The Growth of Companies as a Function of Total Assets

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Abstract: - Total Assets determines the size of the companies and allows classifying them by economic relevance in every industry. However, the path of company growth, measured by Total Assets, might be different depending on the type of industry and the size of companies. Accordingly, this research focuses on identifying the trend in Total Assets growth across industries and company size by finding a function that fits industries-company-size combinations. The method is analytical, deductive and empirical; it is a cross-sectional analysis with six industries in two years (three for every year) with four different company sizes, based on Total Assets, grouped into the categories of micro, small, medium or big companies, for a total of 24 industry-company-size-year combinations. Every combination of industry-company-size is analyzed to see which function yields the best fit. The functions are: 1) Linear, 2) Logarithmic, 3) Inverse, 4) Quadratic, 5) Cubic, 6) Compound, 7) Power, 8) S, 9) Growth, 10) Exponential, and 11) Logistic. The test consists of statistical regression analysis, ANOVA significance test and explained variance. The cubic function gives the best results in all industry-company-size combination for the two years. Other functions are relevant in some, but not all, combinations of categories. The conclusion is that cubic function provides the best fit for Total Assets company growth across industry-company-size combinations for the two years. Cubic function properties are described for future applications.

Key-Words: - Total Assets; company size; financial statements; industry classification.

1 Introduction

Total Assets are not only the company resources to making a profit, but also they define the company and its position in an economy. It is also an essential part of the accounting equation, the final computation of the sources and use of thereof. Accounting equation is regarded as merely a formula or, on the contrary, as a real-world relationship [1] with a considerable importance in the practice and education of financial accounting [see 2, 3, 4, 5, 6, pp. 101–105, 7, 8, 9, 10, 11, 12, 13, see 14, 15 for some discussion].

The link of total Assets to stakeholder's equity and liabilities adds great relevance to identify the management of debts and stakeholder investment return, i.e., the position in debt-risk ranks. The complexity of the Total Assets-Claims on Assets link has been pointed out in many occasions [see 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 31] too.

All of that reveals the nature of Total Assets and the need of analyzing its qualities. Besides, as an index to define the position of companies, it is crucial in bankruptcy prediction [31, 32, 33, 34, 35]. Moreover, assets management is both crucial [36, 37, 38, 39] and complex [e.g. 40, 41, 42].

According to all of the previously mentioned, analyzing company growth in a Total Assets scale acquires relevance and requires more in-depth analysis, so this research addresses the identification of the trend in Total Assets growth across industry-company size combinations. The paper is organized as follows. Section 2 provides the research problem and its justification. Section 3 introduces the methodological details and procedures. Section 4 includes the results. Finally, conclusions are in Section 5.

2 Problem Formulation

Due to the significance of Total Assets in earning management, performance [43, 44, 45 46], and liquidity [47], its evolution will determine the company position in any industry.

The evolution of Total Assets in a sector or industry usually requires a time series approach to analyzing several companies along the time. That allows observing specific companies evolution and their change in position according to Total Assets. However, determining the position of companies based on their Total Assets figures can take advantage of the natural classification in any sector or industry. That means taking the companies of any industry as ranked by their Total Assets under a cross-sectional view.

In this regard, once the classification is available by ranking the companies according to their position in an increasing sequence of Total Assets figures, it is also relevant to find the model that fits that increase. That model should explain the data sequence within any industry but should also take into consideration that particularities are different among industries.

Several functions can model a data sequence. To this, assumptions about data require taking the company as a random sample of a population occupying the full range of the industry. Then, the analysis can fit several functions to data and identify which one is the best.

Therefore, the purpose of this research is to find the best function fitting Company Total Assets growth data across industries and company size.

3 Problem Solution

2.1 Method

The method in this research was analytical, deductive and empirical. Financial Statements data from Superintendencia de Sociedades (Superintendence of Societies) of Colombia [48] provided the financial accounting records of economic activities in industries and the companies reports of Total Assets.

This study is part of a more comprehensive one regarding Total Assets growth and the model giving the best fit for that growth. For that purpose, this research takes six industries, based on Total Assets; every one of them with different Total Assets average than the other in ascendant order. The intention is to provide a test to see the differences and similarities in the function that applies to Total Assets growth within industries.

