Approximating the ruin probability of finite-time surplus process with Adaptive Moving Total Exponential Least Square

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Abstract: - The adaptive moving total least square (AMTLS) method has been used for curve fitting. In this study, AMTLS method is used for the ruin probability fitting and estimation of the ruin probability on an arbitrary initial capital in finite-time surplus process (or risk process). But in reality, it is difficult and complicated to find a fitting method for an appropriate estimate in order to obtain the best performance. So, a new method is developed to estimate the ruin probability of finite-time surplus process. This new method is called adaptive moving total exponential least square (AMTELS) method that applies AMTLS method with least-square fitting exponential. Claim data of motor insurance company from Thailand has used in risk process for the ruin probability fitting. Both AMTLS and AMTELS methods consider weighted function for the distance between node and point with a different constant value d. These methods are compared the performance by using the mean squared error (MSE) and the mean absolute error (MAE) that is, the error between the real ruin probability value that is obtained by the explicit formula and the ruin probability fitting value. With these data, the ruin probability approximating examples are given to prove that AMTELS method shows the better performance than AMTLS method. Moreover, AMTELS method with the narrow value d shows the better performance than AMTELS method with the wide value d.

Key-Words: - Adaptive moving total least square, exponential claim, least-square fitting exponential, moving total least squares, the ruin probability fitting, weighted function

1 Introduction

Nowadays, the insurance is an important part for ensuring in life and asset. Consequently, the study in insurance has many interesting problems. For instance, the quantity of the risk is measured by the probability of insolvency in business, it is so-called the ruin probability. The ruin probability has been introduced by [1]. In the discrete-time risk process, ruin probability of exponential claim severities is easily calculated by using explicit formulas in [2]; see [3]. See also [4] for the finite-time ruin probability under proportional reinsurance in discrete-time surplus process. Moreover, explicit formulas for ruin probability of a discrete-time risk process with proportional reinsurance and investment for exponential and Pareto distribution is presented in [5]. But ruin probability is difficult to find the explicit formulas for other distribution claim severities, such as Weibull, log-normal, Pareto distributions and etc. However, the ruin probability is approximated by using the simulation technique when parameters of claim size distributions are known; see [6], [7]. Although the simulation technique can be used for the calculation of approximations for the finite-time ruin probability, the simulation technique uses a long time.

To solve this problem, many methods have been proposed for an approach in regression analysis to approximate the fitting value provided by a model. Moving Total Least Squares (MTLS) as an approximation method has been used for data fitting; see [8]. Reference [9] has been used MTLS method for generating a surface approximating form the given data. The Adaptive Moving Total Least Squares (AMTLS) method is suggested by [10] for curve fitting.

In this paper, a ruin probability fitting approach called adaptive moving total exponential least square (AMTELS) method is proposed. AMTLS and AMTELS method are compared to estimate ruin probability of finite-time surplus process. A concise description is given for the ruin probability in Section 2. In Section 3, both AMTLS and AMTELS methods are presented in detail. Ruin probability approximating examples are given in Section 4 for comparing between AMTLS and AMTELS methods and conclusions are shown in Section 5.

2 Ruin probabilities with exponential claims

We assume that the claim size process $\{X_n : n \in \mathbb{N}\}$ is independent and identically distributed (i.i.d.), and we consider the ruin probability of finite-time surplus process with exponential claim size distribution. The exponential distribution with intensity parameter $\lambda_0 > 0$ has the probability density function is given by

$$f_X(x;\lambda_0) = \lambda_0 e^{-\lambda_0 x},\ x \geq 0. \tag{1}$$

The cumulative distribution function of the exponential distribution is obtained by:

$$F_X(x;\lambda_0) = 1 - e^{-\lambda_0 x},\ x \geq 0. \tag{2}$$

Reference [2] proposed the explicit formulas for finite-time ruin probability of exponential claim severities with intensity is obtained by

$$\psi_n(u) = \sum_{k=1}^{n} \frac{[\lambda_0(u+kc)]^{k-1}}{(k-1)!} e^{-\lambda_0(u+kc)} \frac{(u+c)}{(u+kc)}, \tag{3}$$

where u is the initial surplus and $n = 0,1,2,...$. $c > 0$ is the constant premium rate and can be calculated in terms of the expected value principle, i.e., $c = (1+\theta)E[X_1]$ where $\theta > 0$ is the safety loading of insurer and $E[X_1] = 1/\lambda_0$.

