Optimal Location and Sizing of Capacitors in Radial Distribution Networks Using an Exact MINLP Model for Operating Costs Minimization

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Abstract: This paper presents an exact mixed-integer nonlinear programming (MINLP) model for optimal sizing and location of capacitor banks in radial distribution networks (RDN). The total operating costs of the RDN are minimized taking into account the classical restrictions of the electrical system, which are modeled by using a power flow formulation. Only fixed capacitor banks are considered in this paper. A commercial optimization package known as general algebraic modeling system (GAMS) is employed to implement the MINLP, ant its solution is obtained by CONOPT package. The effectiveness and robustness of the proposed mathematical model is tested in two RDNs with 10 and 33 nodes respectively. Additionally, three different demand scenarios are considered for simulation analysis.

Key–Words: Capacitors, commercial optimization package, exact mathematical model, demand scenarios, mixed-integer nonlinear programming (MINLP), power losses minimization, radial distribution network (RDN).

Nomenclature and notations

Acronyms

CONOPT	:	Optimization package for MINLP problems.
EDC	:	Electric distribution company.
GAMS	:	General algebraic modeling system.
MINLP	:	Mixed-integer nonlinear programming.
RDN	:	Radial distribution network.

Sets

Ω_N	:	Set that contains all nodes of the system.
Ω_T	:	Set that contains all periods of time of the load curve.

Parameters and sub-indexes

C_E	:	Average cost of energy [$/Wh].
C_I	:	Cost of each kVA reactive installed [$/VAr].
C_L	:	Location cost of a bank of capacitors [$].
C_O	:	Operating and maintenance cost of a bank of capacitors[$].
i, j	:	Sub-indexes associated with the nodes of the RDN.
$N^C_{\max}$	:	Maximum number of capacitor banks allowed in the system.
$P^D_{i,t}$	:	Active power demanded in the node i at the period of time t [W].
$Q^D_{i,t}$	:	Reactive power demanded in the node i at the period of time t [VAr].
$Q^{C,\max}_i$	:	Maximum reactive power penetration permitted in the node i [VAr].
T_t	:	Duration of the load curve in the period t [h].
t	:	Subindex associated with the periods of time.
$V^{\max}_i$	:	Maximum voltage regulation allowed in the node i [V].
$V^{\min}_i$	:	Minimum voltage regulation allowed in the node i [V].
Y_{ij}	:	Magnitude of the admittance associated with the i, j position in the admittance matrix [S].
ϕ_{ij}	:	Angle of the admittance associated with the i, j position in the admittance matrix [rad].

Variables

$P_{i,t}^{G}$: Active power generated by the conventional generator located at the note i at the period of time t [W].

$Q_{i,t}^{G}$: Reactive power generated by the conventional generator located at the note i at the period of time t [VAr].

Q_{i}^{C} : Reactive power generated by the bank of capacitors located at the note i [VAr].

$V_{i,t}$: Magnitude of the voltage in the node i at the period of time t [V].

$V_{j,t}$: Magnitude of the voltage in the node j at the period of time t [V].

$\theta_{i,t}$: Angle of the voltage in the node i at the period of time t [V].

$\theta_{j,t}$: Angle of the voltage in the node j at the period of time t [V].

x_i : Decision binary variable that defines if a bank of capacitors is located in the node i ($x_i = 1$); it takes zero otherwise.

1 Introduction

1.1 General context

Electric distribution networks correspond to the final component in the productive chain generation-transmission-commercialization-distribution [1, 2]. It is the connection of the largest amount of end users, for this reason it is one of the most important and complex component of the productive chain, since its infrastructure is enormous in comparison to the other components of the electric power system [3].

