System Identification and Model Validation of Electro-Hydraulic Actuator for Quarter Car System

NORLELA ISHAK, MAZIDAH TAJJUDIN, RAMLI ADNAN
Frontier Materials and Industry Application, UiTM-RMI-CoRe FMIA
Faculty of Electrical Engineering,
Universiti Teknologi MARA,
40450 Shah Alam, Selangor, MALAYSIA
norlelaishak@salam.uitm.edu.my
HASHIMAH ISMAIL
Faculty of Engineering,
Universiti Selangor, MALAYSIA
hashimah@unisel.edu.my

Abstract - Electro-Hydraulic actuator is common tools used in the industries. This is due to accurate positioning toward the load and fast response make it as major instruments for the industries process. This paper presents experimental work on non-recursive identification of electro-hydraulic actuator system that represented by a discrete-time model in open-loop configuration. A least square method is used to estimate the unknown parameters of the system based on auto regression with exogenous input (ARX) model. The plant mathematical model was approximated using system identification by aid of System Identification Toolbox of Matlab from open-loop input-output experimental data. These models have been validated by R^2 or best fitting criterion, root mean square error and correlation analysis to determine the adequate model for representing the EHA system. By using pole-placement method, this controller is designed for the model chosen through simulation in Matlab-Simulink. The results show that the model chosen which is applied with the proposed controller is able to perform position tracking with high accuracy.

Key-words: - System Identification, electro-hydraulic actuator, best fitting criterion, correlation analysis, root mean square error, pole-placement

1 Introduction

Electro-Hydraulic actuator (EHA) is the important tools used in the industries because the system can give fast response and accurate positioning toward the loads [1][2]. Although, this system commonly inside industries processes such as aircraft manufacturing [3], food production and automotive assembly [4], but there are some disadvantages using this system such as uncertainties, highly nonlinearities, time varying characteristic and compact structure [5].

In general, it is difficult to establish or identify an accurate dynamics model of electro-hydraulic system because of the system inherently have many uncertainties, highly nonlinearities and time varying which makes the modeling and controller design are more complicated.

To overcome the problem, system identification technique is proposed in this research. This method is developed based on i nput-output measurement for modeling the electro-hydraulic system which describes the dynamical behavior consist of the hydraulic actuator and the proportional valve.

System identification has gain much interest in many engineering applications where the parameter of the system can be estimated using recursive and non-recursive manner such as least squares, recursive least squares, recursive instrumental variables and recursive maximum likelihood methods. System identification can be divided into non-parametric and parametric estimation methods. For non-parametric method such as transient and correlation analysis, the result is easy to obtain but the derived model will be rather inaccurate and sensitive to noise [6], whilst parametric is an estimation method based on user-specified models or ready-made models [7] to estimate the model and give an accurate results.

There are numbers of approaches that can be used to identify the model of the system [8]. To achieve the model required, there are two methods which are physical law modeling method and system identification. Physical Law modeling such as Newton's Law required high level of knowledge hydraulic cylinder system by deriving mathematical model and the model is hard to achieve but different with System Identification also known as 'black box modeling' [2] using ARX which is easier to obtain the model [9]. System Identification able to capture and insights and unmodeled dynamic unlike the first principle model. It is reliable to used and able to approximate the model using input and output of electro-hydraulic and no prior to understand the system.

A guide on how to choose a correct model is by applying Parsinomy Principle [10][6]. Parsinomy Principle states that, out of two identifiable model structure that fit certain data, the model with simpler form will be chosen. Thus, model with less parameters, while the accuracy does not significantly improved in high order, is the model of the system. Lower degree and less parameter will make the computation and controller design on the model a lot easier. This criterion has to be taken into account while selection most suitable model for the EHA system.

According to Akaike's [11][6], the selection of model order which normally used in the black box system identification such as Akaike's information criterion (AIC) and Akaike's final prediction error (FPE). These techniques are the most popular validation methods in system identification field for model structure selection of black box systems. Besides that, the best fitting criterion can also be used. These criterions show the preciseness of the approximation model as compared to the true value model [10][L.jung].

The model can be further evaluated by another validation technique is correlation function. The model accepted definition of strong in the auto correlation function (ACF) analysis is where the correlation is beyond the range of confident interval. The ACF is for estimate the whiteness of the residual. Meanwhile, the CCF is used to explore the correlation in residual between input and output. In the analysis, two percent out of one auto correlation (ACF) have been set [4].

For good correlation test defined when the correlation are white noise which mean all the correlation are in the confident interval except correlation at lag 0[12]. But, when less than half of the correlation is beyond the confident interval, the correlation is moderate and when more than half of the correlation is beyond the confident interval it defined as bad. The range of cross correlation is set to be between -0.1 to 0.1[13] The data should be in the confident interval which is 95%.

