Two-Stage Kalman Filter for Estimation of Wind Speed and UAV States by using GPS, IMU and Air Data System

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Abstract: - In this study, an estimation algorithm based on a two-stage Kalman filter (TSKF) was developed for wind speed and Unmanned Aerial Vehicle (UAV) motion parameters. In the first stage, the wind speed estimation algorithm is developed by the usage of GPS measurements and dynamic pressure measurements. For this purpose, Extended Kalman Filter (EKF) was designed. The wind speed is estimated by the EKF using GPS and pitot tube measurements. Here, the wind speed components and pitot scale factor are considered as state vector variables. As there is no information of the state dynamics, dynamic equations are expressed in this case by random walk process. In the second stage, the estimation of the state parameters of the UAV dynamic model was made based on the Air Data System (ADS) and IMU measurements by using the Linear Kalman filter (LKF). The second stage filter uses ADS pitot tube, angle of attack and side sleep angle measurements, IMU attitude angle and velocity measurements, and the first stage EKF estimates of the wind speed values.

Key-Words: - Unmanned Aerial Vehicle, State estimation, Kalman filter, Wind speed, GPS, Pitot tube, Air Data System

1 Introduction

In this work, the wind field is estimated for both horizontal and vertical wind using GPS and pitot tube measurements. Estimation of the wind field is useful in UAV applications for various objectives such as dropping objects, target tracking, automatic control, trajectory optimization, and air traffic control [1]. There is some existing work in the area of wind estimation. A Kalman-like filter is derived in [2] for wind velocity estimation based on magnetic heading, true airspeed, and radar measurements. This filter is called the velocity bias filter for wind estimation.

In [3] the problem of aircraft wind velocity estimation is performed using the aircraft dynamic response rather than the wind triangle relationship. In this method, it is assumed that the mathematical model of the aircraft is perfectly known. The primary limitation of these types of wind estimation methods is the requirement of a known aircraft model. This can be very limiting for some type of aircrafts where the model has not yet been derived, or additional system uncertainties have been introduced. For the wind velocity estimation purpose, in [1] the GPS velocity components are properly related to the body-axis velocity components through the consideration of wind. It considers the wind triangle of airspeed, ground speed, and wind speed. Because of this adjustment, the estimated attitude states correspond to the actual orientation of the aircraft with respect to the fixed Earth. It was shown that this attitude estimation method is effective in distinguishing the roll and yaw angles of the aircraft, and providing a reasonable estimate of the local wind field.

A horizontal wind estimation method using the Extended Kalman Filter (EKF) is presented in [4]. In this work the horizontal wind speed is predicted using a random walk noise assumption, and then these states were regulated through the wind triangle comparison of ground speed from GPS and air speed.

The study [5] describes the extended Kalman filter based method to estimate the airflow angles and three-dimensional wind speed under constant wind condition. In addition, it can correct the scaling error of the airspeed of an aircraft. It uses the airspeed measurements, constant wind condition and the sideslip angle computed from GPS/INS navigation data and stability and control derivatives estimated from flight data. Estimated wind speed can also be used to reconstruct the GPS/INS navigation system by correcting the airspeed in the case of GPS failure.

In this study, an estimation algorithm based on a two-stage Kalman filter (TSKF) was developed for wind speed and UAV motion parameters. In the first stage, the wind speed estimation EKF is developed by the usage of GPS measurements and dynamic pressure measurements. In the second stage, the estimation of the state parameters of the UAV dynamic model was made based on the Air Data System (ADS) and IMU measurements by using the Linear Kalman filter (LKF).

2 Problem Formulations

This work considers the estimation of aircraft body-axis velocity components (u, v, w), Euler attitude angles (φ, θ, ψ), and three-axis wind velocity components (μx, μy, μz). This estimation is performed through the functional fusion of inertial measurement unit (IMU) measurements of three-axis accelerations (a_x, a_y, a_z) and angular rates (p, q, r), GPS velocity components (V_x, V_y, V_z), and ADS measurements from pitot tube (V_{pitot}) and wind vane measurements of angle of attack (α) and sideslip angle (β).