Typically the electric distribution networks have radial topology and for this reason this systems are widely known as radial distribution networks or RDNs [4]. The radial configuration causes high levels of active and reactive power losses, degradation in the voltage profiles, mainly generated by inductive loads connected to the distribution levels [2]. Nevertheless, these systems have the responsibility to attend all end-users guaranteeing reliability, security and quality conditions in all possible operating scenarios [5, 6]. In this context, multiple research papers are focused to improve their operating conditions. One of the most classical and popular methodology to reduce power losses and improve the voltage profile it is the location and sizing of capacitors in the RDN [7, 8, 9].

Notwithstanding there exist multiple references in specialized literature of this topic, we are mainly interested in present an exact MINLP formulation to formulate this problem. Additionally, the solution of the model is obtained by using a commercial optimization package known as GAMS [10, 11], which implies the possibility to scale this model for any number of nodes in the RDN, at the same time that it can be implemented for any electric distribution company (EDC) in a short period of time. The main advantage of use commercial software is that it has the possibility to make multiple simulation cases in few time and also it can guarantee in some cases the global solution of the optimization problem under analysis [12].

1.2 Motivation

Electrical distribution networks need to increase constantly their infrastructure to support all active and reactive power consumption. This load behavior is intimately related with the economic growth of society, is this, the gross domestic product of each country [2, 3]. To support this economic growing, the electric distribution companies (EDC) are obligate to expand and operate their electrical networks under reliability, security and quality their electrical networks [11].

One of the most classical ways to improve the quality of the electrical service by the EDC is installing shunt components in their electrical networks [8]. The shunt capacitor banks correspond to an economic and reliable alternative to reduce power losses and improve the voltage profile, taking advantage of the RDN loads typically have a resistive-inductive behavior [13].

We are mainly interested to formulated a MINLP model for optimal location and sizing of fixed capacitor banks in RDN under different cases of load consumption.

1.3 Related works

In specialized literature, the optimal placement and sizing of capacitor banks in RDN has been widely explored. Some authors have proposed metaheuristics optimization models, such as: genetic [14] and taboo search algorithms [15], crow search algorithm [16], particle swarm optimization [17] and gravitational search algorithms [18], whale optimization [19] and ant colony algorithms [20], among others [21, 22, 23, 24, 25] .

Other approaches prefer use heuristic algorithms [26, 27] based on expert knowledge by using artificial intelligent as are the cases of neural networks [28] and fuzzy logic approaches [13]. There also exist strategies based on combinations between heuristics and metaheuristics techniques with optimal power flow models [29].

Notwithstanding, in specialized literature there exists few papers that presents exact mathematical models. In this way, there are some linear approximations [30] and simplified nonlinear models to formulated the problem under study in this paper [31].

1.4 Contribution and scope

This paper focuses in the possibility to improve the quality service provide by the EDC to the end-users. For this reason we analyze electrical networks with radial topology, because these networks correspond to the topology more employed in distribution levels [4]. In this way, we proposed a MINLP model for optimal placement and sizing of shunt capacitor banks in RDN.

As simulation cases we consider three load consumption cases, in order to highlighted the direct dependence between the capacitor banks and their sizing whit the power losses and voltages' profile in the distribution grid [11]. To the best of the authors' knowledge, this optimization problem has not been analyzed considering different load behaviors, which produces a clear opportunity of investigation.

This investigation only considers shunt fixed capacitor banks location [13] under three possible load curves. The period of analyzed, correspond a to a year of continuous operation. Additionally the capacitor banks are considered as reactive power generator to simplify the power flow analysis.

1.5 Organization

The paper is organized as follows: Section 2 presents the full MINLP formulation of the problem of optimal location and sizing of capacitors in RDNs. Section 3 shows the main characteristics of the two RDN test feeder analyzed. The results and computational implementation is presented in Section 4; following by the conclusions in Section 5 and the reference list, respectively.