2 Experimental Design

The experimental hardware that was utilized as a part of these studies is an EHA system that is indicated in Fig. 1. The hydraulic cylinder was held in vertical position. This is a difficult issue as effect of gravity is considered. The EHA system includes single-ended cylinder kind of actuator. The bidirectional cylinder has 150 mm stroke length, however, the unit measurement is in inches. Thus the stroke length is 5.9 inches. The wire displacement sensor is mounted at the highest point of piston rod. The flow of the fluid is controlled by electronic control valve.

The completed diagram for hardware set-up in these real-time studies of an electro-hydraulic system that is shown in Fig. 2. The data acquisition system (DAQ) for the EHA system workbench makes use of a PCI-1716 card manufactured by Advantech Automation that is compatible with the MATLAB real time Windows target Toolbox.

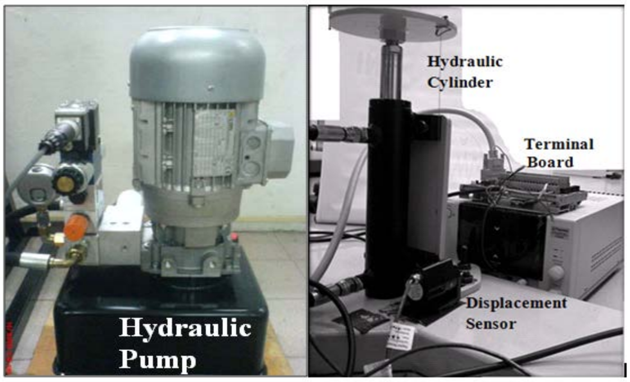

Fig. 1 Electro-Hydraulic Actuator (EHA)

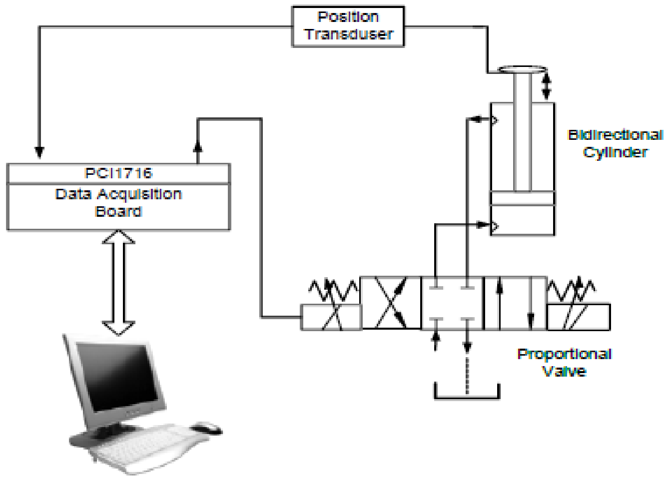

Fig. 2 Experimental setup for electro-hydraulic actuator system

3 Results And Discussion

3.1 Data Collection

During the system identification process, a set of stimulus-response or input signal has to be obtained based on generated by computer. Input signal is used to excite the system and produce response signal. The input signal used in system identification plays an important role to trigger the system so that the output signal contains good information as much as possible and ensure identifiability. Thus, input signal can have a significant influence on the estimation result.

Due to the limitation of the EHA system in this studies, sum of sinusoidal signal is found to be suitable for this identification process [14]. This type of signals also has been used in [15] in the identification process particularly for EHA system. The input signal in this study was generated using three different frequencies that based on equation 1.

$$u(k) = \sum_{i=1}^{p} a_i \cos \omega_i t_s k \qquad (1)$$

Where a_i is amplitude, ω_i is frequencies (rad/sec), t_s is the sampling time (sec) and k is integer. For this particular study, three frequencies within the range of 0.01 Hz to 1 Hz will be used in order to generate a multi-sine signal.

From equation 1, when using three different frequencies for input signal, the models that can be obtained are limited to second and third order only. Higher-order models may produce unstable output. In this case, the third-order model will represent the nearest model of true plant.

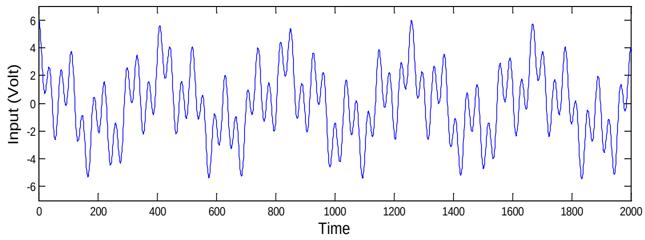

Fig. 3 Input signal for model identification

The output signal of the plant obtained using the input signal of Fig. 3 is sampled at 40 ms, is given in Fig. 4. The input and output signals of Fig. 3 and Fig. 4 have to be divided into two parts, i.e. (1 to 1000) samples and (1001 to 2000) samples. The first part of the input-output signals will be used to obtain the plant model and the second part of the input-output signals will be used to validate the obtained model.

