Crude oil market dynamics through TVEC-Copula-DCC-GARCH models: improving the variance reduction of hedging strategies.

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Abstract: As observed in several studies, crude oil returns are leptokurtic and skewed, thus the traditional assumption of normality may be unrealistic. However Copula functions are perfect to deal with this characteristics and TVEC model is very efficient in capturing market instability. This paper proposes to analyze the applicability of TVEC-Copula-DCC-GARCH methodology to model crude oil volatility. To provide an empirical test, we estimate an out-sample hedge during European Crisis, comparing its results and efficiency with other models analyzed in a previous study. Results show that the Copula-DCC-GARCH presents a superior fit, which promotes a more efficient and accurate hedge.

Keywords: TVEC-Copula-DCC-GARCH, political and economic instability, crude oil regimes, dynamic volatility, improved hedge ratios.

1 Introduction

Since the industrial revolution, economic growth is guided by energy supply. In this scenario, oil market has continually grown to become the world's biggest commodity market. Thus, it is impossible to separate industrial development and oil supply. Large variations in the price of this commodities cause large effects on production costs. Since the 1980s, the volatility of this market became higher. After two gulf wars, decisions from the Organization Petroleum Exporting Countries (OPEC), terrorist attacks and economic crises, the modeling of crude oil volatility has been the target of several studies.

Usually, previous studies applied some variations of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models developed by Bollerslev (1986) to estimate crude oil price volatility, assuming absence of shifts in crude oil volatility, which means that the volatility oscillates within a constant range, being a stationary process. It is well known that

volatility of asset prices is substantially affected by sudden changes or regime shifts, corresponding to economic and political events.

However, recent researches indicate a non-linear behavior of crude oil series, highlighted by the presence of structural breaks and different regimes (Salisu and Fasanya (2013); Ewing and Malik (2010)). Furthermore, the skewed, leptokurtic and asymmetrically dependent characteristics of crude oil returns, make the assumption of normality seem unrealistic.

In this study we propose to consider all those characteristics in the search for an efficient approach to model crude oil volatility. Thus, we tested the series for the presence of different regimes and applied copula based model to estimate the volatility, considering the presence of such regimes. Our first objective is to identify which autoregressive model presents the best fit to model the conditional mean for the crude oil spot and future returns of WTI and Brent, which are the main benchmark for North American and European markets. After that, we propose to model the conditional volatility using a Copula-DCC-GARCH model to deal with the characteristics mentioned. As a way to test the efficiency

of this model in crude oil market, we performed an out-sample prediction test and estimated hedge reasons, comparing the effectiveness of this strategy with Chang, MacAleer and Tansuchat (2011) results.

This kind of information can supply investors with accurate information, contributing to their strategy's choice, being especially useful to investors who desire to determine hedge reasons among different types of crude oil kinds. Furthermore, by determining models with a better fit for energy assets, we can supply investors, policy makers and researchers with better tools, which provide more precise panoramas about the relations between crude oil and fuel volatilities.

2 Review of previous studies

The volatility series behavior has been the target of several researches. Ewing et al. (2002) examined the univariate and bivariate time-series properties of oil and natural gas index returns, and looked at how the volatility in oil and natural gas sectors changes over time and across markets. They estimated a multivariate General Autoregressive Conditional Heterocedasticity (GARCH) model to simultaneously estimate the mean and conditional variance of daily returns in natural gas and oil markets, thus avoiding the generated regressor problem associated with the two-step estimation process found in many earlier studies (Pagan, 1984). In addition, it was used the BEKK specification of the multivariate GARCH model, which does not impose the restriction of constant correlation between index returns.

Through a similar approach, Jin et al. (2012) analyzed the integration between future contracts of WTI, Dubai and Brent using the VAR-BEKK model. They quantified persistence and size of those connections through Volatility Impulse Response Functions (VIRF) for two historical shocks, the 2008 financial crisis and BP Deepwater Horizon oil spill. They observed that Dubai and Brent crude are highly responsive to market shocks and the WTI crude proved to be the least responsive of the three benchmarks. Through simulations, they show that only large shocks will result in an increase in expected conditional volatilities.