The analysis makes a further distinction among company size, based on the classification categories of micro, small, medium or big enterprise. Company size categories, in Colombia, are defined according to Total Assets by the following rule: a) Microenterprises. Total Assets up to 500 minimum legal wages; b) Small enterprises. Greater than 500 up to 5.000 minimum legal wages; c) Medium enterprises. Greater than 5.000 up to 30.000 minimum legal wages; and d) Big enterprises. Greater than 30.000 minimum legal wages. Besides, the analysis starts with combinations of industry and company size in the year 2010 and then conducts a replication with different industry-company-size combinations in the year 2015.

The approach is different from a time series analysis, where companies are analyzed along the time. Here, growth is the difference in Total Assets size within the industry companies or across the whole economy. The analysis is a cross-sectional with selected cases (industries); it means that results for all the companies are aggregated in every combination of industry and company size and replicated for new combinations and period.

That leads to a combination of two factors in a year with replication in another year, introducing industry and company size as variables, and the time as the replication variable.

The six industries are different in Total Assets size and their activities too, so they are a purposive sample with a wide range or maximum variation cases. There are three selected industries in the year 2010, which, according to ISIC (International Standard Industrial Classification) [49] are (data in thousands of Colombian pesos):

a) G5121. Wholesale companies (Total Assets average = 10378790.16, S.D. = 33569296.17).
b) F4521. Construction works of housing (Total Assets average = 13004015.37, S.D. = 32646026.41).
c) K7499. Other business activities (Total Assets average = 14372673.99, S.D. = 50247100.10).

The other three selected industries in the year 2015 are (data in thousands of Colombian pesos):

a) G4663. Wholesale of construction materials, hardware, paints, glass products, plumbing and heating equipment and materials (Total Assets average = 9337364.03, S.D. = 16394787.66).
b) L6810: Real estate activities carried out with own or leased property (Total Assets average = 16854841.35, S.D. = 39982505.33).
c) B0910. Support activities for the extraction of oil and natural gas (Total Assets average = 28582723.41, S.D. = 88285299.18).

Table 1 shows the factors and number of companies (N) in every combination; note that every combination includes the whole population of companies for that combination.

TABLE 1. RESEARCH DESIGN

YEAR			Company Size			
2010	**Industry**	**N**	**1. Micro**	**2. Small**	**3. Medium**	**4. Big**
	1. G5121	312	1.1	1.2	1.3	1.4
	2. F4521	923	2.1	2.2	2.3	2.4
	3. K7499	478	3.1	3.2	3.3	3.4
2015	**Industry**	**N**	**1. Micro**	**2. Small**	**3. Medium**	**4. Big**
	1. B0910	160	1.1	1.2	1.3	1.4
	2. G4663	489	2.1	2.2	2.3	2.4
	3. L6810	2737	3.1	3.2	3.3	3.4

All financial information is already recorded by Superintendence of Societies.

A set of functions enter the analysis to model the Total Assets growth within sectors; they are: 1) Linear, 2) Logarithmic, 3) Inverse, 4) Quadratic, 5) Cubic, 6) Compound, 7) Power, 8) S, 9) Growth, 10) Exponential, and 11) Logistic. The analysis tries to fit all of the functions to every industry-company-size combination by year; the explained variance and goodness-of-fit measures allow identifying the function, or functions, providing the best explanation of company growths within sectors.

2.2 Descriptive Analysis and Functions

Descriptive data for industries and company size for the years 2010 and 2015 are in Table 2 and 3. Total Assets average by company size is different in every industry, as well as the number of companies by size.

Despite these industries are ordered by average when looking at the tables their average by industry and company size is different and they do not seem to have the same order as the industry average, which is due to the standard deviation (S.D.) which is larger in some industries than in others. Nevertheless, they happen to have very different activities and total assets; that is the reason why they were selected.

TABLE 2. TOTAL ASSETS DESCRIPTIVE DATA BY INDUSTRY AND COMPANY SIZE[a] FOR THE YEAR 2010

Industry/ Company Size	N	Mín.	Máx.	$\bar{x}$	SD
G5121/1	8	19134.0	376458.0	185879.6	119505.8
G5121/2	160	407358.0	3869780.0	1651458.8	973330.2
G5121/3	123	3908685.0	23393861.0	9306618.8	4879416.0
G5121/4	21	24074271.0	428621098.0	87035617.5	102676421.5
F4521/1	50	16920.0	384954.0	222125.6	110578.9
F4521/2	390	392470.0	3898461.0	1901152.5	981858.3
F4521/3	375	3910305.0	23246258.0	9631533.9	4856935.1
F4521/4	108	23476859.0	493757147.0	70725233.3	71896659.4
K7499/1	29	4677.0	387770.0	259827.4	116333.0
K7499/2	223	410359.0	3883786.0	1717781.0	1012984.4
K7499/3	169	3911025.0	23420022.0	9074660.8	5052985.1
K7499/4	57	25226891.0	683050163.0	86770532.0	123548950.0

a: figures in thousands.