As the dynamics of the local wind field is unpredictable because of its random nature, the wind velocity dynamics are modeled using random walk process [4]

$$\begin{bmatrix} \dot{\mu}_N(t) \\ \dot{\mu}_E(t) \\ \dot{\mu}_D(t) \end{bmatrix} = w_p(t) \tag{1}$$

where w_p is the zero-mean Gaussian noise vector with the process noise covariance matrix Q_p.

The UAV state equation is given in a discrete-time linear state space format

$$x(k) = f\left[x(k-1), u(k-1), w(k-1)\right] \tag{2}$$

where f is the nonlinear discrete-time state transition function, u is the control input vector, which is composed of the control surface deflections and wind velocity, w is the system noise vector with covariance matrix Q.

In order to define the velocity components in the Earth-fixed frame, a transformation needs to be done using the matrix $A(\phi,\theta,\psi)$ from the body frame as [1]

$$\begin{bmatrix} V_N^{GPS} \\ V_E^{GPS} \\ V_D^{GPS} \end{bmatrix} = A(\phi,\theta,\psi)\begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} \mu_N \\ \mu_E \\ \mu_D \end{bmatrix} \tag{3}$$

A simple output equation can be defined using the x-body axis airspeed measurements of the pitot tube, which is mounted on the aircraft along the longitudinal axis, as,

$$V_{pitot} = u \tag{4}$$

By using the body-axis velocity components, angles of attack and sideslip can be calculated as,

$$\alpha = \arctan\left(\frac{w}{u}\right) \tag{5}$$

$$\beta = \arcsin\left(\frac{v}{\sqrt{u^2 + w^2 + v^2}}\right) \tag{6}$$

Then, the measurement equations can be defined in the following form

$$z(k) = h\left[x(k)\right] + v_m(k) \tag{7}$$

where h is the nonlinear measurement function and v_m is the zero-mean Gaussian measurement noise vector with covariance matrix R. The measurement vector z comprises of GPS velocity measurements, Pitot tube airspeed, and wind vane angle of attack and side sleep angle measurements.

It is required to estimate the wind speed, ADS scale factor and UAV states on the measurements (7).

3 UAV State Estimation Filter

Let us consider the flight dynamics of UAV described by the linear state equation as,

$$x(k+1) = Ax(k) + Bu(k) + Gw(k) \tag{8}$$

and measurement equation

$$z(k) = H(k)x(k) + V(k)\,, \tag{9}$$

where $x(k)$ is the vector of system state; A is the transition matrix of the system; B is the control distribution matrix; u(k) is the control input vector; $w(k)$ is the random vector of disturbances (system noise); G is the transition matrix of system noise; *z(k)* is the vector of measurements; $H(k)$ is the matrix of measurements of the system; and $V(k)$ is the random vector of measurement noise. Assume that random vectors $w(k)$ and $V(k)$ are Gaussian white noise. Their mean values and covariance are determined by the expressions

$$\begin{aligned} &E\left[w(k)\right] = 0; E\left[V(k)\right] = 0; \\ &E\left[w(k)w^T(j)\right] = Q(k)\delta(kj); \\ &E\left[V(k)V^T(j)\right] = R(k)\delta(kj). \end{aligned} \tag{10}$$

Here E is the operator of statistical expectation; T is the sign of transposition; and $\delta(kj)$ *is* the Kronecker delta symbol. Note that $\{w(k)\}$ and $\{V(k)\}$ are assumed mutually uncorrelated.

Apparently [6], the optimum linear Kalman filter (LKF) that estimates the state vector of the system (1) is expressed with the following recursive equations system:

Equation of the estimation value,

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)\left[z(k) - H(k)\hat{x}(k/k-1)\right] \tag{11}$$

where $\hat{x}(k/k-1) = A\hat{x}(k-1/k-1) + Bu(k-1)$ is the extrapolation value, $K(k)$ is the gain matrix of the optimum linear Kalman filter:

$$K(k) = P(k/k-1)H^T(k)\left[H(k)P(k/k-1)H^T(k) + R(k)\right]^{-1} \tag{12}$$

The innovation sequence

$$\Delta(k) = z(k) - H(k)\hat{x}(k/k-1) \tag{13}$$

The innovation covariance

$$P_\Delta(k) = H(k)P(k/k-1)H^T(k) + R(k) \tag{14}$$

The normalized innovation

$$\Delta(k) = \left[H(k)P(k/k-1)H^T(k) + R(k)\right]^{-\frac{1}{2}}\Delta(k) \tag{15}$$

The covariance matrix of the filtering error is,

$$P(k/k) = \left[I - K(k)H(k)\right]P(k/k-1) \tag{16}$$

where I is the identity matrix.