Malik and Hammoudeh (2007) applied multivariate GARCH model with BEEK specification to simultaneously estimate the mean and conditional variance of the oil daily returns for the US and Gulf equity markets. They specifically used this model to capture the volatility and shock transmission among markets since shocks can spillover from one country to another because these countries process common oil-related information. Using daily returns from February 14, 1994 to December 25, 2001, it was examined the volatility and shock transmission mechanism among US equity, global crude oil market, and equity markets of Saudi Arabia, Kuwait, and Bahrain. The results show significant volatility transmission among those markets and are important for building accurate asset pricing models and forecasting future volatility in equity and oil markets.

However, a shortcoming of those approaches is that, even though models deal well with the clustering problem of volatility in data and mitigates the problems of fat tails, GARCH models commonly assume that no shift in volatility occurs, which means that the volatility oscillates within a constant range, being a stationary process. To deal with this problem, Kang et al. (2011) analyzed the volatility considering structural breaks in the series. The ICSS algorithm was used to identify discrete sub-periods of the changing volatility of crude oil returns and BEEK specification of bivariate GARCH (1,1) model was used to estimate the variance changes. The results reveal five structural changes in the period from January 5, 1990 to March 27, 2009, reducing the degree of persistence in the conditional variance of both returns, concluding that ignoring structural changes may distort the direction of information inflow and volatility transmission between crude oil markets.

Salisu and Fasanya (2013) employed the tests developed by Narayan and Popp (2010) and Liu and Narayan (2010) to detect structural breaks in oil price volatility. This form of the model, therefore, permits shifts in the trend function to have a gradual effect on oil price changes. This methodology allowed them to detect two structural breaks (1990 and 2008) in WTI and Brent series, referring to Iraq/Kuwait conflict and the global financial crisis, respectively. After that, they tested four GARCH models, GARCH (1,1), GARCH-M (1,1), TGARCH(1,1) and EGARCH (1,1), to model the oil price volatility, concluding that EGARCH (1,1) presents the best fit. These evidences suggest that oil volatility is not uniform in time, presenting persistence and leverage effects. This kind of evidence shows some necessity to investigate crude oil volatility from a time variable point of view since its behavior is changed because of the considered time window.

Vivian and Wohar (2012) analyzed the presence of volatility breaks in energy commodities, among others. They employed Garch (1,1) to model the volatility series and de ICSS algorithm to identify structural breaks in WTI, Brent, Fuel Oil, Heat Oil and Gas in the period between January 2nd 1985 to July 30th 2010. Their results aim for the presence of three breaks in Brent series, in 7/31/1990, 3/19/1991 and 1/8/1996, being the 2008 financial crises insufficient to generate a new break. The other fuels analyzed did not present any structural break. During the sub-periods, none of the commodities had a significant decline in volatility persistence. Those results are quite different to Ewing and Malik (2010) who found evidences of three structural breaks in WTI volatility series in August 29, 1994, January 8, 1996 and June 13, 2005.

In his work, Chang, McAleer and Tansuchat (2011) analyzed the performance of several multivariate volatility models namely CCC, VARMA-GARCH, for the crude oil spot and futures returns of WTI and Brent. The hedging effectiveness indicates that diagonal BEKK (BEKK) is the best model to estimate optimal hedge ratios in terms of reducing the variance of the portfolio.

All volatility models previously mentioned present the same issue; they are estimated under the assumption of multivariate normality – maximum likelihood (ML) – or based on a mixture of elliptical distributions– quasi maximum likelihood (QML). According to Cherubini (2012), this assumption is unrealistic, as evidenced by numerous empirical studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent. These difficulties can be treated as a problem of copulas.

The concept of copula was introduced by Sklar (1959). A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector in smaller relations of dependence, which is the copula. A complete revision of copula methods can be found in Cherubini (2012). Thus, the joint distribution of the asset return can be specified with full flexibility, which is more realistic.

Following this approach, Reboredo (2011) examined the dependence structure between crude oil benchmark prices using copulas. Testing Gaussian copula, Student-t copula, Clayton copula, Gumbel copula, SJC copula, TVP Gaussian copula and TVP Student-t copula, he identified the Student-t copula as the best fit and concluded the hypothesis that the oil market behaves like a common market.

The Copula-DCC-GARCH model was proposed with a financial application by Jondeau and Rockinger (2006). Some posterior studies employed the Copula-DCC-GARCH model because of its advantages. Fantazzini (2009) performed Value at Risk simulations. Aas and Berg (2009), Ausin and Lopes (2010) and Hafner and Reznikova (2010) investigated dependence structures between financial assets. Righi and Ceretta (2011a) identified structural changes in volatility in European markets. Righi and Ceretta (2011b) performed daily risk predictions for a global portfolio.