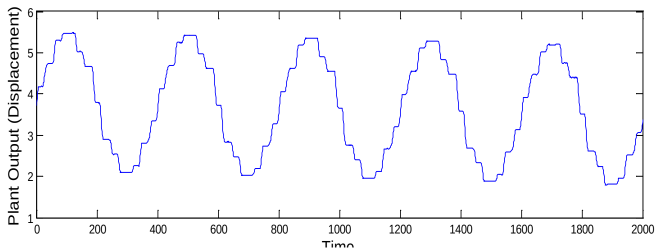

Fig. 4 Output signal of the plant using 30ms sampling time and input signal of Fig.4 for model identification

Using Matlab System Identification Toolbox, the first part of the input-output signal produces a plant model, ARX331 in the form of discrete-time open-loop transfer function as follows:

$$\frac{B_c(z^{-1})}{A_c(z^{-1})} = \frac{0.0038z^{-1} + 0.0029z^{-2} - 0.0055z^{-3}}{1 - 0.907z^{-1} + 0.9414z^{-2} - 0.0341z^{-3}} \qquad (2)$$

The second part of input-output signals was used to validate the obtained model of equation 2. The second part of the input signal was used as an input to the model and the output from the model will be compared with the second part of the output signal. The result can be seen from Fig. 5.

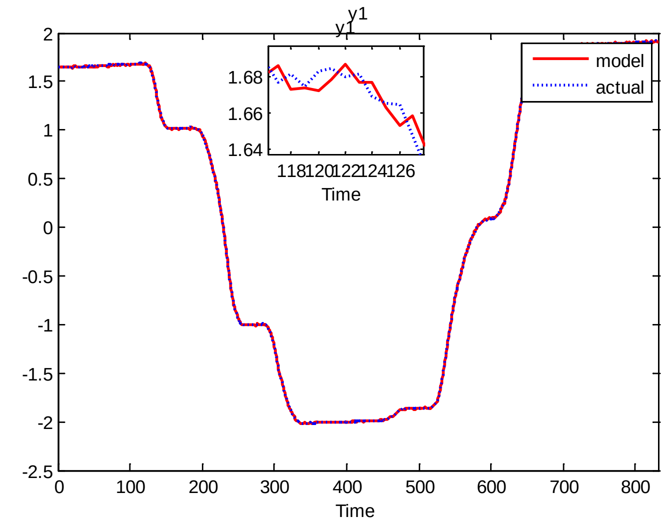

Fig. 5 The comparison between model and actual plant signal

3.2 Model Validation

During the system identification process, four model structure including the 2nd, 3rd, 4th and 5th order for the black box model of the EHA system were performed in the estimation process. From the observation through graphical approach in Fig.6, it showed that the third order model is adequate to represent the underlying system. Even though the second order can be considered to represent the EHA system, through the statistical approach and first principle modeling, it was found that the third order of estimated model is the closest to the actual model. Higher-orders model may produce unstable output.

Besides, the improvement of the estimated model is insignificant for higher order model although it produced a better R^2 value than the third order model. As a rule of thumb, it is not recommended to use model with more parameters if there is an adequate model with less parameters that can be employed. Also based on a parsimony model as discussed by Soderstrom and Stoica [6], the third order can be concluded as the most suitable model to represent the EHA system.

The FPE between the outputs of the actual plant and estimated model, belonging to models with different orders have been compared and tabulated in Table 1.

Table. 1 Model order selection

Model Order	R^2 Value	FPE	Loss Function
2rd	0.8593	3.702e-05	3.680e-05
3rd	0.9116	3.703e-05	3.670e-05
4th	0.9130	3.039e-05	3.003e-05
5th	0.9148	2.824e-05	2.782e-05

The R^2 or best fitting criterion and its loss function were analysed from one of the data sets in order to show the third model is adequate to represent the EHA system. R^2 value or best fitting criterion values are tabulated in Table 1 and showed about 90% of the actual model can be presented by the estimated model. It also has been proven by the FPE value and its loss function which produced a very small value.

