# Risk reducing actions: efficiency evaluation 

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#### Abstract

This paper reflects the results of research related to evaluating possible consequences of actions directed at the enterprise risk reduction. These results are based on applying the generalization of the classic approach to decision making in condition of uncertainty to multicriteria problems and permit one to provide a rational way to choose robust actions with the presence of multiple scenarios. The choice of the robust actions are associated with the use of so-called choice criteria of Wald, Laplace, Savage, and Hurwicz. Considering these choice criteria as objective functions permits one to apply the Bellman-Zadeh approach to decision making in a fuzzy environment to choose a rational solution alternative. The approach presented in this paper was applied in a case study to improve indicators of an enterprise liquidity risk. The results of the paper are of a general character and can be used to analyze various types of enterprise risks by considering diverse indicators.


Key-Words: - Risk reduction, Robust solutions, Multicriteria decision making, Information uncertainty, Multiple scenarios, Payoff matrices.

## 1 Introduction

The present paper presents the results of research related to evaluating possible consequences of realizing different actions directed at the enterprise risk reduction with the goal to choose the most robust actions. We speak about the necessity of evaluating of particular (monocriteria) consequences and aggregated (multicriteria) consequences. This necessity is associated with the following considerations.

As it is indicated in [1], to describe or to measure the risk or, in fact, to judge it as big or as small, it is possible to use several metrics. As examples, the following characteristics can be indicated [1]:

1. The combination of probability and magnitude/severity of consequences.
2. The triplet $\left(t_{s}, p_{s}, c_{s}\right)$, where $t_{s}$ is the $s$ th scenario, $p_{s}$ is the probability of that scenario, and $c_{s}$ is the consequence of the $s$ th scenario, $s=1, \ldots, S$.
3. The triplet $\left(C^{\prime}, Q, K\right.$, , where $C^{\prime}$ is some specified consequences, $Q$ is a measure of uncertainty associated with $C^{\prime}$ (typically probability) and $K$ the background knowledge that supports $C^{\prime}$ and $Q$ (which includes a judgment of the strength of this knowledge).

Thus, the risk is characterized by more than one indicator (criterion). Taking this into account, the present paper reflects results associated with evaluating the consequences of realizing different actions directed at reducing the risks on the basis of generalizing the classical approach to decision making in conditions of uncertainty [2-4], taking into account the suggestions of [5]. The results of [2-5] are associated with the analysis of particular and aggregated payoff matrices.

## 2 Classic approach to dealing with the uncertainty of information

The classic approach [6-8] to dealing with the uncertainty of information is based on the assumption that the analysis is carried out for a given number $K$ of solution alternatives $X_{k}, k=1, \ldots, K \quad$ and $\quad$ a given number $\quad S$ of representative combinations of initial data (the states of nature or scenarios) $Y_{s}, s=1, \ldots, S$, which define the corresponding payoff matrix, presented in Table 1. The payoff matrix reflects effects (or consequences) of one or other action $X_{k}, k=1, \ldots, K$ to the corresponding state of nature.

The analysis of payoff matrices and choice of the rational solution alternatives are associated with the use of so-called choice criteria. In this work, we discuss the use of the choice criteria of Wald, Laplace, Savage, and Hurwicz, whose application is justified in $[6-8]$. These criteria are based on applying the following characteristic estimates for the given solution alternative (Table 2):

- the minimum objective function (indicator, criterion, etc.) level

$$
\begin{equation*}
F^{\min }\left(X_{k}\right)=\min _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right) \tag{1}
\end{equation*}
$$

which is the most optimistic estimate if the objective function is to be minimized or the most pessimistic estimate if the objective function is to be maximized;

- the maximum objective function (indicator, criterion, etc.) level

$$
\begin{equation*}
F^{\max }\left(X_{k}\right)=\max _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right) \tag{2}
\end{equation*}
$$

which is the most optimistic estimate for the maximized objective function or the most pessimistic estimate if the objective function is to be minimized;