The covariance matrix of the extrapolation error is,

$$P(k/k-1) = AP(k-1/k-1)A^T + GQ(k-1)G^T \tag{17}$$

4 Wind Velocity Estimation Filter

Nonlinear state-space formulations of wind velocity estimation problem of unmanned aerial vehicle are discussed. As the formulation uses the relationship of the wind triangle, it is necessary to have knowledge of both the ground and air speed. The formulations considered use the pitot-static tube air speed and the global positioning system (GPS) velocity estimates.

Here, state vector includes north (μ_N), east (μ_E), down (μ_D) components of the wind velocity and the scale factor (ζ).

The effects of the sideslip angle, angle of attack and the air density parameters can be estimated by using the scale factor. State-space system is composed of state vector x, input vector u, and output vector y [7]

$$x = \left[\mu_N \quad \mu_E \quad \mu_D \quad \zeta\right]^T \tag{18}$$

$$u = \left[V_N^{GPS} \quad V_E^{GPS} \quad V_D^{GPS}\right]^T \tag{19}$$

$$y = P_d \tag{20}$$

Here V_N^{GPS}, V_E^{GPS}, V_D^{GPS} are the ground velocity measurement components by GPS, P_d is the dynamic pressure measurements by pitot-static tube. As there is no information of the state dynamics, dynamic equations are expressed by random walk process:

$$x(k) = x(k-1) + w_p(k-1) \tag{21}$$

where w_w is the zero-mean Gaussian system noise vector with the process covariance matrix Q_w. Output vector can be composed by using wind triangle relationship.

$$\vec{V}_{air} = \vec{V}_{ground} - \vec{V}_{wind} \tag{22}$$

$\vec{V}_{air}, \vec{V}_{ground}, \vec{V}_{wind}$ represent air, ground and wind velocity vectors in NED coordinate system respectively. Let's take square of the each side of the equation's L_2 norm as,

$$\begin{gathered}\left\|\vec{V}_{air}\right\|_2^2 = \left\|\vec{V}_{ground} - \vec{V}_{wind}\right\|_2^2 \\ V_{air}^2 = \left(V_N^{GPS} - \mu_N\right)^2 + \left(V_E^{GPS} - \mu_E\right)^2 + \left(V_D^{GPS} - \mu_D\right)^2\end{gathered} \tag{23}$$

The air speed of pitot-static tube $\left(V_{pitot}\right)$ can be expressed in terms of total air speed $\left(V_{air}\right)$, angle of attack (α) and sideslip angle (β),

$$V_{pitot} = V_{air} \cos\alpha \cos\beta \tag{24}$$

Dynamic pressure can be written considering the Bernoulli equation as:

$$P_d = \frac{\rho}{2} V_{pitot} \tag{25}$$

ρ represents the air density. So, the scale factor (ζ) can be written as:

$$\zeta = \frac{\rho}{2}\left(\cos\alpha\right)^2\left(\cos\beta\right)^2 \tag{26}$$

If all of these combined, the output vector equation becomes:

$$P_d = \zeta\left[\left(V_N^{GPS} - \mu_N\right)^2 + \left(V_E^{GPS} - \mu_E\right)^2 + \left(V_D^{GPS} - \mu_D\right)^2\right] + v_p \tag{27}$$

where v_p is zero-mean Gaussian measurement noise with the measurement covariance matrix R_p.

Let's move linear state model (21) into (8) form

$$x(k+1) = Ax(k) + Gw_p(k) \tag{28}$$

where A and G are the 4x4 unit matrices.