TABLE 3. TOTAL ASSETS DESCRIPTIVE DATA BY INDUSTRY AND COMPANY SIZE[a] FOR THE YEAR 2015

Industry/ Company Size	N	Mín.	Máx.	$\bar{x}$	SD
G4663/1	5	235814.0	377306.0	302856.8	55218.5
G4663/2	214	460771.0	3864165.0	2015781.1	941774.8
G4663/3	233	3912987.0	23394030.0	9303192.9	5091168.9
G4663/4	37	23848579.0	179898276.0	53119881.5	34221942.3
L6810/1	17	16896.0	390169.0	268563.0	118144.3
L6810/2	795	392405.0	3904261.0	2351920.5	928232.3
L6810/3	1520	3909306.0	23325841.0	9871162.0	4906432.1
L6810/4	405	23508469.0	704884767.0	72230104.2	83970089.6
B0910/1	3	216364.0	300075.0	247274.7	45948.4
B0910/2	44	522697.0	3557865.0	1787680.1	936228.7
B0910/3	81	3975835.0	23273115.0	10356991.5	5576847.3
B0910/4	32	24515583.0	845299821.0	114216240.1	174245087.1

a: figures in thousands.

Table 4 shows the type of regression and functions used in the computation [see 50].

2.3 Analysis by industry-company-size and year

The function $f: A \rightarrow \mathbb{N}$ provides an order, such as for every pair of companies in every industry-company-size combination Total Assets A_i, A_j, $A_i \leq A_j$; then, Total Assets of industries have an order such as $A_1 \leq A_2 \dots A_{n-1} \leq A_n$ in every industry and company size, and, despite some companies are intertwined due to large variance in industry Total Assets, they, in general, have an order too. Therefore, in Table 4, x is the sequence number in the ordered industry Total Assets, and y is company´s Total Assets.

TABLE 4. TYPE OF FUNCTIONS TO FIT TOTAL ASSETS GROWTH

Type of regression	*Function*
1. Linear	$y = \beta_0 + \beta_1 x$
2. Logarithmic	$y = \beta_0 + \beta_1 \ln(x)$
3. Inverse	$y = \beta_0 + \beta_1 / x$
4. Quadratic	$y = \beta_0 + \beta_1 x + \beta_2 x^2$
5. Cubic	$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$
6. Compound	$y = \beta_0 \beta_1^x$
7. Power	$y = \beta_0 x^{\beta_1}$
8. S	$y = \exp(\beta_0 + \beta_1 / x)$
9. Growth	$y = \exp(\beta_0 + \beta_1 x)$
10. Exponential	$y = \beta_0 e^{\beta_1 x}$
11. Logistic	$y = ((1 / u) + \beta_0 \beta_1^x)^{-1}$

Results for every industry and company size are in Tables 5-28.

All tables include ANOVA (Analysis of Variance) test showing the significance coefficient (p) of the model and the explained variance coefficient R^2. ANOVA $F = (SSr \;\; df_e / SS_e \, df_r)$; R^2 = 1- (SSe – SS_t); where SSr: Regression Sum of Square, SS_T: Total Sum of Squares; SS_e: Residual Sum of Squares. Beta coefficients β_0, β_1, β_2, β_3 are those of Table 4.

TABLE 5. RESULTS FOR INDUSTRY G5121 COMPANY SIZE 1 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.97	***	-30667.	48121.5		
Logarithmic	0.86	***	-22944.2	157534.5		
Inverse	0.65	*	296958.8	-326960.9		
Quadratic	0.98	***	2555.0	28188.3	2214.8	
Cubic	0.98	***	-21794.7	53521.8	-4426.0	491.9
Compound	0.84	***	28535.9	1.4		
Power	0.96	***	24489.4	1.3		
S	0.95	***	12.9	-3.1		
Growth	0.84	***	10.3	0.3		
Exponential	0.84	***	28535.9	0.3		
Logistic	0.84	***	$3.504\cdot10^{-5}$	0.7		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 6. RESULTS FOR INDUSTRY G5121 COMPANY SIZE 2 FOR THE YEAR 2010