3 Methodology

Data about crude oil prices were taken from Energy Information Administration of the US Department of Energy. They are given in US dollar per barrel for West Texas Intermediate (WTI), Brent (North Sea- Europe) and Cushing (OK Crude Oil Future). The data constitute daily closing prices over the period from May 20, 1987 to June 19, 2012, totalizing 6296 observations.

We analyzed 5538 observations (May 20, 1987 to June 19, 2009) to calculate the estimates, during this period it was possible find much time of regular behavior and low regime, and different clusters of instability, an important feature to be captured by the model. Remainder observations were used to test the predictability of the model (June 19, 2009 to June 19, 2012), at this period the market was passing through a very turbulent moment, when very precise models and hedge operations are more necessary than ever.

Autoregressive models have been used as an alternative to estimate the conditional mean between two assets. Vector Autoregressive model (VAR) efficient to capture the linear interdependencies among multiple time series in short-run. However when the time series are cointegrated they share a common stochastic drift which means, they present a long term relationship. In this case the Vector Error Corretion model (VEC) promotes a long-run adjustment through the Error Correction Term (ECT)

Both VAR and VEC models present a good fit to linear relations, nevertheless they may ignore important non-linear dependence structures. The Threshold cointegration study was suggested by Balke and Fomby (1997), as an alternative to combine non-linearity and cointegration. Autoregressive process with threshold can be exposed as a Threshold Vector Error Correction (TVEC):

$$\begin{bmatrix} \Delta O_t^L \\ \Delta F_t^{j,L} \end{bmatrix} = \begin{bmatrix} C_1^L \\ C_1^L \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{p} \beta_1^L \Delta O_{t-n} \\ \sum_{i=1}^{p} \beta_1^L \Delta F_{t-n}^j \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{p} \beta_2^L \Delta F_{t-n}^j \\ \sum_{i=1}^{p} \beta_2^L \Delta O_{t-n} \end{bmatrix} + \begin{bmatrix} ECT_{t-1}^L \\ ECT_{t-1}^L \end{bmatrix} + \omega_t \longrightarrow ECT_{t-1}^L \leq y, \tag{1}$$

$$\begin{bmatrix} \Delta O_t^H \\ \Delta F_t^H \end{bmatrix} = \begin{bmatrix} C_1^H \\ C_1^H \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{p} \beta_1^H \Delta O_{t-n} \\ \sum_{i=1}^{p} \beta_1^H \Delta F_{t-n} \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{p} \beta_2^H \Delta F_{t-n} \\ \sum_{i=1}^{p} \beta_2^H \Delta O_{t-n} \end{bmatrix} + \begin{bmatrix} ECT_{t-1}^H \\ ECT_{t-1}^H \end{bmatrix} + \omega_t \longrightarrow ECT_{t-1}^H \geq y, \tag{2}$$

In the current study C is constant, ΔO and ΔF are the log-returns of crude oil spot and future price variation, are the residuals, L and H denote low and high regimes respectively and y is the threshold.

In order to determine which autoregressive presents the best fit to model the conditional mean, we applied the The Hansen and Seo (2002) methodology to verify the

presence of linear or non-linear trend (Table 3). The test is very conclusive, pointing out for the necessity of a threshold model.

Subsequently, using the residuals, the conditional covariance matrix is estimated with a Copula-DCC-GARCH model, which is proposed with a financial application by Jondeau and Rockinger (2006), represented

by formulation (4), which is able to deal with the

$$H = D'_t \, R_t \, D_t \tag{3}$$

Where, $D_t = diag(\, h_{1,t}^{1/2} ... h_{N,t}^{1/2} \,);$

$$R_t = diag(\, q_{11,t}^{-1/2} ... q_{NN,t}^{-1/2} \,) Q_t diag(\, q_{11,t}^{-1/2} ... q_{NN,t}^{-1/2} \,); \tag{4}$$

$$\tag{5}$$

Since the square matrix of order N symmetric and positive defined $Q_t (q_{ij,t})$ has the form proposed in (7).

$$Q_t = (1 - \alpha - \beta)\hat{Q} + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1}, u_{i,t} = \omega_{i,t} / \sqrt{h_{i,t}} \sim skew - t_{v,\gamma}; \tag{6}$$

and is the *NxN* matrix composed by unconditional covariance of ; and are non-negative scalar parameters satisfying. The definition of residuals joint distribution extends the traditional DCC developed by Engle (2002), through copulas, which allow more flexibility in describing the data, once copulas are estimated separately from marginal. We tested the fit of six-dimensional copulas: Normal, Student's t, Clayton, Gumbel and Frank. Cherubini et al. (2012) presented the definition of these copulas.