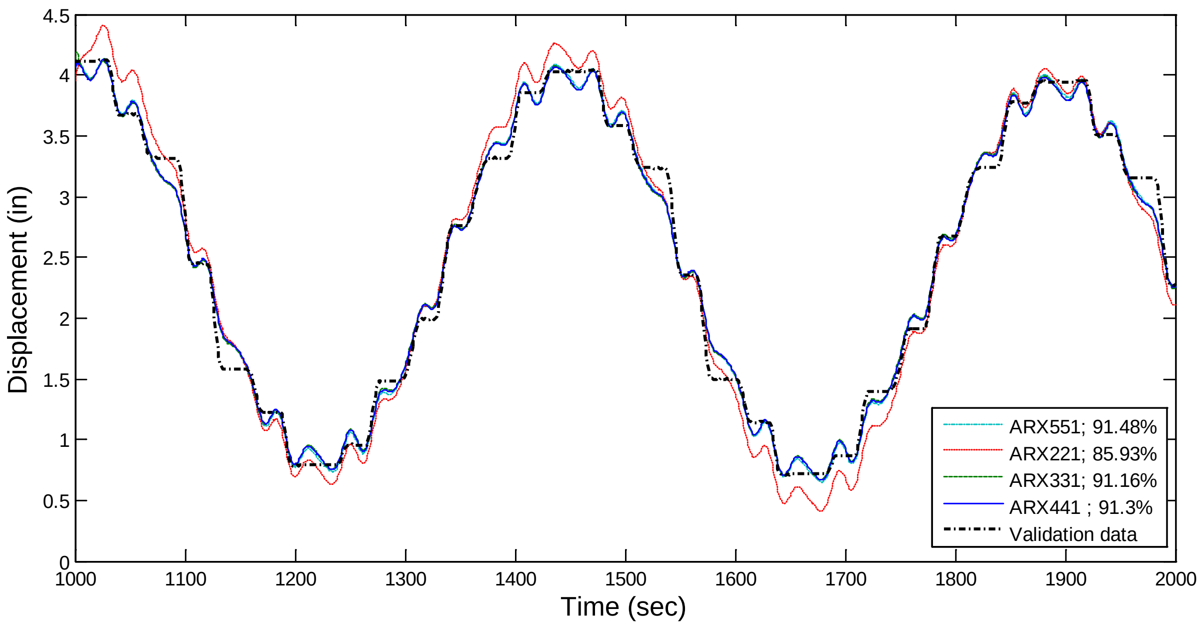

Fig. 6 Simulated output from different model order

Besides best fitting, autocorrelation of residuals and cross correlation analysis are considered as w ell. A good model should also having good autocorrelation and cross correlation analysis, for more stability of model.

In this study, the correlation analyses are conducted to observe the whiteness test of its residual and the relation between the error and the input signals. Fig. 7 to Fig. 10 showed that the CCF and ACF of the estimated models formed four data sets.

Fig. 7 to Fig. 10 showed the auto correlation and cross correlation when using ARX221 to ARX551 with sampling time 40ms. In this case, the autocorrelation are clearly exceeding the confident interval bounds for a white noise autocorrelation at many lags. Therefore, the ACF is said to be moderate and the CCF are good. From the validation process, all the estimated models are statistically acceptable. This stage is responsible for the approval of the model's adequacy to represent the underlaying system.