	R^2	P	β_0	β_1	β_2	β_3
Linear	0.92	***	32452.7	20111.9		
Logarithmic	0.60	***	-1601606.5	794054.1		
Inverse	0.11	***	1770564.9	-3369628.0		
Quadratic	0.99	***	621016.7	-1686.8	135.4	
Cubic	1.00	***	379566.8	16034.0	-138.9	1.1
Compound	0.99	***	485334.0	1.0		
Power	0.83	***	127346.9	0.6		
S	0.22	***	14.3	-3.0		
Growth	0.99	***	13.1	0.01		
Exponential	0.99	***	485334.0	0.01		
Logistic	0.99	***	$2.060\cdot10^{-6}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 7. RESULTS FOR INDUSTRY G5121 COMPANY SIZE 3 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.88	***	1336282.4	128553.8		
Logarithmic	0.56	***	-5598613.3	3882356.8		
Inverse	0.11	***	9955750.3	-14803703.8		
Quadratic	0.97	***	4727499.8	-34224.6	1312.7	
Cubic	0.99	***	3010355.0	128662.3	-1958.0	17.6
Compound	0.99	***	3470275.0	1.0		
Power	0.74	***	1419182.1	0.5		
S	0.17	***	16.0	-1.9		
Growth	0.99	***	15.1	0.0		
Exponential	0.99	***	3470275.0	0.0		
Logistic	0.99	***	$2.882\cdot10^{-07}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 8. RESULTS FOR INDUSTRY G5121 COMPANY SIZE 4 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.55	***	-47606314.2	12240175.6		
Logarithmic	0.31	**	-65063945.2	70385214.7		
Inverse	0.11		114315323.8	-157151566.5		
Quadratic	0.79	***	81057442.3	-21324282.6	1525657.2	
Cubic	0.89	***	-27099354.4	31614123.1	-4352429.6	178123.8
Compound	0.88	***	14288167.4	1.1		
Power	0.62	***	10083879.0	0.8		
S	0.27	**	18.2	-2.0		
Growth	0.88	***	16.5	0.1		
Exponential	0.88	***	14288167.4	0.1		
Logistic	0.88	***	$6.999\cdot10^{-8}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 9. RESULTS FOR INDUSTRY F4521 COMPANY SIZE 1 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.96	***	32582.1	7433.1		
Logarithmic	0.89	***	-126394.3	117364.3		
Inverse	0.39	***	261581.8	-438479.7		
Quadratic	0.99	***	-12092.1	12587.8	-101.1	
Cubic	0.99	***	-14181.0	13056.7	-123.8	0.3
Compound	0.79	***	52317.3	1.1		
Power	0.98	***	13058.4	0.9		
S	0.65	***	12.5	-4.0		
Growth	0.79	***	10.9	0.1		
Exponential	0.79	***	52317.3	0.1		
Logistic	0.79	***	$1.911\cdot10^{-5}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 10. RESULTS FOR INDUSTRY F4521 COMPANY SIZE 2 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.99	***	211268.4	8643.9		
Logarithmic	0.70	***	-2295052.1	843262.9		
Inverse	0.09	***	1977782.6	-4566439.2		
Quadratic	1.00	***	473325.8	4632.8	10.3	
Cubic	1.00	***	445674.9	5476.1	4.9	0.01
Compound	0.97	***	589578.2	1.0		
Power	0.89	***	92781.5	0.6		
S	0.17	***	14.4	-3.9		
Growth	0.97	***	13.3	0.01		
Exponential	0.97	***	589578.2	0.01		
Logistic	0.97	***	$1.696\cdot10^{-6}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 11. RESULTS FOR INDUSTRY F4521 COMPANY SIZE 3 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.88	***	1749668.3	41924.8		
Logarithmic	0.54	***	-8529613.3	3678371.2		
Inverse	0.06	***	9944753.6	-18055157.1		
Quadratic	0.97	***	5103528.4	-11452.3	142.0	
Cubic	0.99	***	3080668.7	52680.5	-283.9	0.8
Compound	0.98	***	3789624.7	1.0		
Power	0.74	***	1080307.2	0.4		
S	0.10	***	16.0	-2.4		
Growth	0.98	***	15.2	0.0		
Exponential	0.98	***	3789624.7	0.0		
Logistic	0.98	***	$2.639\cdot10^{-7}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 12. RESULTS FOR INDUSTRY F4521 COMPANY SIZE 4 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.54	***	-20871507.5	1680674.1		
Logarithmic	0.27	***	-79302593.2	40413456.0		
Inverse	0.04	***	77223870.8	-133331484.8		
Quadratic	0.78	***	59771736.5	-2718048.3	40355.3	
Cubic	0.88	***	-4756160.3	4226425.5	-118190.0	969.7
Compound	0.88	***	17030644.7	1.0		
Power	0.56	***	6646595.3	0.6		
S	0.12	***	17.9	-2.1		
Growth	0.88	***	16.7	0.01		
Exponential	0.88	***	17030644.7	0.01		
Logistic	0.88	***	$5.872\cdot10^{-8}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 13. RESULTS FOR INDUSTRY K7499 COMPANY SIZE 1 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.84	***	71480.1	12556.5		
Logarithmic	0.95	***	-69454.5	134010.3		
Inverse	0.64	***	324668.9	-474650.8		
Quadratic	0.96	***	-25973.0	31418.4	-628.7	
Cubic	0.98	***	-75974.5	49873.2	-2140.9	33.6
Compound	0.55	***	50206.3	1.1		
Power	0.88	***	11601.8	1.2		
S	0.91	***	12.9	-5.1		
Growth	0.55	***	10.8	0.1		
Exponential	0.55	***	50206.3	0.1		
Logistic	0.55	***	$1.992\cdot10^{-5}$	0.9		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 14. RESULTS FOR INDUSTRY K7499 COMPANY SIZE 2 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.96	***	-6978.5	15399.6		
Logarithmic	0.64	***	-2024070.2	845918.5		
Inverse	0.09	***	1819738.3	-3797876.0		
Quadratic	1.00	***	422115.6	3957.1	51.1	
Cubic	1.00	***	387225.9	5805.6	30.5	0.1
Compound	0.99	***	453221.6	1.0		
Power	0.82	***	89682.8	0.6		
S	0.17	***	14.3	-3.3		
Growth	0.99	***	13.0	0.01		
Exponential	0.99	***	453221.6	0.01		
Logistic	0.99	***	$2.206\cdot10^{-6}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 15. RESULTS FOR INDUSTRY K7499 COMPANY SIZE 3 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.84	***	1045885.5	94456.2		
Logarithmic	0.50	***	-6522408.3	3757862.3		
Inverse	0.08	***	9578891.1	-14923619.9		
Quadratic	0.96	***	5099149.4	-47763.6	836.6	
Cubic	0.99	***	2955501.9	101357.6	-1349.9	8.6
Compound	0.97	***	3338417.5	1.0		
Power	0.69	***	1270925.4	0.4		
S	0.13	***	16.0	-2.0		
Growth	0.97	***	15.0	0.01		
Exponential	0.97	***	3338417.5	0.01		
Logistic	0.97	***	$2.995\cdot10^{-7}$	1.0		