- the average objective function (indicator, criterion, etc.) level

$$
\begin{equation*}
\bar{F}\left(X_{k}\right)=\frac{1}{S} \sum_{s=1}^{S} F\left(X_{k}, Y_{s}\right) \tag{3}
\end{equation*}
$$

- the maximum regret level:

$$
\begin{equation*}
A^{\max }\left(X_{k}\right)=\max _{1 \leq s \leq S} A\left(X_{k}, Y_{s}\right) \tag{4}
\end{equation*}
$$

where $A\left(X_{k}, Y_{s}\right)$ is an over-expenditure which takes place under combination of the state of nature $Y_{s}$ and the choice of the solution alternative $X_{k}$ instead of the solution alternative that is locally optimal for the given $Y_{s}$.

To determine the regrets $A\left(X_{k}, Y_{s}\right)$, one needs to define the minimum value of the objective function (indicator, criterion, etc.) if it is to be minimized (as in Table 1) for each combination of the state of nature $Y_{s}$ (for each column of the payoff matrix):

$$
\begin{equation*}
F^{\min }\left(Y_{s}\right)=\min _{1 \leq k \leq K} F\left(X_{k}, Y_{s}\right) \tag{5}
\end{equation*}
$$

On the other hand, if the objective function (indicator, criterion, etc.) is to be maximized, it is necessary to define its maximum value for each combination of the state of nature $Y_{s}$ (for each column of the payoff matrix):

$$
\begin{equation*}
F^{\max }\left(Y_{s}\right)=\max _{1 \leq k \leq K} F\left(X_{k}, Y_{s}\right) \tag{6}
\end{equation*}
$$

The regret for any solution alternative $X_{k}$ and any state of nature $Y_{s}$ can be evaluated as

$$
\begin{equation*}
A\left(X_{k}, Y_{s}\right)=F\left(X_{k}, Y_{s}\right)-F^{\min }\left(Y_{s}\right) \tag{7}
\end{equation*}
$$

if the objective function (indicator, criterion, etc.) is to be minimized or

$$
\begin{equation*}
A\left(X_{k}, Y_{s}\right)=F\left(X_{k}, Y_{s}\right)-F^{\min }\left(Y_{s}\right) \tag{8}
\end{equation*}
$$

if it is to be maximized.
Table 1. Payoff matrix

|  | $Y_{1}$ | $\ldots$ | $Y_{2}$ | $\ldots$ | $Y_{S}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | $F\left(X_{1}, Y_{1}\right)$ | $\ldots$ | $F\left(X_{1}, Y_{s}\right)$ | $\ldots$ | $F\left(X_{1}, Y_{S}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{k}$ | $F\left(X_{k}, Y_{1}\right)$ | $\ldots$ | $F\left(X_{k}, Y_{s}\right)$ | $\ldots$ | $F\left(X_{k}, Y_{S}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{K}$ | $F\left(X_{K}, Y_{1}\right)$ | $\ldots$ | $F\left(X_{K}, Y_{s}\right)$ | $\ldots$ | $F\left(X_{K}, Y_{S}\right)$ |
|  | $F^{\min }\left(Y_{1}\right)$ | $\ldots$ | $F^{\min }\left(Y_{s}\right)$ | $\ldots$ | $F^{\min }\left(Y_{S}\right)$ |

Table 2. Matrix with characteristic estimates

|  | $F^{\min }\left(X_{k}\right)$ | $\bar{F}\left(X_{k}\right)$ | $A^{\max }\left(X_{k}\right)$ | $F^{\max }\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | $F^{\min }\left(X_{1}\right)$ | $\bar{F}\left(X_{1}\right)$ | $A^{\max }\left(X_{1}\right)$ | $F^{\max }\left(X_{1}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{k}$ | $F^{\min }\left(X_{k}\right)$ | $\bar{F}\left(X_{k}\right)$ | $A^{\max }\left(X_{k}\right)$ | $F^{\max }\left(X_{k}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{K}$ | $F^{\min }\left(X_{K}\right)$ | $\bar{F}\left(X_{K}\right)$ | $A^{\max }\left(X_{K}\right)$ | $F^{\max }\left(X_{K}\right)$ |

The choice criteria, which are based on the use of the characteristic estimates are represented below under the assumption that the objective function is to be minimized.