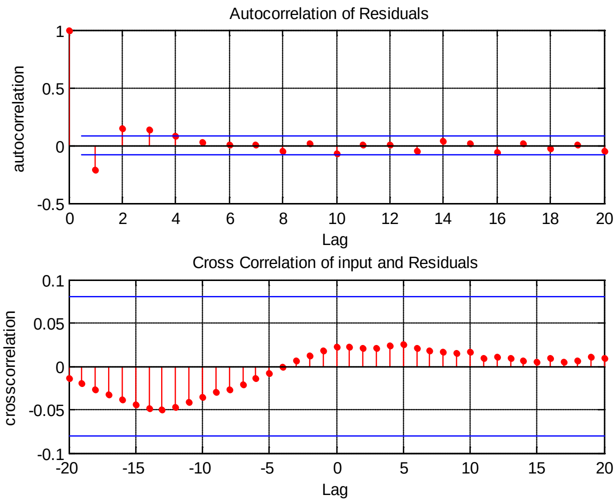

Fig. 7 Correlation analysis for ARX221

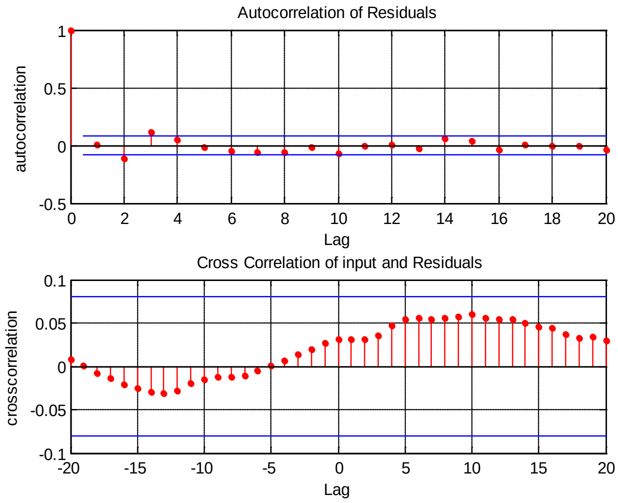

Fig. 8 Correlation analysis for ARX331

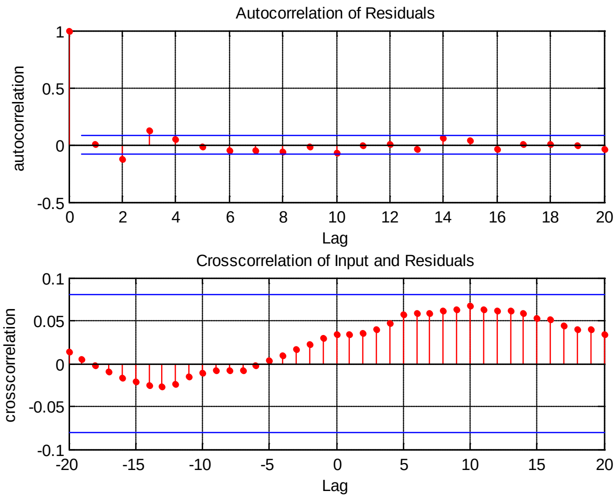

Fig. 9 Correlation analysis for ARX441

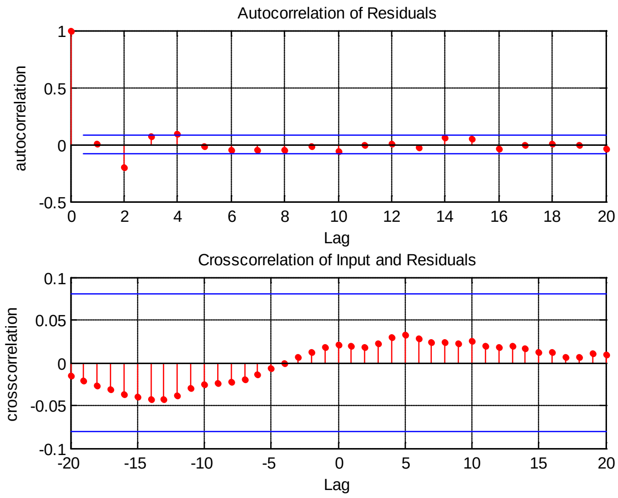

Fig. 10 Correlation analysis for ARX551

In this study, histogram is referred to as frequency distribution. It is a method to summarize a data distribution into several intervals and the number of data points in each interval is represented as a b ar length. The histogram is usually coupled with residual mean and variance so as to evaluate the distribution property of residual.

The residual histogram of a good model is expected to be in Gaussian distribution, zero mean, small variance and symmetric as shown in Fig. 11 t o Fig. 14. T he histograms of the residual show that the Gaussian distributions emerged from the plotted graph and the estimated model generally are acceptable as a good model where the residuals are determined to be white noise.