*: $p \leq ,05$; **: $p \leq ,01$; ***: $p \leq ,001$

TABLE 16. RESULTS FOR INDUSTRY K7499 COMPANY SIZE 4 FOR THE YEAR 2010

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.33	***	-36976760.6	4267148.0		
Logarithmic	0.17	**	-87564055.6	56333930.7		
Inverse	0.04		99796933.3	-160402410.2		
Quadratic	0.53	***	93983395.6	-9050834.0	229620.4	
Cubic	0.70	***	-52873613.7	20074518.8	-1014930.6	14305.2
Compound	0.80	***	19775639.9	1.04		
Power	0.55	***	9670834.9	0.6		
S	0.18	***	18.1	-2.0		
Growth	0.80	***	16.8	0.04		
Exponential	0.80	***	19775639.9	0.04		
Logistic	0.80	***	$5.057\cdot10^{-8}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 17. RESULTS FOR INDUSTRY B0910 COMPANY SIZE 1 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.83		163563.7	41855.5		
Logarithmic	0.70		205829.2	69393.4		
Inverse	0.58		308846.0	-100753.2		
Quadratic	1.00	[a]	273012.0	-89482.5	32834.5	
Cubic	1.00	[a]	224203.4	0.001	-15974.1	8134.8
Compound	0.84		176360.7	1.2		
Power	0.72		207919.1	0.3		
S	0.60		12.7	-0.4		
Growth	0.84		12.1	0.2		
Exponential	0.84		176360.7	0.2		
Logistic	0.84		$5.670\cdot10^{-6}$	0.9		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$; a: not enough cases for the analysis.