The choice criterion of Wald uses the estimate $F^{\text {max }}\left(X_{k}\right)$ and permits one to choose the solution alternatives $X^{W}$, for which this estimate is minimum:

$$
\begin{equation*}
\min _{1 \leq k \leq K} F^{\max }\left(Y_{s}\right)=\min _{1 \leq k \leq K} \max _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right) . \tag{9}
\end{equation*}
$$

The choice criterion of Laplace uses the estimate $\bar{F}\left(X_{k}\right)$ and is oriented to choose the solution alternatives $X^{L}$, for which this estimate is minimum:

$$
\begin{equation*}
\min _{1 \leqslant k \leqslant K} \bar{F}\left(X_{k}\right)=\min _{1 \leqslant \leqslant K K} \frac{1}{S} \sum_{s=1}^{S} F\left(X_{k}, Y_{s}\right) \tag{10}
\end{equation*}
$$

The choice criterion of Savage is associated with the use of estimate $A^{\text {max }}\left(X_{k}\right)$ and allows one to choose the solution alternatives $X^{s}$, for which this estimate is minimum:

$$
\begin{equation*}
\min _{1 \leq k \leq K} A^{\max }\left(X_{k}\right)=\min _{1 \leq k \leq K} \max _{1 \leq s \leq S} A\left(X_{k}, Y_{s}\right) . \tag{11}
\end{equation*}
$$

Finally, the choice criterion of Hurwicz utilizes a convex combination of $F^{\max }\left(X_{k}\right)$ and $F^{\min }\left(X_{k}\right)$ and permits one to choose the solution alternatives $X^{H}$, for this combination is minimum:

$$
\begin{align*}
& \min _{1 \leq k K K}\left[\alpha F^{\max }\left(X_{k}\right)+(1-\alpha) F^{\min }\left(X_{k}\right)\right]=  \tag{12}\\
& \min _{1 \leq k \leq K}\left[\alpha \max _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right)+(1-\alpha) \min _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right)\right.
\end{align*}
$$

where $\alpha \in[0,1]$ is the index "pessimism-optimism" whose magnitude is defined by the decision maker.

The advantages and disadvantages of the choice criteria considered above are discussed in $[4,5,8]$. These criteria have found the broad practical applications (for instance, [8]) for monocriteria decision making under uncertainty.

## 3 Multicriteria decision making as applied to generalizing the classic approach to dealing with the uncertainty of information

In the sequel, we present the information on the use of the Bellman-Zadeh approach to decision making in a fuzzy environment $[9,10]$ for generalizing the classic approach to deal with information uncertainty.

When using the Bellman-Zadeh approach for analyzing multiobjective problems [4,11], objective functions $F_{p}(X), p=1, \ldots, q$ are replaced by fuzzy sets $A_{p} X \in L, p=1, \ldots, q$, where $\mu_{A_{p}}(X)$ is the membership function of $A_{p}=\left\{X, \mu_{A_{p}}(X)\right\}, A_{p}$ [4,10].

A fuzzy solution $D$ is defined as $D=\bigcap_{p=1}^{q} A_{p}$ with the membership function

$$
\begin{equation*}
\mu_{D}(X)=\min _{1 \leq p \leq q} \mu_{A_{p}}(X), \quad X \in L . \tag{13}
\end{equation*}
$$

The use of (13) allows one to get the solution
$\max \mu_{D}(X)=\max _{X \in L} \min _{1 \leq p \leq q} \mu_{A_{p}}(X)$.
(14)

Therefore, the problem of multicriteria decision making is reduced to search for

$$
\begin{equation*}
X^{0}=\arg \max _{X \in L} \min _{1 \leq p \leq q} \mu_{A_{p}}(X) \tag{15}
\end{equation*}
$$