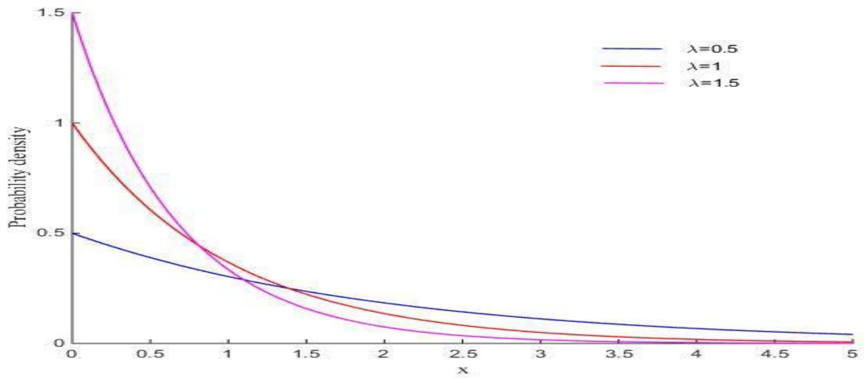

Fig.1 Probability density function of exponential distribution

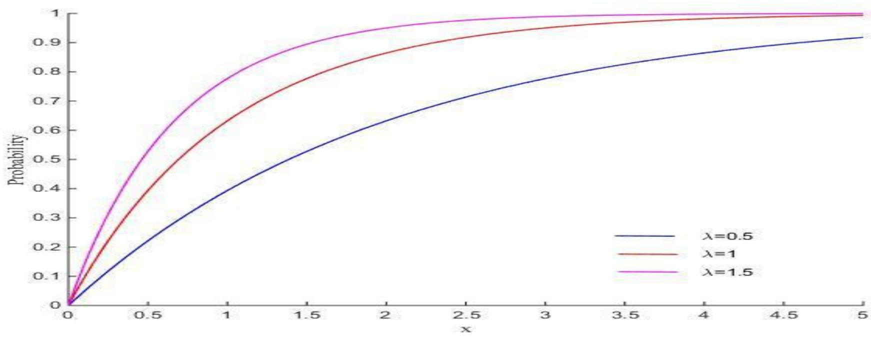

Fig.2 Cumulative distribution function of exponential distribution

3 Description of the estimators

In this section, we are describe AMTLS and AMTELS methods

3.1 Adaptive moving total least squares (AMTLS) method

In AMTLS approximation, a function for the local approximation at x is defined as

$$J(a_x,b_x)=\sum_{I=1}^{n} w(x-x_I)\frac{\lambda^2 a_x^{\,2}+1}{a_x^{\,2}+1}(a_x x_I+b_x-y_I)^2 \tag{4}$$

where $\{(x_I,y_I): I=1,2,3,...,n\}$ is the set of discrete point to be fitted and $w(x-x_I)$ is a weighted function. a_x and b_x are the coefficient of (4) in the sense of the total least squares by minimizing the weighted sum of squared distances. The parameters of local approximants, a_x and b_x can be obtained by solving a system of nonlinear equations as follows:

$$\begin{aligned}\frac{\partial J}{\partial a_x} &= (a_x^{\,5}\lambda^2+2a_x^{\,3}\lambda^2+a_x)\sum_{I=1}^{n} wx_I^{\,2}\\ &+(3a_x^{\,2}b_x\lambda^2-a_x^{\,2}b_x+a_x^{\,4}b_x\lambda^2+b_x)\sum_{I=1}^{n} wx_I\\ &+(a_x^{\,2}-3a_x^{\,2}\lambda^2-a_x^{\,4}\lambda^2-1)\sum_{I=1}^{n} wx_I y_I\\ &+2(a_xb_x-a_xb_x\lambda^2)\sum_{I=1}^{n} wy_I+(a_x\lambda^2-a_x)\sum_{I=1}^{n} wy_I^{\,2}\\ &+(a_xb_x^{\,2}\lambda^2-a_xb_x^{\,2})\sum_{I=1}^{n} w\\ &=0.\end{aligned} \tag{5}$$

and

$$\frac{\partial J}{\partial b_x}=a_x\sum_{I=1}^{n} wx_I+b_x\sum_{I=1}^{n} w-\sum_{I=1}^{n} wy_I=0. \tag{6}$$

In AMTLS method, a parameter λ associated with the direction of local approximants is introduced to assign the direction local approximants (see Fig. 3).