TABLE 18. RESULTS FOR INDUSTRY B0910 COMPANY SIZE 2 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.96	***	183101.3	71314.6		
Logarithmic	0.70	***	-741889.4	888154.2		
Inverse	0.26	***	2070450.2	-2845337.5		
Quadratic	0.99	***	591419.3	18055.8	1183.5	
Cubic	0.99	***	516495.1	36980.5	143.9	15.4
Compound	0.98	***	582119.8	1.0		
Power	0.88	***	281914.3	0.6		
S	0.41	***	14.5	-2.2		
Growth	0.98	***	13.3	0.04		
Exponential	0.98	***	582119.8	0.04		
Logistic	0.98	***	$1.718\cdot10^{-6}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 19. RESULTS FOR INDUSTRY B0910 COMPANY SIZE 3 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.92	***	1040497.6	227231.6		
Logarithmic	0.60	***	-5783076.0	4701538.7		
Inverse	0.14	***	11365279.9	-16407038.0		
Quadratic	0.99	***	4531139.0	-25104.0	3077.3	
Cubic	1.00	***	3619822.6	104306.2	-844.1	31.9
Compound	0.99	***	3557056.2	1.0		
Power	0.76	***	1567652.5	0.5		
S	0.22	***	16.1	-2.0		
Growth	0.99	***	15.9	0.02		
Exponential	0.99	***	3557056.2	0.02		
Logistic	0.99	***	$2.811\cdot10^{-7}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 20. RESULTS FOR INDUSTRY B0910 COMPANY SIZE 4 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.43	***	-87810945.4	12244071.9		
Logarithmic	0.22	**	-130868278.4	96161118.4		
Inverse	0.06		143497689.1	-230875317.9		
Quadratic	0.73	***	139564300.8	-27880971.6	1215910.4	
Cubic	0.89	***	-84305591.1	47768467.3	-4427868.4	114015.7
Compound	0.82	***	14972521.4	1.1		
Power	0.55	***	8497081.1	0.8		
S	0.21	**	18.3	-2.2		
Growth	0.82	***	16.5	0.19		
Exponential	0.82	***	14972521.4	0.1		
Logistic	0.82	***	$6.679\cdot10^{-8}$	0.9		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 21. RESULTS FOR INDUSTRY G4663 COMPANY SIZE 1 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.95	**	200519.6	34112.4		
Logarithmic	0.87	*	225151.3	81154.7		
Inverse	0.73		369446.7	-145817.4		
Quadratic	0.96	*	221528.6	16104.7	3001.3	
Cubic	0.97		186231.8	65688.3	-15907.7	2101.0
Compound	0.96	**	213015.7	1.1		
Power	0.91	**	230074.7	0.3		
S	0.79	*	12.8	-0.5		
Growth	0.96	**	12.3	0.1		
Exponential	0.96	**	213015.7	0.1		
Logistic	0.96	**	$4.694 \cdot 10^{-6}$	0.9		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 22. RESULTS FOR INDUSTRY G4663 COMPANY SIZE 2 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.98	***	395226.2	15074.9		
Logarithmic	0.72	***	-1658438.4	838325.2		
Inverse	0.14	***	2131366.1	-4160300.9		
Quadratic	0.99	***	626221.5	8658.4	29.8	
Cubic	1.00	***	468555.7	17357.1	-71.1	0.3
Compound	0.96	***	722576.2	1.0		
Power	0.91	***	176232.0	0.5		
S	0.25	***	14.5	-3.2		
Growth	0.96	***	13.5	0.01		
Exponential	0.96	***	722576.2	0.01		
Logistic	0.96	***	$1.384 \cdot 10^{-6}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 23. RESULTS FOR INDUSTRY G4663 COMPANY SIZE 3 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.85	***	1160075.8	69599.3		
Logarithmic	0.50	***	-7455615.6	3751959.7		
Inverse	0.06	***	9718253.0	-16036917.4		
Quadratic	0.98	***	5301895.4	-36149.3	451.9	
Cubic	1.00	***	3422993.5	59185.4	-564.4	2.9
Compound	0.97	***	3476131.2	1.0		
Power	0.68	***	1203925.5	0.4		
S	0.11	***	16.0	-2.1		
Growth	0.97	***	15.1	0.01		
Exponential	0.97	***	3476131.2	0.01		
Logistic	0.97	***	$2.877 \cdot 10^{-7}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 24. RESULTS FOR INDUSTRY G4663 COMPANY SIZE 4 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.67	***	3973546.1	2586649.2		
Logarithmic	0.40	***	-14014944.4	25007281.2		
Inverse	0.13	*	60947028.4	-68927404.7		
Quadratic	0.86	***	39950073.0	-2948201.1	145654.0	
Cubic	0.94	***	10841376.6	5674415.5	-414128.7	9820.7
Compound	0.89	***	19993321.3	1.0		
Power	0.64	***	13305890.4	0.5		
S	0.25	**	17.8	-1.4		
Growth	0.89	***	16.8	0.04		
Exponential	0.89	***	19993321.3	0.04		
Logistic	0.89	***	$5.002 \cdot 10^{-8}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 25. RESULTS FOR INDUSTRY L6810 COMPANY SIZE 1 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.86	***	72947.0	21735.1		
Logarithmic	0.96	***	-18322.8	145561.8		
Inverse	0.75	***	356085.8	-432581.6		
Quadratic	0.97	***	-29147.3	53975.4	-1791.1	
Cubic	0.98	***	-59994.8	72014.9	-4226.5	90.2
Compound	0.62	***	68918.1	1.1		
Power	0.90	***	31686.7	1.0		
S	0.97	***	13.0	-3.4		
Growth	0.62	***	11.1	0.1		
Exponential	0.62	***	68918.1	0.1		
Logistic	0.62	***	$1.451 \cdot 10^{-5}$	0.9		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 26. RESULTS FOR INDUSTRY L6810 COMPANY SIZE 2 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	1.00	***	745588.8	4036.0		
Logarithmic	0.81	***	-2467175.3	847880.3		
Inverse	0.08	***	2406962.4	-6030482.7		
Quadratic	1.00	***	666519.4	4631.3	-0.8	
Cubic	1.00	***	599323.3	5641.1	-3.9	0.0
Compound	0.91	***	968384.3	1.0		
Power	0.95	***	145050.0	0.5		
S	0.17	***	14.6	-4.4		
Growth	0.91	***	13.8	0.001		
Exponential	0.91	***	968384.3	0.001		
Logistic	0.91	***	$1.033 \cdot 10^{-6}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 27. RESULTS FOR INDUSTRY L6810 COMPANY SIZE 3 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.91	***	1763757.0	10660.6		
Logarithmic	0.56	***	-13560251.8	3701949.1		
Inverse	0.02	***	9991034.8	-23052432.6		
Quadratic	0.98	***	4768738.5	-1185.6	7.8	
Cubic	1.00	***	3390157.9	9673.0	-10.1	0.01
Compound	0.99	***	3804594.2	1.0		
Power	0.73	***	621511.0	0.4		
S	0.04	***	16.0	-3.0		
Growth	0.99	***	15.2	0.001		
Exponential	0.99	***	3804594.2	0.001		
Logistic	0.99	***	$2.628 \cdot 10^{-7}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