To obtain (15), one needs to build the membership functions $\mu_{A_{p}}(X), p=1, \ldots, q$, which reflect a degree of achieving own optima by $F_{p}(X), X \in L, p=1, \ldots, q$. This condition is satisfied if one chooses [4,11]:

$$
\begin{equation*}
\mu_{A_{p}}(X)=\left[\frac{\max _{X \in L} F_{p}(X)-F_{p}(X)}{\max _{X \in L} F_{p}(X)-\min _{X \in L} F_{p}(X)}\right]^{\lambda_{p}} \tag{16}
\end{equation*}
$$

for minimized objective functions or

$$
\begin{equation*}
\mu_{A_{p}}(X)=\left[\frac{F_{p}(X)-\min _{X \in L} F_{p}(X)}{\max _{X \in L} F_{p}(X)-\min _{X \in L} F_{p}(X)}\right]^{\lambda_{p}} \tag{17}
\end{equation*}
$$

for maximized ones. In (16) and (17), $\lambda_{p}, p=1, \ldots, q$ are the importance factors for the corresponding objective functions.

4 Choice criteria as objective functions in multicriteria decision making under information uncertainty
The results of Section 3 have served [2-4] for generalizing the classical approach to dealing with the uncertainty of information, described in Section 2, to analyze multicriteria problems. However, the results of $[5,12]$ indicate some limitations of [2-4]. One way to overcome these limitations is based on considering the choice criteria (9)-(12) for a given objective function in an environment with several states of nature $Y_{s}, s=1, \ldots, S[5,12]$.

Therefore, considering the choice criteria of Wald, Laplace, Savage, and Hurwicz, respectively, as objective functions, one can obtain the following correlations:

$$
\begin{align*}
& F^{W}\left(X_{k}\right)=F^{\max }\left(X_{k}\right)=\max _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right),  \tag{18}\\
& F^{L}\left(X_{k}\right)=\bar{F}\left(X_{k}\right)=\frac{1}{S} \sum_{s=1}^{S} F\left(X_{k}, Y_{s}\right),  \tag{19}\\
& F^{S}\left(X_{k}\right)=A^{\max }\left(X_{k}\right)=\max _{1 \leq s S S} A\left(X_{k}, Y_{s}\right), \tag{20}
\end{align*}
$$

and

$$
\begin{align*}
& F^{H}\left(X_{k}\right)=\alpha F^{\max }\left(X_{k}\right)+(1-\alpha) F^{\min }\left(X_{k}\right)= \\
& \alpha \max _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right)+(1-\alpha) \min _{1 \leq s \leq S} F\left(X_{k}, Y_{s}\right) . \tag{21}
\end{align*}
$$

This consideration of the choice criteria of the classic approach permits one to construct $q$ problems, generally, including four or less objective functions (if not all choice criteria are used in the analysis) as follows:

$$
\begin{equation*}
F_{r, p}(X) \rightarrow \underset{X \in L}{\operatorname{extr}, \quad r=1, \ldots, t \leq 4, p=1, \ldots, q, ~} \tag{22}
\end{equation*}
$$

where the objective functions are $F_{1, p}(X)$ $=F_{p}^{W}\left(X_{k}\right), \quad F_{2, p}(X)=F_{p}^{L}\left(X_{k}\right), F_{3, p}(X)=F_{p}^{S}\left(X_{k}\right)$, and $F_{4, p}(X)=F_{p}^{H}\left(X_{k}\right)$.

Applying (16) to construct the membership functions for $F_{r, p}(X), r=1, \ldots, t, p=1, \ldots, q$, one can solve the problem (14) for the solution alternatives $X_{k}, k=1, \ldots, K$. The analysis, realized in this way, guarantees the choice of the rational solution alternatives in accordance with the principle of the Pareto optimality [5] and allows one to overcome the limitations of the generalization of the classic approach to deal with information uncertainty to multicriteria decision making, as previously discussed. Considering this, the matrix with the characteristic estimates (Table 2) is presented as the
matrix with the choice criteria estimates for $p=1, \ldots, q$, in Table 3 .