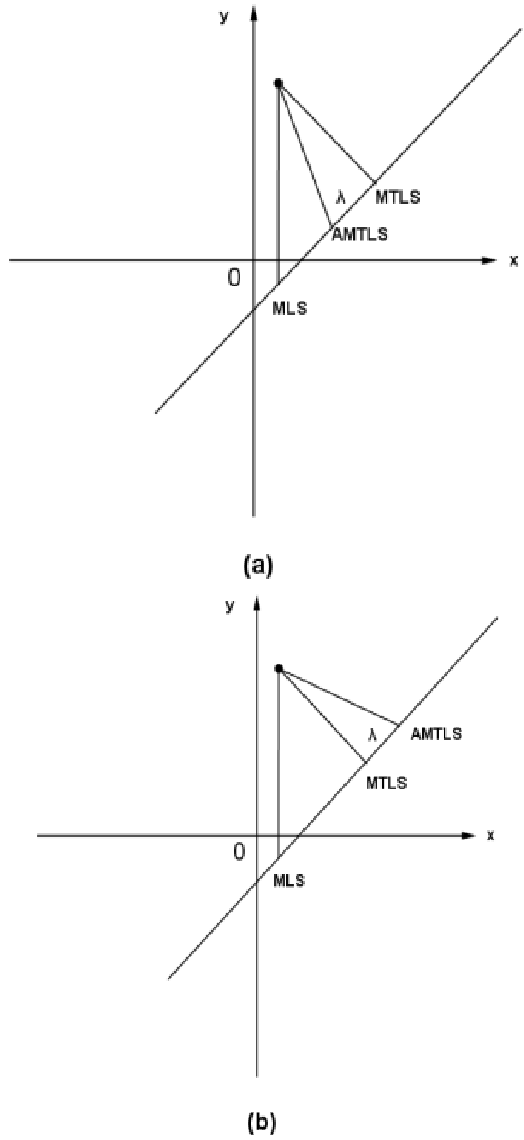

Fig. 3 Local approximants of AMTLS

In case of $\lambda=0,$ we see that (4) is called the moving total least squares (MTLS) method and is a function for local approximants as shown in (7):

$$J(a_x,b_x)=\sum_{I=1}^{n} w(x-x_I)\frac{(a_x x_I+b_x-y_I)^2}{a_x^{\,2}+1}. \tag{7}$$

In case of $\lambda=1,$ we see that (4) is called the moving least squares (MLS) method, define a function for local approximants as:

$$J(a_x, b_x) = \sum_{I=1}^{n} w(x - x_I)(a_x x_I + b_x - y_I)^2. \quad (8)$$

3.2 Adaptive Moving Total Exponential Least Square (AMTELS) method

This paper develops the least-square fitting exponential with AMTLS method and is called Adaptive Moving Total Exponential Least Square (AMTELS) method which its algorithm is described as follows:

The least-square fitting exponential is in the form

$$Y = Ae^{BX}. \quad (9)$$

Then, take logarithm of both sides, we obtain

$$\ln Y = \ln A + BX. \quad (10)$$

For convenient, let $y = \ln Y, x = X, b = \ln A$ and $a = B$.

Now, the model is changed into the form $y = ax + b$. Applying the least square criterion (10) to this situation requires the minimization of

$$J(a_x, b_x) = \sum_{I=1}^{n} w(x - x_I)\frac{\lambda^2 a_x{}^2 + 1}{a_x{}^2 + 1}(a_x x_I + b_x - y_I)^2 \quad (11)$$

where $y_I = \ln Y$ and $x_I = X$. Then, $\hat{Y} = \exp(y)$, where $\hat{Y}$ is the ruin probability fitting value in AMTELS method.

The weighted function is important framework of function for local approximants of both AMTLS and AMTELS methods. Throughout this research, we use the following weighted function:

$$w(x - x_I) = \begin{cases} \dfrac{e^{-(|x-x_I|/\alpha d)^2} - e^{-1/\alpha^2}}{1 - e^{-1/\alpha^2}} & \text{if } |x - x_I| \le d, \\ 0 & \text{elsewhere,} \end{cases} \quad (12)$$

which given by [11], where α is a parameter determining the weighted function's shape. The value of d has to be large enough to include a minimum number of nodes as mentioned above.