2 Mathematical model

This section presents the mixed-integer nonlinear programming model for optimal placement and dimensioning of capacitor in radial distribution networks. We consider fixed capacitor banks in this formulation [13]. This mathematical model takes into account a minimization objective function and the classical constraints of the steady state operation of electrical grids, such as: voltage regulation, active and reactive power balance and physical constraints of the capacitors and generators [11]. The mathematical model is defined as follows:

$$\min z = \begin{cases} C_E \sum\limits_{t\in\Omega_T} \sum\limits_{i\in\Omega_N} \left(P_{i,t}^G - P_{i,t}^D\right) T_t \\ C_I \sum\limits_{i\in\Omega_N} Q_i^C + (C_O + C_L) \sum\limits_{i\in\Omega_N} x_i \end{cases} \tag{1}$$

s.t.

$$P_{i,t}^G = \begin{cases} V_{i,t} \sum\limits_{j\in\Omega_N} V_{j,t} \cos\left(\theta_{i,t} - \theta_{j,t} - \phi_{ij}\right) \\ +P_{i,t}^D \quad \begin{Bmatrix} \forall i \in \Omega_N \\ \forall t \in \Omega_T \end{Bmatrix} \end{cases} \tag{2}$$

$$Q_{i,t}^G = \begin{cases} V_{i,t} \sum\limits_{j\in\Omega_N} V_{j,t} \sin\left(\theta_{i,t} - \theta_{j,t} - \phi_{ij}\right) \\ +Q_{i,t}^D - Q_i^C \quad \begin{Bmatrix} \forall i \in \Omega_N \\ \forall t \in \Omega_T \end{Bmatrix} \end{cases} \tag{3}$$

$$V_i^{\min} \leq V_{i,t} \leq V_i^{\max} \quad \begin{Bmatrix} \forall i \in \Omega_N \\ \forall t \in \Omega_T \end{Bmatrix} \tag{4}$$

$$0 \leq Q_i^C \leq x_i Q_i^{C,\max} \quad \left\{ \forall i \in \Omega_N \right\} \tag{5}$$

$$\sum_{i\in\Omega_N} x_i \leq N_{\max}^C \tag{6}$$

In the above formulation, the objective function defined by (1) corresponds to an operative and investment costs for one year of operation. Its first component represents the cost of active power losses, the second component calculates the cost of investment and maintenance of each bank of capacitors installed in the distribution network [32]. Equations (2) and (3) allow to calculate the active and reactive power balance in each node of each period of time. On the other hand, (4) guarantees the voltage regulation in all nodes of the distribution network for each period of time. Finally, (5) defines the maximum number of condensers available and (6) shows the binary nature of the capacitor decision variables.

Notice that the mathematical formulation presented from (1) to (6) is clearly a MINLP model, since the power flow variables are continuous and the decision variable of location of a condenser in the i node is binary.

3 Test system and simulation cases

To validate the proposed mathematical model, we use two RDNs. Those networks have been widely employed in specialized literature for different optimization problems. The information of theses RDNs will be presented in the following sections.

3.1 First RDN test system

This RDN test feeder is conformed by 10 nodes, 9 lines and operates with a voltage level of 23 kV. The total active and reactive power demanded by the loads are 12.368 MW and 4.186 MVAr, respectively [13]. The line connections and their parameters, active and reactive power consumed by each node and its voltage profile are presented in Fig. 1 and Table 1, respectively.

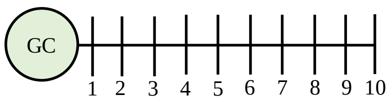

Figure 1: Single-phase diagram for the first test feeder

This RDN has an active and reactive power losses equivalent to 861.44 kW and 1049.80 kVAr, respectively [13]. This power losses have been computed considering load peak operation. Additionally, it should be noted that the 10 node has the worst voltage regulation with a 16.87% of deviation.

3.2 Second RDN test system

This RDN test feeder is conformed by 33 nodes, 32 lines and operates with a voltage level of 12.66 kV. The total active and reactive power demanded by the loads are 3715 kW and 2300 kVAr, respectively [16]. The line connections and their parameters, active and reactive power consumed by each node and its voltage profile are presented in Fig. 2 and Table 2, respectively.