Therefore, using $q$ matrices for the choice criteria estimates, we can construct $q$ modified matrices of the choice criteria estimates when applying (16), as shown in Table 4.

Finally, with those $q$ modified matrices of the choice criteria estimates, after applying (14), we can construct the aggregated matrix of the choice criteria estimates, as it is given in Table 5. This matrix includes the estimates calculated on the basis of (14) and used to choose the solution alternatives.

Table 3. Matrix of choice criteria estimates for the $p$ th objective function

|  | $F_{p}^{W}\left(X_{k}\right)$ | $F_{p}^{L}\left(X_{k}\right)$ | $F_{p}^{s}\left(X_{k}\right)$ | $F_{p}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | $F_{p}^{W}\left(X_{1}\right)$ | $F_{p}^{L}\left(X_{1}\right)$ | $F_{p}^{s}\left(X_{1}\right)$ | $F_{p}^{H}\left(X_{1}\right)$ |
| $\ldots$ | ... | ... | ... | ... |
| $X_{k}$ | $F_{p}^{W}\left(X_{k}\right)$ | $F_{p}^{L}\left(X_{k}\right)$ | $F_{p}^{s}\left(X_{k}\right)$ | $F_{p}^{H}\left(X_{k}\right)$ |
| ... | $\cdots$ | $\ldots$ | ... | $\ldots$ |
| $X_{K}$ | $F_{p}^{W W}\left(X_{K}\right)$ | $F_{p}^{L}\left(X_{K}\right)$ | $F_{p}^{s}\left(X_{K}\right)$ | $F_{p}^{H}\left(X_{K}\right)$ |
|  | $\min _{1 \leq k \leq K} F_{p}^{W}\left(X_{k}\right)$ | $\min _{1 \leq K K} F_{p}^{L}\left(X_{k}\right)$ | $\min _{1 \leq 1 \leq K} F_{p}^{s}\left(X_{k}\right)$ | $\min _{1 \leq \leq K} F_{p}^{H}\left(X_{k}\right)$ |
|  | $\max _{1 \leq \leqslant \leqslant K} F_{p}^{W}\left(X_{k}\right)$ | $\max _{1 \leq<K} F_{p}^{L}\left(X_{k}\right)$ | $\max _{1 \leq \leq K} F_{p}^{s}\left(X_{k}\right)$ | $\max _{1 \leq k<K} F_{p}^{H}\left(X_{k}\right)$ |

Table 4. Modified matrix of choice criteria estimates for the $p$ th objective function

|  | $\mu_{A_{p}}^{W}\left(X_{k}\right)$ | $\mu_{A_{p}}^{L}\left(X_{k}\right)$ | $\mu_{A_{p}}^{S}\left(X_{k}\right)$ | $\mu_{A_{p}}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | $\mu_{A_{p}}^{W}\left(X_{1}\right)$ | $\mu_{A_{p}}^{L}\left(X_{1}\right)$ | $\mu_{A_{p}}^{S}\left(X_{1}\right)$ | $\mu_{A_{p}}^{H}\left(X_{1}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{k}$ | $\mu_{A_{p}}^{W}\left(X_{k}\right)$ | $\mu_{A_{p}}^{L}\left(X_{k}\right)$ | $\mu_{A_{p}}^{S}\left(X_{k}\right)$ | $\mu_{A_{p}}^{H}\left(X_{k}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{K}$ | $\mu_{A_{p}}^{W}\left(X_{K}\right)$ | $\mu_{A_{p}}^{L}\left(X_{K}\right)$ | $\mu_{A_{p}}^{S}\left(X_{K}\right)$ | $\mu_{A_{p}}^{H}\left(X_{K}\right)$ |