4 Simulation study

This experiment uses data of motor insurance company in Thailand. The purpose of this paper is to compare the performance of approximation of the ruin probability between AMTLS method and AMTELS method for exponential claim. The first operation, we consider the data and histogram of the claim size as shown in Fig.4, Fig.5, respectively. We found that scale parameter of exponential claim as 5.5168×10^{-6}, i.e., $X_1 \sim \exp(5.5168 \times 10^{-6})$.

After that, we determine

$n = 356,$

$c = (1 + 0.1)(1/5.5168 \times 10^{-6}),$

and then calculate the finite-time ruin probability $y_I, I = 1, 2, ..., 55,$ with exponential claim size by (3) for initial capital x_I $(x_I = 0, 0.1, 0.2, ..., 5).$

Next, we approximate the ruin probability of interested initial capital as $x = 0.00, 0.01, 0.02, ..., 5.00$ by AMTLS method with weighted function for the distance between node and point with a constant value $d = 0.5$, $\alpha = 0.5$ and $\lambda = 0$. The ruin probability function of finite-time surplus process with exponential claims is plotted in Fig.6.

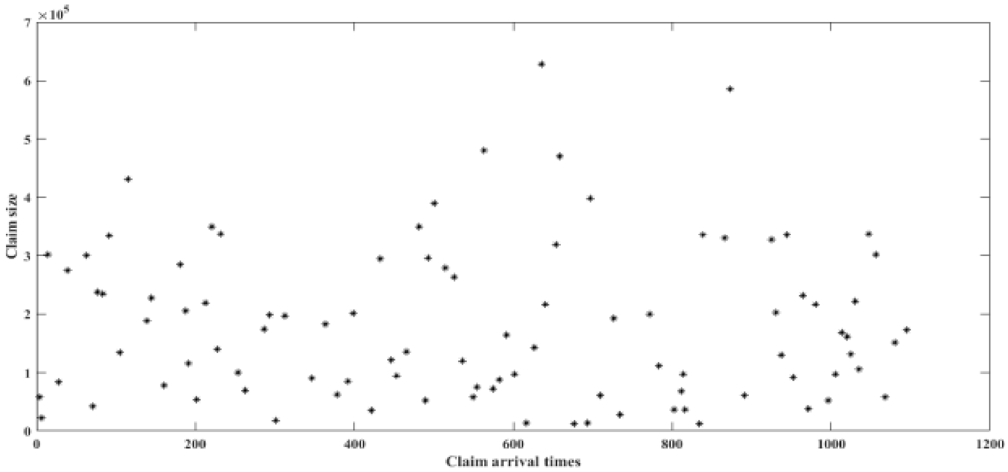

Fig.4 The claim size of motor insurance in Thailand

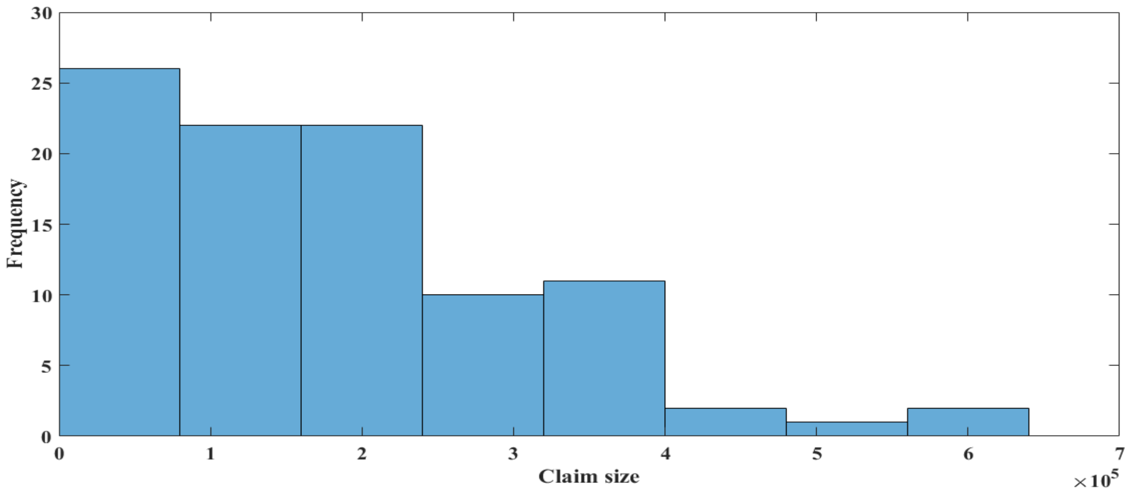

Fig.5 Histogram of claim data

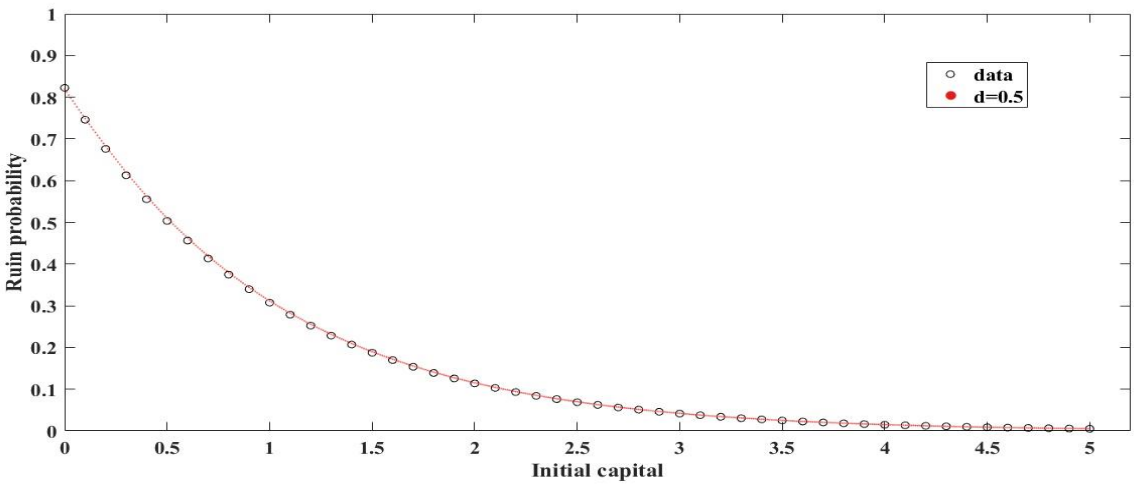

Fig.6 The approximation ruin probability

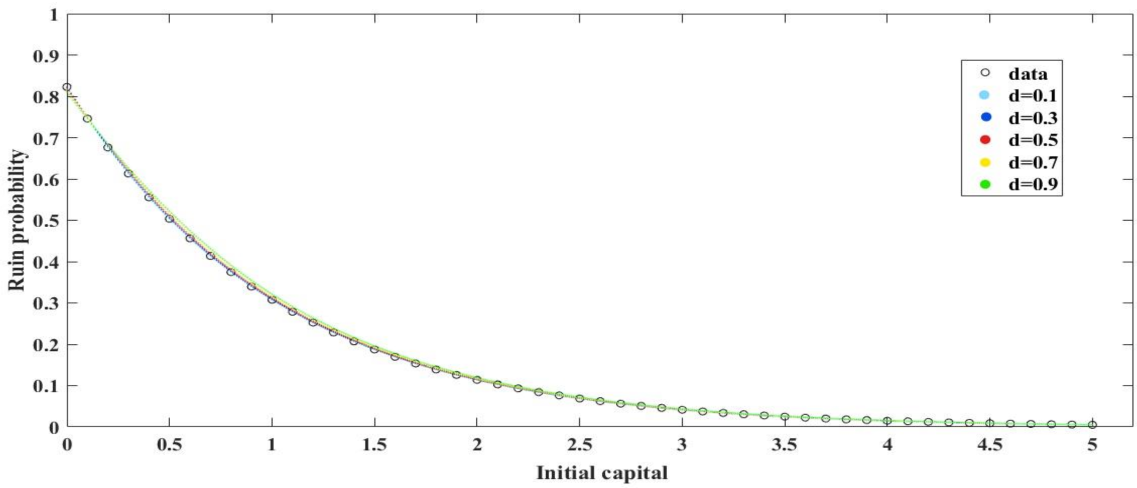