The Akaike criterion (AIC) was employed to select number of lags which best fitted the data. The model parameters were estimated through Quasi Maximum Likelihood (QML) in two steps. (Jondeau et al. (2007)). In the first stage, the conditional variance is estimated by means of a univariate GARCH model, respectively, for each asset. In the second step, the parameters for the conditional correlation, given the parameters of the first stage, are estimated. To validate the model, we use the *Q* statistic in linear and squared residuals in order to test the

null hypothesis that the data is random against the alternative of non-randomness.

Based on estimates of the conditional covariance matrices, we tested the out-sample models' predictive power and calculated the optimal hedge ratio for each asset, defined by the relation:

$$\delta = \frac{cov(\, r_{s,t}r_{f,t}\,)}{var(\, \sigma_{f,t}^2 \,)} \tag{7}$$

Where $\delta_{sf,t}$ is the dynamic hedge ratio between the active spot (*s*) and future (*f*) at time *t*; $\sigma_{f,t}^2$ is the variance of future asset at time *t*; $cov(r_{s,t}, r_{f,t})$ is the covariance between spot and future assets at time.

4 Results

Table 1 presents the descriptive statistic, WTI and Brent represent log-return of spot prices and Future represents the log-return of OK Crude Oil Future Contract prices.

Table 1: Descriptive Statistics of log-returns of WTI, Brent and Future prices in the period from May 20, 1987 to June 19, 2009.

	WTI	Future	Brent
Observations	5,538	5,538	5,538
Minimum	-0.406	-0.328	-0.361
Maximum	0.188	0.121	0.181
Mean	0.000	0.000	0.000
Median	0.000	0.000	0.000
Stdev	0.025	0.020	0.020
Skewness	-0.861	-0.967	-0.967
Kurtosis	15.946	15.471	15.471

As it can be seen, WTI presents the highest standard deviation, been a little more volatile than Brent. The kurtosis analysis shows that WTI, Brent, and the Future contract returns present heavier tails, suggesting that the copula model is a good approach.

To be estimated any autoregressive model is necessary that time series be stationary. To verify such characteristic

we apply Dickey-Fuller with GLS (ADF-GLS) constant unit root test in the log of WTI, Brent and crude oil future prices series (*WTI, Brent* and *Future* respectively) and in their difference (*dWTI, dBrent* and *dFuture* respectively) (Table 2).

Table 2: Dickey-Fuller with GLS constant unit root test in the log and in the log-return of WTI, Brent and crude oil future prices series.

Series	test statistic	critical values 5%	p-value: Z(t)
WTI	-0.8341	-1.94	0.354
Brent	-0.632	-1.94	0.443
Future	-1.859	-1.94	0.365
dWTI	-4.930	-1.94	0.000
dBrent	-5.100	-1.94	0.000
dFuture	-5.115	-1.94	0.000

Note: "*d*" denote differenced time series.

As it can be observed the log-return of the series are stationary, what enables the use of the cited autoregressive models. To verify the existence of a long-run relation and the need of employ a model with ECT, we apply the two-step Engle–Granger approach to test for a cointegration relationship between WTI or Brent spot prices and future price "*F*". At first, we run a par of simple regressions.

$$WTI = \alpha + \beta F_t^{\ j} + \varepsilon_{wti} \tag{8}$$

$$Brent = \alpha + \beta F_t^{\ j} + \varepsilon_{Brent} \tag{9}$$

At Second, we apply ADF-GLS unit root test in the residuals of each regressions, (see Table 3). According to Engle-Granger if the leveled series present unit-root and its respective the residual ε_i is stationary the analyzed series are cointegrated.

Table 3: Dickey-Fuller with GLS constant unit root test in residuals (ε_{wti} and ε_{Brent}) of equations (8) and (9).

Series	test statistic	critical values 5%	p-value: Z(t)
ε_{wti}	-4.837	-1.94	0.000
ε_{Brent}	-2.786	-1.94	0.005

Note: ε_{wti} and ε_{Brent} are the error term of the regression between the respective crude oil and the future contract prices according to equations (8) and (9).

