \beta = 1$ (Fig. 1a), the KF produces optimal estimates, which cannot be improved. A bit better performance of the H_∞ filter (dashed) is rather due to finite data. The UFIR filter (dashed) is inherently worst here.

Even for small $\alpha = 0.5$ and $\beta = 2$ (Fig. 1b), the KF becomes worst. The H_∞ filter improves the performance with $\theta_{\mathrm{opt}}(k)$ to be more accurate than the UFIR filter. By $\theta_{\mathrm{opt}}(k = 0)$ and $N_{\mathrm{opt}}(k = 0)$ applied to all k, the H_∞ filter rapidly diverges, while the UFIR estimate saves the performance. In a span of $0 \leqslant k \leqslant 20$, factor θ_{opt} rapidly reduces from 0.043 to 0.00023, while $N_{\mathrm{opt}} = 29$ holds for $0 \leqslant k \leqslant 12$ and increases to 33 at $k = 20$. The UFIR filter is thus more robust to errors in $N_{\mathrm{opt}}(k)$ than the H_∞ filter in θ_{opt}.

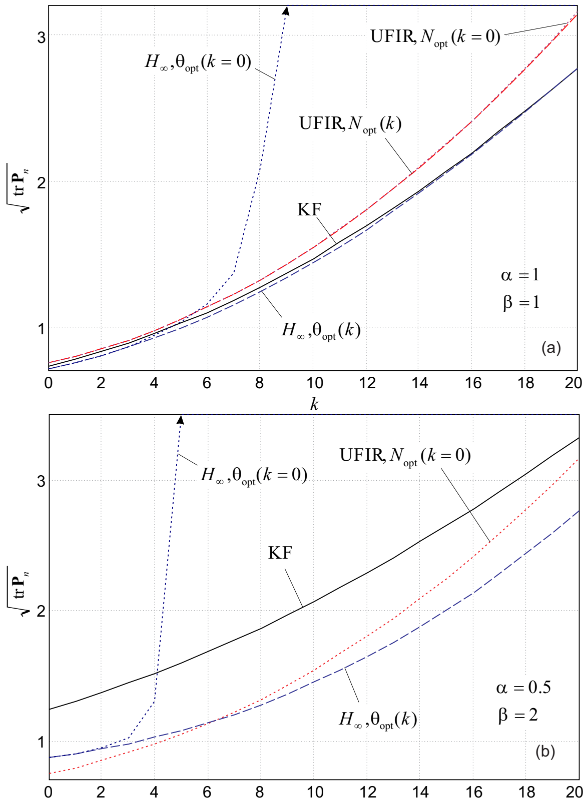

Figure 1: Effect of the k-step-lag on the RMSE $\sqrt{\mathrm{tr}\, P_n}$ of the UFIR filter, KF (solid), and H_∞ filter for $\alpha^2 Q = \alpha^2 \check{Q}$ and $\beta^2 R = \beta^2 \check{R}$: (a) $\alpha = \beta = 1$ and (b) $\alpha = 0.5$ and $\beta = 2$. Tuning factors θ and N_{opt} are specified by minimizing the MSE at 1) $k = 0$ (dotted) and 2) each k (dashed).

4.2 Inaccurate Error Matrices

Allow $Q = \check{Q}$ and $R = \check{R}$ and substitute in the algorithms as $\alpha^2 Q$ and $\beta^2 R$. Investigate effect, which α and β take on the estimation errors. Because $K^{\mathrm{UF}}(k)$ is $\{\alpha, \beta\}$-invariant, set $\Delta K^{\mathrm{UF}}(k) = 0$. Figure 2 sketches $\Delta K_1(k)$ related to the first state as function of k. For $\alpha = \beta = 1$, we have $\Delta K_1^{\mathrm{KF}} = \Delta K_1^{\mathrm{UF}} = 0$ and $\Delta K_1^{\infty} = 0$ with $\theta = 0$. If $\alpha < 1$ and/or $\beta > 1$, extra errors produced by the KF are compensated in the H_∞ filter (Fig. 2) with $\theta = 0.08$ at $k = 4$, $\theta = 0.0239$ at $k = 8$, $\theta = 0.00898$ at $k = 12$, etc. By $\theta(k)$, the H_∞ filter becomes as robust as the UFIR filter, which is $\{\alpha, \beta\}$-invariant. However, the H_∞ filter diverges with k when θ is set constant (Fig. 2). Note that the H_∞ filter is not able to reduce ΔK^∞ caused by $\alpha > 1$ and/or $\beta < 1$ with $\theta > 0$, because modelling errors are not maximized here, as required.