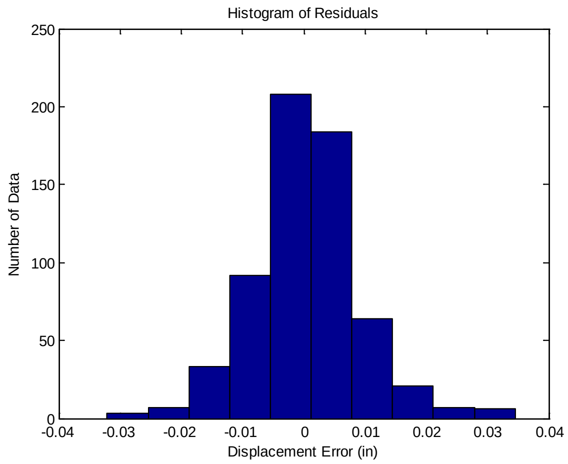

Fig. 11 Histogram of residuals for ARX221

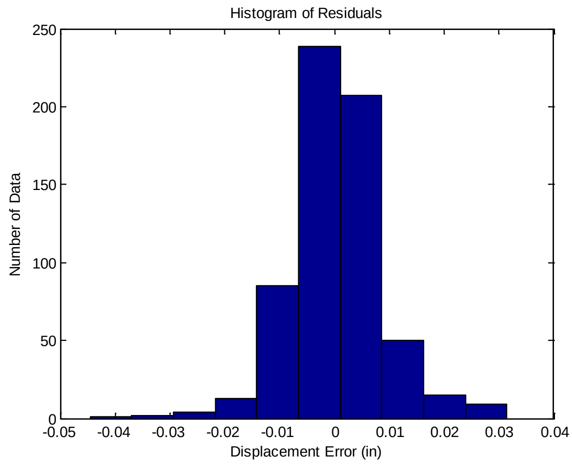

Fig. 12 Histogram of residuals for ARX331

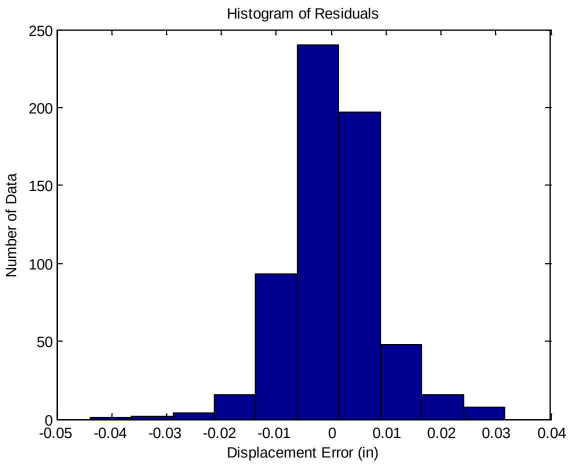

Fig. 13 Histogram of residuals for ARX441

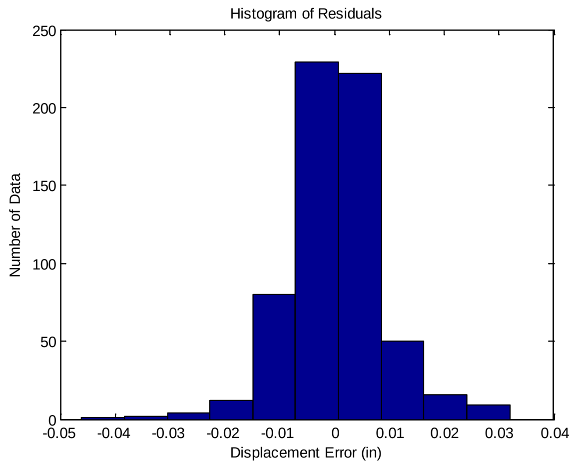

Fig. 14 Histogram of residuals for ARX551

3.3 Controller Design

To show that the model obtained is controllable, pole-placement method [8,9] was used for the design of feedback controller as in Fig. 15. This method enables all poles of the closed-loop to be placed at desired location and providing satisfactory and stable output performance.

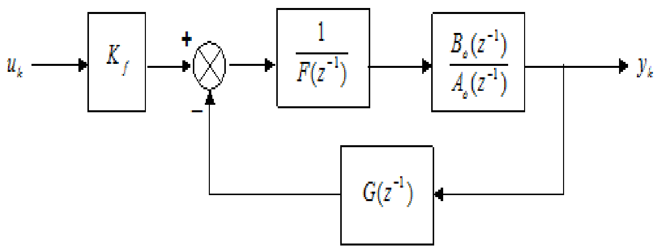

Fig. 15 Feedback controller using pole-placement method

The closed-loop transfer function of the feedback system in Fig.15 is given by

$$\frac{C(z^{-1})}{R(z^{-1})}=\frac{K_f B_o(z^{-1})}{A_o(z^{-1})F(z^{-1})+B_o(z^{-1})G(z^{-1})} \tag{3}$$

where

$$\begin{aligned}
A_o(z^{-1}) &= 1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}+\cdots+a_nz^{-n}\\
B_o(z^{-1}) &= b_1z^{-1}+b_2z^{-2}+b_3z^{-3}+\cdots+b_mz^{-m}\\
F(z^{-1}) &= 1+f_1z^{-1}+f_2z^{-2}+f_3z^{-3}+\cdots+f_mz^{-m-1}\\
G(z^{-1}) &= g_0+g_1z^{-1}+g_2z^{-2}+g_3z^{-3}+\cdots+g_{n-1}z^{-n-1}
\end{aligned}$$

Using Diophantine equation, $F(z^{-1})$ and $G(z^{-1})$ can be solved from:

$$A_o(z^{-1})F(z^{-1})+B_o(z^{-1})G(z^{-1})=T(z^{-1}) \tag{4}$$

and $T(z^{-1})$ is location of poles that reuqired. Using $T(z^{-1})=1+t_1z^{-1}$, only one pole position is considered at $t_1=-p$ which is inside a unity circle. Other poles cancelled each other. The range of p is $0<p<1$. For slow response, p is set small. The forward gain is given as $\mathrm{K_f}=\frac{\mathrm{Sum(T)}}{\mathrm{Sum(B_o)}}$. From equation (4) the following matrix equation can be derived:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0\\
a_1 & 1 & 0 & 0 & 0 & 0 & b_2 & b_1 & 0 & 0 & 0 & 0 & 0\\
a_2 & a_1 & 1 & 0 & 0 & 0 & b_3 & b_2 & b_1 & 0 & 0 & 0 & 0\\
a_3 & a_2 & a_1 & 1 & 0 & 0 & b_4 & b_3 & b_2 & b_1 & 0 & 0 & 0\\
a_4 & a_3 & a_2 & a_1 & 1 & 0 & b_5 & b_4 & b_3 & b_2 & b_1 & 0 & 0\\
a_5 & a_4 & a_3 & a_2 & a_1 & 1 & \vdots & b_5 & b_4 & b_3 & b_2 & b_1 & 0\\
\vdots & a_5 & a_4 & a_3 & a_2 & a_1 & b_m & \vdots & b_5 & b_4 & b_3 & b_2 & b_1\\
a_n & \vdots & a_5 & a_4 & a_3 & a_2 & 0 & b_m & \vdots & b_5 & b_4 & b_3 & b_2\\
0 & a_n & \vdots & a_5 & a_4 & a_3 & 0 & 0 & b_m & \vdots & b_5 & b_4 & b_3\\
0 & 0 & a_n & \vdots & a_5 & a_4 & 0 & 0 & 0 & b_m & \vdots & b_5 & b_4\\
0 & 0 & 0 & a_n & \vdots & a_5 & 0 & 0 & 0 & 0 & b_m & \vdots & b_5\\
0 & 0 & 0 & 0 & a_n & \vdots & 0 & 0 & 0 & 0 & 0 & b_m & \vdots\\
0 & 0 & 0 & 0 & 0 & a_n & 0 & 0 & 0 & 0 & 0 & 0 & b_m
\end{bmatrix}
\begin{bmatrix} f_1\\ f_2\\ f_3\\ f_4\\ \vdots\\ f_{m-1}\\ g_0\\ g_1\\ g_2\\ g_3\\ g_4\\ \vdots\\ g_{n-1}\end{bmatrix}
=
\begin{bmatrix} t_1-a_1\\ -a_2\\ -a_3\\ -a_4\\ -a_5\\ \vdots\\ -a_n\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix} \tag{5}$$

Let $E\cdot M = D$ where E is a Sylvester Matrix given by:

$$E=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0\\
a_1 & 1 & 0 & 0 & 0 & 0 & b_2 & b_1 & 0 & 0 & 0 & 0 & 0\\
a_2 & a_1 & 1 & 0 & 0 & 0 & b_3 & b_2 & b_1 & 0 & 0 & 0 & 0\\
a_3 & a_2 & a_1 & 1 & 0 & 0 & b_4 & b_3 & b_2 & b_1 & 0 & 0 & 0\\
a_4 & a_3 & a_2 & a_1 & 1 & 0 & b_5 & b_4 & b_3 & b_2 & b_1 & 0 & 0\\
a_5 & a_4 & a_3 & a_2 & a_1 & 1 & \vdots & b_5 & b_4 & b_3 & b_2 & b_1 & 0\\
\vdots & a_5 & a_4 & a_3 & a_2 & a_1 & b_m & \vdots & b_5 & b_4 & b_3 & b_2 & b_1\\
a_n & \vdots & a_5 & a_4 & a_3 & a_2 & 0 & b_m & \vdots & b_5 & b_4 & b_3 & b_2\\
0 & a_n & \vdots & a_5 & a_4 & a_3 & 0 & 0 & b_m & \vdots & b_5 & b_4 & b_3\\
0 & 0 & a_n & \vdots & a_5 & a_4 & 0 & 0 & 0 & b_m & \vdots & b_5 & b_4\\
0 & 0 & 0 & a_n & \vdots & a_5 & 0 & 0 & 0 & 0 & b_m & \vdots & b_5\\
0 & 0 & 0 & 0 & a_n & \vdots & 0 & 0 & 0 & 0 & 0 & b_m & \vdots\\
0 & 0 & 0 & 0 & 0 & a_n & 0 & 0 & 0 & 0 & 0 & 0 & b_m
\end{bmatrix} \tag{6}$$

$$\begin{aligned}
M &= \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & \cdots & f_{m-1} & g_0 & g_1 & g_2 & g_3 & g_4 & \cdots & g_{n-1}\end{bmatrix}^T\\
\mathrm{D} &= \begin{bmatrix} (\mathrm{t_1}-\mathrm{a_1}) & -\mathrm{a_2} & -\mathrm{a_3} & -\mathrm{a_4} & -\mathrm{a_5} & \cdots & -\mathrm{a_n} & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}
\end{aligned} \tag{7}$$