Fig.7 The ruin probability fitting of AMTLS method

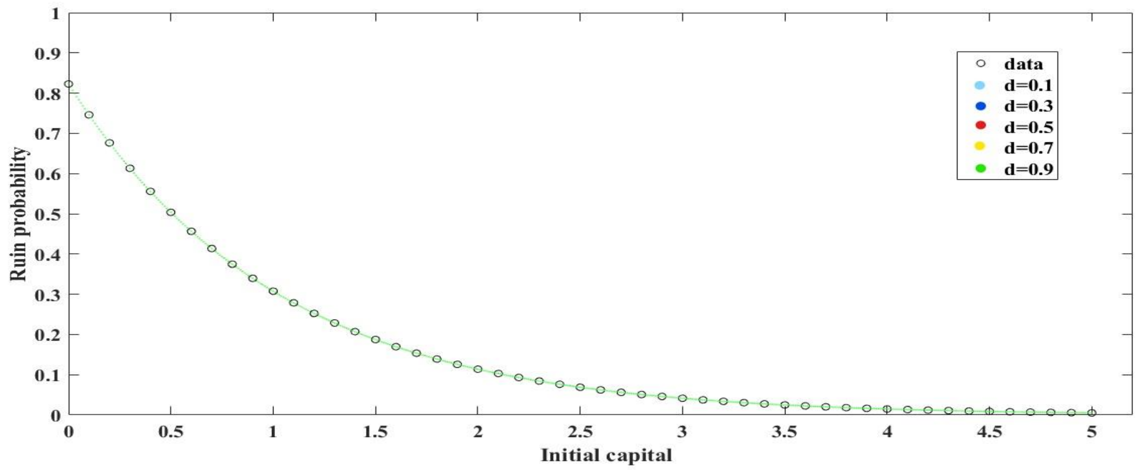

Fig.8 The ruin probability fitting of AMTELS method

From Fig.6, we found that the ruin probability function is also a decreasing function on the initial capital, and has a very clear exponential distribution function feature. Therefore, least square fitting exponential is developed to estimate ruin probability, which is AMTELS method. We now approximate of the ruin probability by AMTLS method and AMTELS method with weighted function that different constant value d ($d = 0.1, 0.3$ $0.5, 0.7,$ and 0.9), and $\alpha = 0.5$. These results are presented in Fig.7 and Fig.8, respectively.

Next, we compare the performance of both methods by using the mean squared error (MSE) and the mean absolute error (MAE) of the real ruin probability value that are obtained by the explicit formula (3) and the ruin probability fitting value:

$$MSE = \frac{1}{501}\sum_{i=1}^{501}(y_i - \hat{y}_i)^2, \tag{13}$$

$$MAE = \frac{1}{501}\sum_{i=1}^{501}| y_i - \hat{y}_i |, \tag{14}$$

where y_i is the real ruin probability value and $\hat{y}_i$ is the ruin probability fitting value for measured data. The experiment result is presented in Table 1.

For the same constant value d, it is clear that both MSE and MAE values of AMTELS method are less than AMTLS method. In the same method, both MSE and MAE values of constant value $d = 0.1$ are less than $d = 0.3, 0.5, 0.7, 0.9$, respectively.

Table 1. Experiment result of ruin probability estimators

d	MSE $(\times 10^{-12})$		MAE $(\times 10^{-6})$	
	AMTLS	AMTELS	AMTLS	AMTELS
0.1	52611.00	4.58	131.78	1.45
0.3	1290500.00	121.21	740.41	8.33
0.5	8315200.00	837.25	1900.00	22.07
0.7	26891000.00	2889.10	3500.00	41.34
0.9	62286000.00	7118.20	5500.00	65.41

5 Conclusion

In this study, AMTELS method is developed and represented to approximate the ruin probability. Its superior features are shown over AMTLS method. Suitability of both methods is judged base on MSE and MAE values for the weighted function with different constant value d. The ruin probability fitting of AMTELS method is given the better performance than the ruin probability fitting of AMTLS method. Furthermore, AMTELS method with the narrow value d shows the better performance than AMTELS method with the wide value d. Therefore, in this study, the appropriate method for approximating the ruin probability of finite-time surplus process is AMTELS method with the narrow value d and it does not take long. Although AMTELS method fit the ruin probability than AMTLS method, two methods don't have the explicit formulas of approximations for the ruin probability. These methods depend on regression analysis to approximate the fitting value.

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