Table 5. Aggregated matrix of the choice criteria estimates

|  | $\mu_{D}^{W}\left(X_{k}\right)$ | $\mu_{D}^{L}\left(X_{k}\right)$ | $\mu_{D}^{S}\left(X_{k}\right)$ | $\mu_{D}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | $\mu_{D}^{W}\left(X_{1}\right)$ | $\mu_{D}^{L}\left(X_{1}\right)$ | $\mu_{D}^{S}\left(X_{1}\right)$ | $\mu_{D}^{S}\left(X_{1}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{k}$ | $\mu_{D}^{W}\left(X_{k}\right)$ | $\mu_{D}^{L}\left(X_{k}\right)$ | $\mu_{D}^{s}\left(X_{k}\right)$ | $\mu_{D}^{H}\left(X_{k}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $X_{K}$ | $\mu_{D}^{W}\left(X_{K}\right)$ | $\mu_{D}^{L}\left(X_{K}\right)$ | $\mu_{D}^{S}\left(X_{K}\right)$ | $\mu_{D}^{H}\left(X_{K}\right)$ |

## 5 Case study

This Section presents a case study associated with the evaluation of actions to reduce the liquidity risk of an enterprise to illustrate the use of the approach presented in this paper. The study is related to an enterprise planning for a 12 -month period from 2016 to 2017. This means that the actions defined in 2016 are to be evaluated for possible future scenarios for 12 months later in 2017. In the first stage, the risk management specialists defined four actions that will be evaluated to attempt to reduce the Liquidity Risk:

- Roll the debt;
- Perform the capital contribution;
- Decrease distribution of dividends.
- Capitalization through dividends from subsidiaries.
The risk management specialists identified the uncertainty variables related to the Liquidity Risk analysis. In particular, the following uncertain variables were selected for the study:
$v_{1}$ - Inflation rate in \% by year;
$v_{2}$ - interest rate in \% by year;
$v_{3}$ - exchange rate BRL/USD;
$v_{4}$ - state debt, since the company is controlled by the state;
$v_{5}$ - profit rate evolution of subsidiaries;
$v_{6}$ - enterprise's rating, by credit rating agencies.
In the second stage, the risk management specialists have been asked to create three scenarios (representing distinct views of the future based on the six variables selected for the study):
- $Y_{1}$ - Optimistic;
- $Y_{2}$ - Balanced;
- $Y_{3}$-Pessimistic.

Each scenario represents a possible future based on the temporal behavior of the uncertainty variables.

The variables $v_{4}, v_{5}$ and $v_{6}$ are defined by two or more components, complicating the quantification of the variable states in the future. Therefore, these variables are represented by the following qualitative estimates, indicating the variables variation in the planning period:
$I C$ - their values will increase;
$S M$ - their values remains the same levels;
$D C$ - their values will decrease.
The three scenarios created by the specialists are presented in Table 6:

Table 6. Scenarios definition

|  | $v_{1}$ | $v_{2}$ | $v_{3}$ | $v_{4}$ | $v_{5}$ | $v_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Y_{1}$ | 2.98 | 10.50 | 2.50 | $D C$ | $I C$ | $I C$ |
| $Y_{2}$ | 5.19 | 12.00 | 3.30 | $S M$ | $S M$ | $S M$ |
| $Y_{3}$ | 6.55 | 18.00 | 5.60 | $I C$ | $D C$ | $D C$ |

In the third stage, the specialists evaluated the possible consequences of the actions in each scenario. The consequences of the actions will be evaluated applying two criteria (indicators), widely used to analyze risks: Probability and Impact. The Table 7 shows the payoff matrix for the criterion of Probability:

Table 7. Payoff matrix for the criterion of Probability, \%

| $Y_{1}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $X_{1}$ | 18.68 | 30.00 | 45.15 |
| $X_{2}$ | 23.33 | 36.92 | 44.44 |
| $X_{3}$ | 19.33 | 30.00 | 45.15 |
| $X_{4}$ | 20.33 | 33.81 | 44.40 |

Based on the expressions (18)-(21), with $\alpha=0.75$ (for the choice criterion of Hurwicz), it is possible to construct Table 8 with the choice criteria values for the indicator of Probability. Considering that this indicator is to be minimized, using (16), it is possible to define the normalized value of the choice criteria for each action, presented in Table 9.