TABLE 28. RESULTS FOR INDUSTRY L6810 COMPANY SIZE 4 FOR THE YEAR 2015

	R^2	p	β_0	β_1	β_2	β_3
Linear	0.43	***	-23771809.8	472915.8		
Logarithmic	0.21	***	-124661769.9	39271800.8		
Inverse	0.01	*	74913065.9	-165078075.8		
Quadratic	0.66	***	65999701.6	-850497.1	3259.6	
Cubic	0.79	***	-14368760.0	1510382.0	-11259.9	23.8
Compound	0.86	***	17277984.0	1.0		
Power	0.53	***	3937013.5	0.5		
S	0.05	***	17.9	-2.6		
Growth	0.86	***	16.7	0.01		
Exponential	0.86	***	17277984.0	0.01		
Logistic	0.86	***	$5.788 \cdot 10^{-8}$	1.0		

*: $p \leq 0.05$; **: $p \leq 0.01$; ***: $p \leq 0.001$

The comparisons among these goodness-of-fit indicators give the relevance of the functions and which one of them yields the best results. The higher explained variance and the lower the significance coefficient, the better.

As shown in Tables 5-28, the cubic function provides the most usually significant regression (*p*) and explained variance; while other regressions are significant in some cases, their combinations of significance and explained variance is lower than that of the cubic function. The function is numbered 5 in Table 4, and their coefficients β_0, β_1, β_2, β_3 are industry and company size dependents.

Table 29 gives a summary of the results; cubic function has appropriate significance and explained variance in 21 out of 24 industry-company-size combinations, being the most relevant in 12, and as relevant as others in 11, while the group of Compound/Growth/Exponential/Logistic functions has a good significance and explained variance in four industry-company-size combinations, two of them along with cubic function.