The measurement equation (27) is nonlinear and can be written in the form:

$$z(k) = h\left[x(k), k\right] + v_p(k) \tag{29}$$

where $h\left[\cdot\right]$ are the nonlinear measurement function, $x(k)$ is the 4 dimensional state vector at time k, $z(k)$ is the scalar measurement, $v_p(k)$ is the zero-mean Gaussian noise with covariance R_p. It is assumed that both noise vectors $w_p(k)$ and $v_p(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean,

$$E\left[w(k)\right] = E\left[v(k)\right] = 0, \quad \forall\ k \tag{30}$$

Filter algorithm based on the described system and measurements in (28)-(29) can be given. The estimation of states (18) can be found based on the Extended Kalman filter (EKF). The estimation value by EKF can be found as,

$$\begin{aligned}\hat{x}(k/k) &= \hat{x}(k/k-1) + K(k) \\ &\times\left\{z(k) - h\left[\hat{x}(k/k-1, k)\right]\right\}\end{aligned} \tag{31}$$

The extrapolation value from the dynamic function can be found as,

$$\hat{x}(k/k-1) = A\hat{x}(k-1/k-1) \tag{32}$$

Filter-gain of the EKF is,

$$\begin{aligned}K(k) &= P(k/k-1)\frac{\partial h^T[\hat{x}(k/k-1), k]}{\partial \hat{x}(k/k-1)} \\ &\times\left\{\frac{\partial h[\hat{x}(k/k-1), k]}{\partial \hat{x}(k/k-1)} P(k/k-1)\frac{\partial h^T[\hat{x}(k/k-1), k]}{\partial \hat{x}(k/k-1)} + R(k)\right\}^{-1}\end{aligned} \tag{33}$$

where $\frac{\partial h[\hat{x}(k/k-1), k]}{\partial \hat{x}(k/k-1)}$ is the partial derivatives of the measurement function with respect to the states.

The covariance matrix of the extrapolation error is,

$$P(k/k-1) = AP(k-1/k-1)A^T + GQ(k-1)G^T \quad (34)$$

The covariance matrix of the filtering error is,

$$P(k/k) = \left[I - K(k)\frac{\partial h[\hat{x}(k/k-1),k]}{\partial \hat{x}(k/k-1)} \right] P(k/k-1) \quad (35)$$

where I is the identity matrix.

The innovation sequence is presented as,

$$\Delta(k) = z(k) - h\left[\hat{x}(k/k-1),k\right] \quad (36)$$

The innovation covariance is,

$$P_\Delta(k) = \frac{\partial h[\hat{x}(k/k-1),k]}{\partial \hat{x}(k/k-1)} P(k/k-1) \times \frac{\partial h^T[\hat{x}(k/k-1),k]}{\partial \hat{x}(k/k-1)} + R(k) \quad (37)$$

The normalized innovation can be expressed in the form,

$$\tilde{\Delta}(k) = [\frac{\partial h[\hat{x}(k/k-1),k]}{\partial \hat{x}(k/k-1)} P(k/k-1) \times \frac{\partial h^T[\hat{x}(k/k-1),k]}{\partial \hat{x}(k/k-1)} + R(k)]^{-0.5} \times \Delta(k) \quad (38)$$

EKF that estimates the state vector of the system (30) is expressed with the formulas (31) - (38).

5 TSKF Simulation Results

The wind speed and longitudinal and lateral motion parameters of UAV are estimated via proposed TSKF. Simulation results are given in Figs. 2-4. Estimations of wind speed, longitudinal motion parameters and lateral motion parameters are presented in Figs. 1, 2, 3 respectively. In these figures blue line shows the actual values and red line - estimated values. The obtained results show that, the wind speed and UAV state estimations converge to actual values.

The normalized innovation corresponding to the first stage filter is presented in Fig 4. Behavior of the normalized innovation approve that the TSKF operates properly.

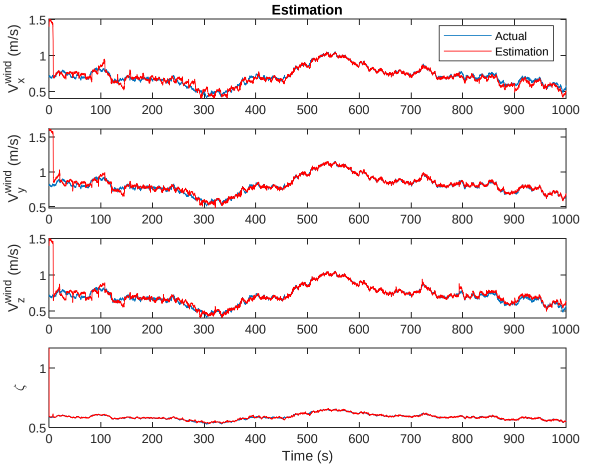

Fig.1. Wind speed estimation results: actual value-blue line; estimated value- red line.