Table 8. Values of the choice criteria for the criterion of Probability $\%$

|  | $F_{p}^{W}\left(X_{k}\right)$ | $F_{p}^{L}\left(X_{k}\right)$ | $F_{p}^{s}\left(X_{k}\right)$ | $F_{p}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | 45.15 | 31.28 | 0.75 | 38.53 |
| $X_{2}$ | 44.44 | 34.90 | 6.92 | 39.16 |
| $X_{3}$ | 45.15 | 31.49 | 0.75 | 38.70 |
| $X_{4}$ | 44.40 | 32.85 | 3.81 | 38.38 |

Table 9. Normalized values of the choice criteria for the criterion of Probability

|  | $\mu_{A_{p}}^{W}\left(X_{k}\right)$ | $\mu_{A_{p}}^{L}\left(X_{k}\right)$ | $\mu_{A_{p}}^{S}\left(X_{k}\right)$ | $\mu_{A_{p}}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | 0.00 | $\mathbf{1 . 0 0}$ | $\mathbf{1 . 0 0}$ | 0.81 |
| $X_{2}$ | 0.95 | 0.00 | 0.00 | 0.00 |
| $X_{3}$ | 0.00 | 0.94 | $\mathbf{1 . 0 0}$ | 0.60 |
| $X_{4}$ | $\mathbf{1 . 0 0}$ | 0.57 | 0.50 | $\mathbf{1 . 0 0}$ |

Let us analyze the solution alternatives for the criterion of Probability. Table 9 shows that the use of the criterion of Wald leads to $X^{W}=\left\{X_{4}\right\}$, since $\mu_{A_{p}}^{W}\left(X_{4}\right)=1$ for its column. The criterion of Laplace produces the selection of $X^{L}=\left\{X_{1}\right\}$, since $\mu_{A_{p}}^{L}\left(X_{1}\right)=1$. It is not difficult to understand that the application of the criterion of Savage leads to $X^{S}=\left\{X_{1}, X_{3}\right\}$. Finally, the criterion of Hurwicz generates $X^{H}=\left\{X_{4}\right\}$. Thus, the solution $X_{2}$ is clearly the worst solution from the point of view of the criterion of Probability, since it was not selected by applying all choice criteria.

Let us consider the payoff matrix for the criterion of Impact, presented in Table 10. Thus, based on the expressions (18)-(21), with $\alpha=0.75$, we can construct the Table 11 with the choice criteria values for the criterion of Impact.

Table 10. Payoff matrix for the criterion of Impact BRL $\times 10^{6}$

|  | $Y_{1}$ | $Y_{2}$ | $Y_{3}$ |
| :---: | :---: | :---: | :---: |
| $X_{1}$ | 10.75 | 15.71 | 21.00 |
| $X_{2}$ | 10.95 | 15.23 | 24.49 |
| $X_{3}$ | 10.75 | 15.48 | 21.24 |
| $X_{4}$ | 14.28 | 15.18 | 17.78 |

Table 11. Values of the choice criteria for the criterion of Impact BRL $\times 10^{6}$

| criterion of Impact BRL×10 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $F_{p}^{W}\left(X_{k}\right)$ | $F_{p}^{L}\left(X_{k}\right)$ | $F_{p}^{S}\left(X_{k}\right)$ | $F_{p}^{H}\left(X_{k}\right)$ |
| $X_{1}$ | 21.00 | 15.82 | 3.22 | 18.44 |
| $X_{2}$ | 24.49 | 16.89 | 6.71 | 21.10 |
| $X_{3}$ | 21.24 | 15.82 | 3.46 | 18.62 |
| $X_{4}$ | 17.78 | 15.75 | 3.53 | 16.91 |

Finally, as the criterion of Impact is to minimized, using (16), it is possible to define the normalized value of the choice criteria for each action, presented in Table 12.