The quadratic function also resulted competitive with appropriate significance and explained variance in eight combinations, seven of them along with cubic function; only one has it as the best fit.

TABLE 29. SUMMARY OF THE BEST FUNCTION FITS

	Company Size			
Industry	1. Micro	2. Small	3. Medium	4. Big
2010/ G5121	Quadratic/ Cubic	Cubic	Cubic/ Compound/ Growth/ Exponential/ Logistic	Cubic
2010/ F4521	Quadratic/ Cubic	Quadratic/ Cubic	Cubic	Cubic/ Compound/ Growth/ Exponential/ Logistic
2010/ K7499	Cubic	Quadratic/ Cubic	Cubic	Compound/ Growth/ Exponential/ Logistic
2015/ B0910	Quadratic/ Cubic	Quadratic/ Cubic	Cubic	Cubic
2015/ G4663	Quadratic	Cubic	Cubic	Cubic
2015/ L6810	Cubic	Quadratic/ Cubic	Cubic	Compound/ Growth/ Exponential/ Logistic

According to these results, the cubic function is the best model for the industry-company-size growth in economic sectors. The cubic function fits, despite the difference in distance between Total Assets scores, and provides a path to growth free from the industry and company size. However those combinations create differences in the coefficients of the function, so they have large influence on them giving the model a second explanation level, once the properties of the function are fully explored.

2.4 Properties of Cubic Function

One of the main properties regarding the cubic function is that there is an approximation cubic-to-normal distribution. Keeping in mind that this analysis used the population in every industry-company-size combination, be μ the mean, σ the standard deviation, α_3 the skewness and α_4 the kurtosis of the distribution of a variable *X*. Then, according to Fleishman [51] and Zhao and Lu [52], the polynomial transformation is:

$$\frac{X-\mu}{\sigma} = S_U(U) = a_{1+}a_2U + a_3U^2 + a_4U^3 \qquad (1)$$

The distribution has CDF: $F(X)=\Phi(U)$, and PDF:

$$f(X) = \frac{\phi(U)}{\sigma(a_{2+}2a_3U + 3a_4U^2)} \qquad (2)$$

Where *F*: X CDF, *f*: X PDF μ: X mean, σ: X standard deviation, Φ: CDF of standard normal random variable *U*, ϕ: PDF of a standard normal random variable *U*; a_1, a_2, a_3, a_4: polynomial coefficients. According to Zhao and Lu [52], equating the first four central moment of $S_U(U)$ to those of $X_s = (X-\mu)/\sigma$ would allow obtaining the parameters a_1, a_2, a_3, and a_4 (see Zhao and Lu for a full explanation [52]). Besides, they develop a system to find the parameters a_1, a_2, a_3, and a_4 for every combination of α_3 and α_4 values, by looking for α_3 and α_4 entry values in a table.

In this research coefficients of the cubic function for the variable *x* are already identified (β_0, β_1, β_2, and β_3 in Tables 5-28), by curvilinear regression. However, the variable is not in standard form, which is required in (1).

Now, using the obtained coefficients, the cubic function can be assumed as comprising a random standard normal variable *U* with μ, σ, α_3 and α_4 moments involving those coefficients; i.e.– it exists a normal standard random variable with the same mean, standard deviation, skewness and kurtosis as that of *x* that fits the obtained cubic equations, and then identify the *u* values. Otherwise, by applying (1) it gives new values for standard *x*.

Besides, the cubic function has two critical points and an inflection point. The first derivate allows identifying the critical points; it is:

$$y' = \beta_1 + 2\beta_2x + 3\beta_3x^2 \qquad (3)$$

Its solution, once provided discriminant Δ ($b^2 - 4ac$) be positive, gives two critical points x_1, x_2 in which the function gets to a local minimum/maximum. The inflection point ($- \beta_2 / 3\beta_3$) signs a change in concavity and builds the symmetry of the function. In this way, the function gives valuable information regarding the growth of companies according to Total Assets in industries.

In summary, the properties of cubic equation make it a base for building a useful classification system for the evolution of the growth of companies.

4 Conclusion

The preliminary results of this research are relevant. They suggest that a function seems to be underlying the increase in Total Assets and the company positions in every industry. Moreover, there seems to be only one relevant function across industries and company size along the years. Other functions were relevant only occasionally. However, they are not disregarded, for now, as it requires other confirmatory analyses.

The results confirm that future research is grounded on an appropriate basis, but it requires more in in-depth analyses, and the research will expand their sample to including more industries and explore in-depth the utility of the Total Assets growth function.

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