Thus, vector F and G can be computed from vector M that given by : $M = E^{-1}\cdot D$. The computed controller parameters are as follows :

$$\begin{gathered}
\mathrm{T} = 1\ {-0.87}\\
\mathrm{K_f} = 98.0695\\
\mathrm{F}\ (\mathrm{z}^{-1}) = 1 + 0.0559\mathrm{z}^{-1} - 0.8545\mathrm{z}^{-2}\\
\mathrm{G}\ (\mathrm{z}^{-1}) = 240.6831 - 148.1202\mathrm{z}^{-1} + 5.5237\mathrm{z}^{-2}
\end{gathered} \tag{8}$$

3.4 Simulation Studies

In this section, simulation results were analysed to show that the model obtained is controllable. A Visual C++ console programming has been developed to perform simulation studies where the parameters computation for the feedback controller were done automatically when the open-loop transfer function parameters was provided.

The simulation for pole placement were done using t_1 = - 0.87 and t_1 = - 0.1. The result shows that the 3rd order model was given the lower RMSE compare with other model order as show in Table 2. The RMSE value for t_1 = 0.1 given higher RMSE because the location of poles was located at the border of unity circle or outside of unity circle, the system became unstable thus causing high error.

Based on the simulation results, the best response is obtained by ARX331 model structure. The result provides the fastest speed without overshoot. This can be observed from Fig. 16.

Table 2
Result of Root Mean Squared Error (RMSE)

Sampling Time	Model Order	RMSE $t_1 = 0.87$	RMSE $t_1 = 0.1$
40ms	2rd	0.957676	4.06643
	3rd	0.78218	3.74671
	4th	2.848916	25.73418
	5th	9.426919	105.265

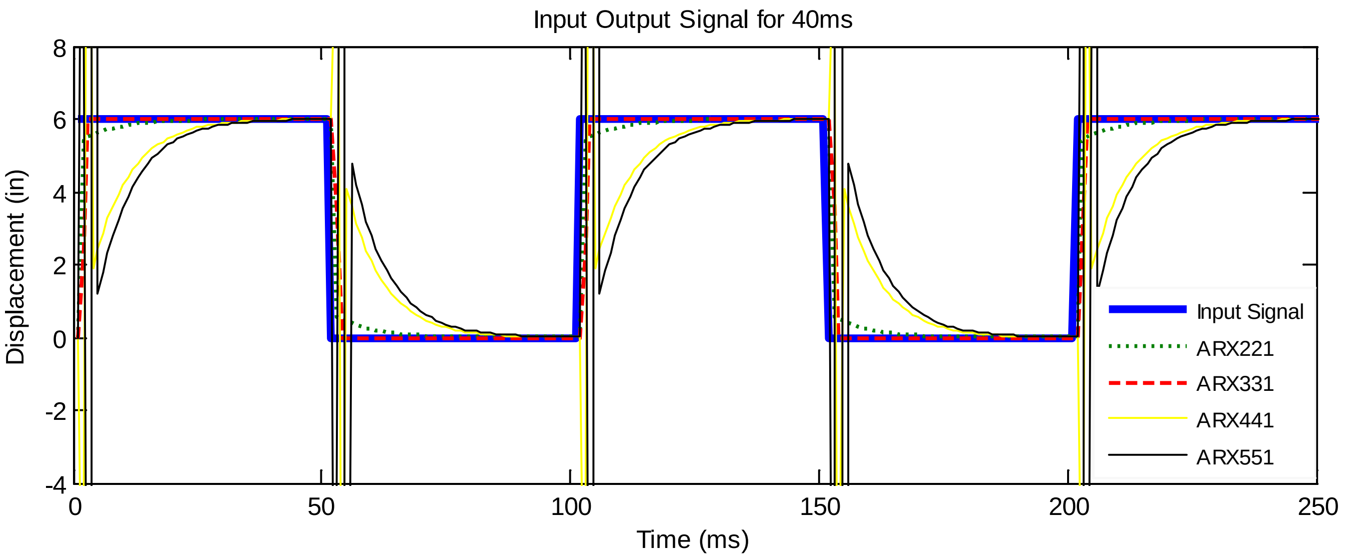

Fig. 16 Simulation results using t_1 = 0.87

4 Conclusions

The model identification using Matlab system identification toolbox to approximate the plant model from input-output experimental data was presented. The ARX331 model used is a good representation of the true electro-hydraulic actuator system. The method of using experimental data to approximate the true model is very much simpler as compared to deriving the mathematical model using physic law. Although it looks simple, trial an error method to use correct frequencies and sampling rates will be the problems and also time consuming to have good and acceptable model. To show that the model obtained is controllable, a controller design using pole-placement method was also applied and presented. The approximated model used in feedback system simulation studies show acceptable performances.

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