Table 12. Normalized values of the choice criteria for the criterion of Impact

|  | $\mu_{A_{p}}^{W}\left(X_{k}\right)$ | $\mu_{A_{p}}^{L}\left(X_{k}\right)$ | $\mu_{A_{p}}^{S}\left(X_{k}\right)$ | $\mu_{A_{p}}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | 0.52 | 0.94 | $\mathbf{1 . 0 0}$ | 0.64 |
| $X_{2}$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $X_{3}$ | 0.48 | 0.93 | 0.93 | 0.59 |
| $X_{4}$ | $\mathbf{1 . 0 0}$ | $\mathbf{1 . 0 0}$ | 0.91 | $\mathbf{1 . 0 0}$ |

The analysis of Table 12 shows that the application of the criterion of Wald permits one to choose the solution alternative $X^{W}=\left\{X_{4}\right\}$. The use of the criterion of Laplace produces the selection of $X^{L}=\left\{X_{4}\right\}$ as well. The utilization of the criterion of Savage generates $X^{S}=\left\{X_{1}\right\}$. Finally, the use of the criterion of Hurwicz also generates $X^{H}=\left\{X_{4}\right\}$. Thus, the analysis of the criterion of Impact indicates that the solutions $X_{1}$ and $X_{4}$ are to be considered as preferable solution alternatives.

Let us consider the multicriteria problem, which is associated with the simultaneous minimization of the indicators of Probability and Impact of the Liquidity Risk. Using (13) we can aggregate the normalized values of the choice criteria given in Table 9 and Table 12 to obtain Table 13. It represents the performance of the actions for the criteria of Probability and Impact simultaneously.

Table 13. Aggregated choice criteria values for the multicriteria analysis

|  | $\mu_{A_{p}}^{W}\left(X_{k}\right)$ | $\mu_{A_{p}}^{L}\left(X_{k}\right)$ | $\mu_{A_{p}}^{S}\left(X_{k}\right)$ | $\mu_{A_{p}}^{H}\left(X_{k}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $X_{1}$ | 0.00 | $\mathbf{0 . 9 4}$ | $\mathbf{1 . 0 0}$ | 0.64 |
| $X_{2}$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $X_{3}$ | 0.00 | 0.93 | 0.93 | 0.59 |
| $X_{4}$ | $\mathbf{1 . 0 0}$ | 0.57 | 0.50 | $\mathbf{1 . 0 0}$ |

Analyzing Table 15 , it is possible to obtain the following results: $\quad X^{W}=\left\{X_{4}\right\}, \quad X^{L}=\left\{X_{1}\right\}$ $X^{S}=\left\{X_{1}\right\}$, and $X^{H}=\left\{X_{4}\right\}$. These results indicate that the actions $X_{2}$ and $X_{3}$ can be excluded from the future consideration. At the same time, the actions $X_{1}$ and $X_{4}$ should be subjected to the additional analysis, based on applying other
decision making techniques, for example, discussed in [4, 12]..

## 6 Conclusion

In this paper, the results related to analyzing the consequences of realizing actions to enterprise risk reduction have been presented. The proposed approach is based on the construction of payoff matrices, which permit one the rational consideration of the uncertainty factor. The analysis of payoff matrices, based on the use of specific choice criteria, allows one to find the robust actions, considering diverse possible future scenarios.

The use of the choice criteria as objective functions permits the correct evaluation of the consequences of the action alternatives. An important benefit of this approach is the possibility to evaluate the action alternatives from the point of view of a particular criterion (indicator) as well as from the point of view of multiple criteria (indicators) on the bases of generalizing the classis approach to considering the information uncertainty with using procedures of multicriteria decision making in a fuzzy environment.

The paper results have been illustrated by the case study related to improving indicators of the enterprise liquidity risk.

The results of the paper demonstrate their applicability to analyze various types of enterprise risks, considering diverse indicators which characterized these risks.

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