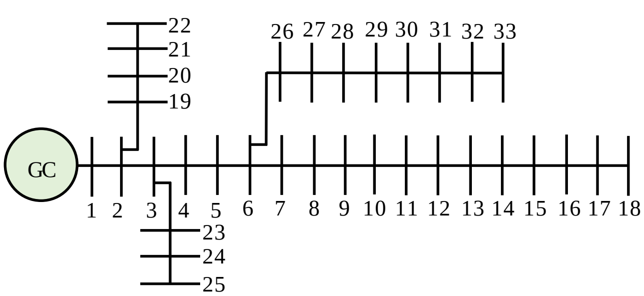

Figure 2: Single-phase diagram for the second test feeder

This RDN has an active and reactive power losses equivalent to 210.99 kW and 143.13 kVAr, respectively. This power losses have been calculated considering load peak operation. Additionally, it should be noted that the 18 node has the worst voltage regulation with a 9.62% of deviation.

3.3 Simulation cases

To evaluated the proposed mathematical model, we consider three possible demand cases during one year of continuous operation, as follows:

i. In the first case a operation under peak load condition it is considered [13].

ii. In the second case are considered that the demand consumption have been discretized in three main periods. The total demand (load peak) appears during 1000 hours/year, the 60% of total load appears during 6760 hours/year, and the rest to the time the total demand decreases until 30% [33].

iii. In the third case we considers that the active and reactive power consumption of the system has a typical daily behavior of Colombian EDC [11]. This behavior is depicted in Fig. 3

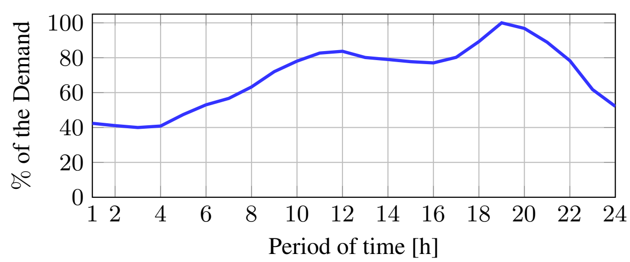

Figure 3: Percentage of demand consumption

3.4 Cost parameters

The calculation of the objective function proposed in the MINLP model defined by (1) is made by using the parameters presented in Table 3.

Table 3: Cost parameters

Parameter	value	unit
C_E	0.06	$/kWh
C_I	3	$/kVAr
C_L	1000	$/year
C_O	300	$/year

The cost parameters above presented have been adapted from [32].

Table 1: Parameters of the 10 nodes test feeder

Node i	Node j	R [Ω]	X [Ω]	P [kW]	Q [kVAr]	V [p.u]
1	2	0.1233	0.4127	1840	460	0.9929
2	3	0.2467	0.6051	980	340	0.9823
3	4	0.7469	1.2050	1790	446	0.9581
4	5	0.6984	0.6084	1598	1840	0.9427
5	6	1.9837	1.7276	1610	600	0.9116
6	7	0.9057	0.7886	780	110	0.9015
7	8	2.0552	1.1640	1150	60	0.8832
8	9	4.7953	2.7160	980	130	0.8527
9	10	5.3434	3.0264	1640	200	0.8313