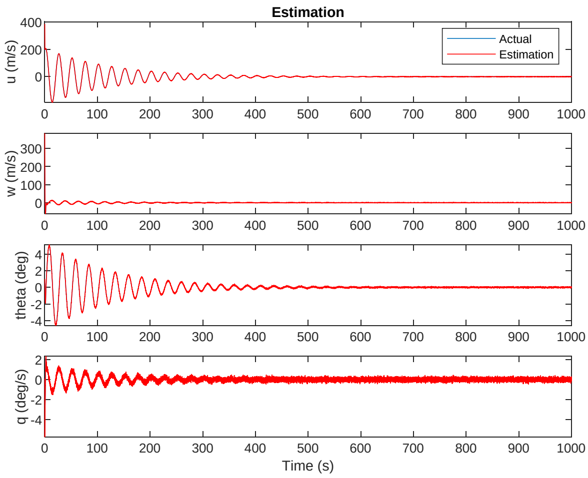

Fig.2. Estimation results of the longitudinal motion parameters: actual value-blue line; estimated value-red line.

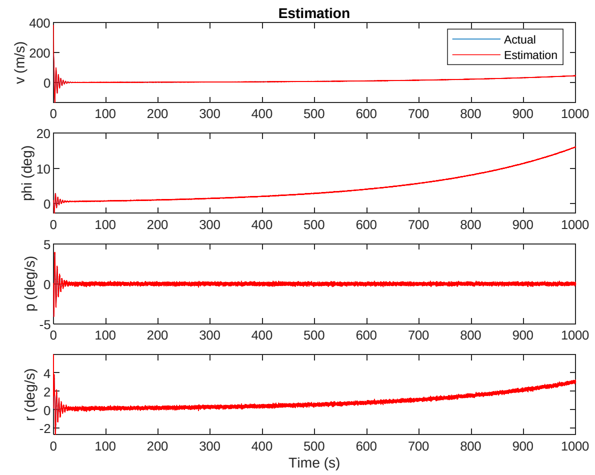

Fig.3. Estimation results of the lateral motion parameters: actual value-blue line; estimated value-red line.

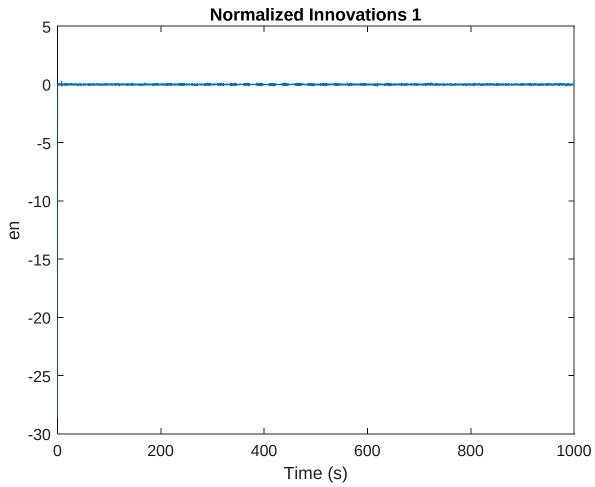

Fig.4. Normalized innovation of the first stage filter

6 Conclusion

In this work a two-stage Kalman filter was developed for estimation of the wind speed and UAV states. In the first stage, the wind speed estimation algorithm is developed by the usage of GPS and dynamic pressure measurements. For this purpose, Extended Kalman Filter based on nonlinear measurements was designed.

In the second stage, the estimation of the state parameters of the UAV dynamic model was made by using the Conventional Linear Kalman filter based on the Air Data System and IMU measurements.

Simulations showed that the proposed two-stage estimation procedure has the ability to estimate the wind speed and UAV states with a high accuracy performance.

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