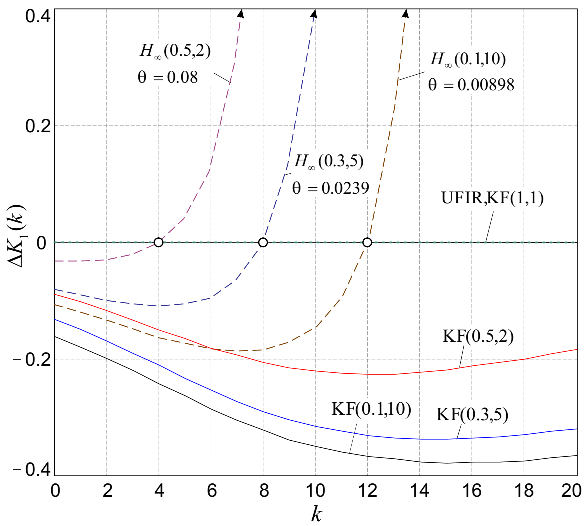

Figure 2: Effect of α and β on $\Delta K_1(k)$ for the KF(α, β) (solid) and $H_\infty(\alpha, \beta)$ filter (dashed). The UFIR filter is $\{\alpha, \beta\}$-invariant (dotted).

4.3 Model Errors

Consider the above model and, in addition to $\{\alpha, \beta\}$, introduce $\{\eta, \mu\}$. Investigate effect of $\{\alpha, \beta, \eta, \mu\}$ on

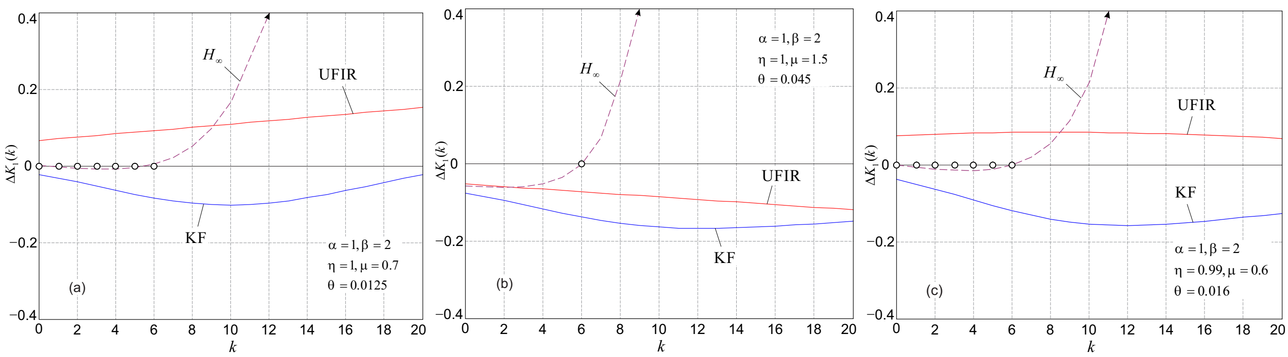

Figure 3: Typical effects of $\{\alpha, \beta, \eta, \mu\}$ and θ on $\Delta K_1(k)$: (a) near full error compensation in the H_∞ filter for $0 \leqslant k \leqslant 6$ and growing errors in the UFIR filter, (b) local error compensation in the H_∞ filter at $k = 6$ with the UFIR filter outperforming the KF, and (c) near full error compensation in the H_∞ filter for $0 \leqslant k \leqslant 6$ with a near constant error in the UFIR filter over k, which can be compensated. The H_∞ filter always diverges with large k.

the bias correction gains as sketched in Fig. 3 for the first state. In Fig. 3a, errors in the UFIR filter grow with k and the H_∞ filter operates "ideally" when $0 \leqslant k \leqslant 6$. In Fig. 3b, the UFIR filter outperforms the KF, while the H_∞ filter compensates errors only at $k = 6$. Finally, in Fig. 3c, the UFIR filter produces almost equal errors for any k, which can be compensated algorithmically.

Several other inferences can also be made. Provided θ_{opt} for some k, the H_∞ filter is not efficient with smaller k values and diverges otherwise. Function $\Delta K^{\text{UF}}(k)$ is more linear than others and can be corrected algorithmically. All filters are highly sensitive to η and the H_∞ filter is efficient only if $0.99 < \eta \leqslant 1$. Factor μ does not affect estimates as much as η and has a much wider allowed range around unity.

5 Conclusions

An analysis of errors provided in this paper for the UFIR filter developed for time-stamped k_n-step-lag discretely-delayed and missing data has demonstrated its higher robustness than in the KF and H_∞ filter. This can be explained by the fact that no information is required by the UFIR filter about noise and initial conditions and the filter becomes blind on given horizons. Therefore, the UFIR estimate is not affected by undesirable factors as much as in the KF and H_∞ filter at any k_n. Extensive simulation based on a two-state space model representing a tracking problem have confirmed higher robustness of the UFIR filter. In fact, as compared to the KF and H_∞ filter, the UFIR filter has demonstrated smaller errors caused by effects of latency, the invariance to errors in the noise statistics and error matrices, and lower sensitivity to model errors caused by mismodeling.

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