Table 2: Parameters of the 33 nodes test feeder

Node i	Node j	R [Ω]	X [Ω]	P [kW]	Q [kVAr]	V [p.u]
1	2	0.0922	0.0477	100	60	0.9970
2	3	0.4930	0.2511	90	40	0.9829
3	4	0.3660	0.1864	120	80	0.9754
4	5	0.3811	0.1941	60	30	0.9679
5	6	0.8190	0.7070	60	20	0.9495
6	7	0.1872	0.6188	200	100	0.9459
7	8	0.7114	0.2351	200	100	0.9323
8	9	1.0300	0.7400	60	20	0.9260
9	10	1.0400	0.7400	60	20	0.9201
10	11	0.1966	0.0650	45	30	0.9192
11	12	0.3744	0.1238	60	35	0.9177
12	13	1.4680	1.1550	60	35	0.9115
13	14	0.5416	0.7129	120	80	0.9092
14	15	0.5910	0.5260	60	10	0.9078
15	16	0.7463	0.5450	60	20	0.9064
16	17	1.2890	1.7210	60	20	0.9044
17	18	0.7320	0.5740	90	40	0.9038
2	19	0.1640	0.1565	90	40	0.9965
19	20	1.5042	1.3554	90	40	0.9929
20	21	0.4095	0.4784	90	40	0.9922
21	22	0.7089	0.9373	90	40	0.9916
3	23	0.4512	0.3083	90	50	0.9793
23	24	0.8980	0.7091	420	200	0.9726
24	25	0.8960	0.7011	420	200	0.9693
6	26	0.2030	0.1034	60	25	0.9475
26	27	0.2842	0.1447	60	25	0.9450
27	28	1.0590	0.9337	60	20	0.9335
28	29	0.8042	0.7006	120	70	0.9253
29	30	0.5075	0.2585	200	600	0.9218
30	31	0.9744	0.9630	150	70	0.9176
31	32	0.3105	0.3619	210	100	0.9167
32	33	0.3410	0.5302	60	40	0.9164

4 Implementation and results

To prove the effectiveness and the robustness of the proposed MINLP model, we use a commercial optimization package known as GAMS. As nonlinear solver the CONOPT package was selected [11]. The simulation were carry out in a personal computer with processor AMD A10-8700 Radeon R6, 8 Gb RAM and Windows 10 Single Language.

4.1 First test system

The Table 4 are presented the cost results for this RDN considering their base cases and the optimal solution obtained by the GAMS implementation. Additionally, the Table 5 present the capacity and the location of each bank of capacitors.

Table 4: Cost results

Case	Initial cost [$]	Final cost [$]	Saving [$]
i	452774.8	409988.4	42786.4
ii	166979.7	160565.1	6414.6
iii	217197.4	206846.3	10351.1

Table 5: Location and size of the capacitor banks

Case	Location [node]	Size [kVAr]
	5	2469.1
i	6	1262.1
	10	376.3
ii	6	1331
iii	6	1628.4

When are analyzed the results presented in Tables 4 and 5, it is possible to note that depending of the demand scenario, the number of capacitors and their sizing presents high changes. In this way, when peak load consumption is considered, the total reactive power installed is 5007.5 kVAr, which produces a net profit of $\$42786.4$. This reduction implies a reduction in the power losses from to 861.44 kW to 749.17 kW. In case of the reactive power, it decreases form 1049.80 kVAr to 900.42 kVAr.

On the other hand, when the second simulation case is observed, it should be noted that it corresponds to the most pessimism scenario, since in this case the lowest net profit is obtained. The total net profit in this case is $\$6414.6$, which implies that the ending active and reactive power losses the period of peak load are reduced to 802.18 kW and 970.51 kVAr, respectively.

The third case of simulation corresponds the intermediate scenario of simulation, the net profit obtained in this case is $\$10351.1$. the ending losses under the peak load hour are 793.59 kW and 958.45 kVAr, respectively.

In this simulation analysis it is clear that for the first test system, the location and sizing of the capacitor banks depends exclusively of the demand behavior; furthermore, for all simulation cases in this test feeder the 6 node corresponds to the best candidate node to allocate reactive compensation.

The behavior of the voltage profile in this system is depicted in Fig. 4.

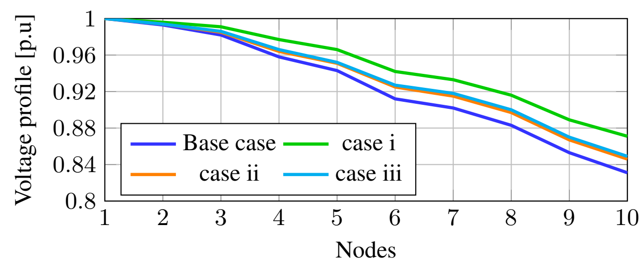

Figure 4: Voltage's profile for 10 nodes RDN for the three demand cases

Notice that, the voltages' profile present the best performance for the first simulation case and the worst performance in the second case of simulation. These situation are waited because the voltage's profile in power systems are related directly with the reactive power injection. Nevertheless, in all simulation cases should be highlighted that the voltage's regulation is superior to the 10%. To fulfill the typical regulation conditions in distribution networks ($\pm10\%$) it is mandatory to rise up the voltage value in the equivalent substation until 1.05 p.u.

4.2 Second test system

The Table 6 are presented the cost results for this RDN considering their base cases and the optimal solution obtained by the GAMS implementation. Additionally, the Table 7 present the capacity and the location of each bank of capacitors.

Table 6: Cost results

Case	Initial cost [$]	Final cost [$]	Saving [$]
i	113037.7	83339.6	29698.1
ii	44559.5	37039.8	7519.7
iii	56934.5	47943.5	8991

Table 7: Location and size of the capacitor banks

Case	Location [node]	Size [kVAr]
i	11	448.9
	30	995.2
ii	30	690.6
iii	32	658.5

Notice that the simulation cases for this test feeder present the same behavior analyzed for the first test feeder, which implies that the first simulation case is the most optimist situation; the second case is the most pessimist demand scenario and the third case correspond to the average behavior, this is easily conclude from Table 6.

First simulation case is installed 1444.5 kVAr in the system, which reduces the total active power losses during the load peak from 210.99 kW to 142.10 kW, additionally, the reactive power losses passed from 143.13 kVAr to 96.56 kVAr.

In the second simulation cases the ending active and reactive power losses are 163.05 kW and 110.96 kVAr, respectively; while third simulation case the active and reactive power losses are 166.31 kW and 113.72 kVAr, respectively.

In Fig. 5 are presented the voltages' profile en each node before and after compensation for each simulation case.

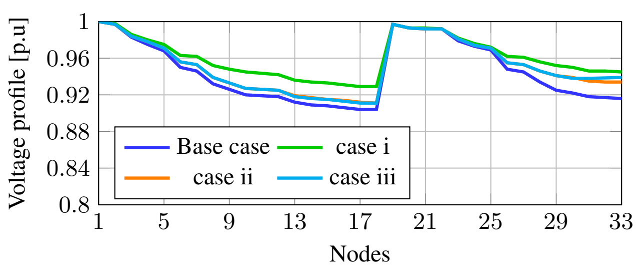

Figure 5: Voltage's profile for 33 nodes RDN for the three demand cases

When the voltages' profile presented in Fig. 5 are observed, it is possible to note, that for this test system all voltages are concatenated between 0.90 p.u and 1.0 p.u. Moreover, the direct relation between the reactive power injection and the voltage's regulation remains in this systems of the same way that occurred for the first test feeder.

5 Conclusions and future works

Optimal placement and sizing of fixed shunt capacitor banks using a MINLP model was presented in this paper. Different scenarios in the demand behavior permitted to observed the direct dependence the location and capacity of the capacitor banks with the active and reactive power losses reduction and the improvement of the voltage's profile.

The total cost reductions define what is the most appropriate location and sizing of capacitor banks in the RDN; nevertheless, the period of time under analysis as well as the energy costs and installation and maintenance costs determine if it a net profit can be achieve by using capacitor banks.

As future work it can be considered the possibility to reduce the active and reactive power losses using a MINLP model for locate and sizing capacitor banks and distributed generators in RDN. The design of distributed generators will also depend of the renewable energy resources available as well as the regulation policies. Additionally, it can be included chemical energy storage devices to improve the daily operation of the RDN by economical dispatch analysis.

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