According to Table 2, WTI, Brent and Future leveled series present unit-root, however the residuals of equations (9) and (10) are stationary (Table 3), indicating that WTI and Brent series are cointegrated with Future series. It reveals the existence of a long-run relationship among these series, and points to the need of an autoregressive model with ECT.

The Hansen and Seo (2002) methodology (Table 4) was employed to test which autoregressive model, VEC or TVEC, is more adequate to model the conditional mean. According to the test, the series present non-linear relations.

Table 4: Hansen and Seo (2002) linearity test.

Relation	Test Statistic	P-Value	Trend
WTI / Future	67.18164	0.00	non-linear
Brent / Future	46.61097	0.02	non-linear

Considering the presence of cointegration and non-linear trend, we conclude that TVEC model presents the best fit for data. TVEC model results (Table 4), were estimated using 10 lags, according to Akaike Criterion (AIC = -10624), the error correction term is significant in the high regime, for the *Future Prices equation*.

Analyzing the results, it can be observed the different behavior of each regime. In the low regime, we can notice the presence of price transmission between spot and future market up to ten lags, and they are capable of influencing each other with different lags. In high regime the same does not happen, price transmission between future and spot prices seems to be weaker and less dependent on the future price than the spot price.

TVEC results for Brent (Table 5) reveal two different relations with past data. In low regime, we have less influence from previous periods and in high regime this dependence is superior, suggesting that in crisis, when volatility is higher and market liquidity is lower, previous periods have more influence on investor analysis.

Table 5: TVEC estimated coefficients and regimes for the relations between *dWTI and dFuture*, and between *dBrent and dFuture*.

	TVEC results for the relations between *dWTI* and *dFuture*				TVEC results for the relation between *dBrent* and *dFuture*.			
	Low regime - 94.2%		High regime - 5.8%			Low regime - 94.7%		High regime - 5.3%	
Coefficient	*dWTI*	*dFuture*	*dWTI*	*dFuture*	**Coefficient**	*dBrent*	*dFuture*	*dBrent*	*dFuture*
ECT_1	-0.001	0.013	-0.510*	-0.085	*ECT_1*	-0.019	0.038	-0.027*	0.012
dWTI_1	0.141*	2.00*	-0.219*	0.235	*dBrent_1*	-0.473*	0.140*	-0.351*	0.016
dFuture_1	-0.154*	-0.239*	-0.060	-0.258*	*dFuture_1*	0.587*	-0.102*	0.566*	0.566*
dWTI_2	-0.034	0.103*	-0.161	0.044	*dBrent_2*	-0.378*	-0.088	-0.166*	-0.166*
dFuture_2	0.006	-0.131*	0.196	0.005	*dFuture_2*	0.473*	0.073	0.220*	-0.094*
dWTI_3	0.025	0.162*	-0.049	0.041	*dBrent_3*	-0.394*	-0.421*	-0.140*	0.008
dFuture_3	0.003	-0.167*	0.047	-0.109	*dFuture_3*	0.467*	0.386*	0.150*	-0.065*
dWTI_4	0.109*	0.187*	-0.056	0.078	*dBrent_4*	0.040	0.125	-0.047*	-0.014
dFuture_4	-0.061	-0.131*	-0.019	-0.110	*dFuture_4*	0.094	-0.007	0.131*	0.074*
dWTI_5	-0.118*	-0.024	-0.223*	-0.043	*dBrent_5*	0.061	0.173*	-0.003	-0.014
dFuture_5	0.062	-0.013	0.166	0.032	*dFuture_5*	-0.078	-0.298*	0.036	0.014
dWTI_6	0.013	0.087*	-0.291*	-0.170	*dBrent_6*	-0.190*	0.016	-0.042	0.011
dFuture_6	-0.0352	-0.101*	0.478	0.292*	*dFuture_6*	0.092	-0.085	0.035	-0.002
dWTI_7	-0.072*	-0.013	-0.181	-0.159	*dBrent_7*	-0.114*	-0.087	0.020	0.030
dFuture_7	0.077*	0.030	0.178	0.108	*dFuture_7*	0.114	0.034	0.005	-0.012
dWTI_8	-0.047	-0.029	-0.311*	-0.217*	*dBrent_8*	0.000	0.077	0.073*	0.096
dFuture_8	0.041	0.037	0.135	0.057	*dFuture_8*	0.066	0.003	-0.090*	-0.103*
dWTI_9	0.068*	0.057*	-0.206	-0.127	*dBrent_9*	-0.115	-0.052	0.082*	0.115*
dFuture_9	-0.082*	-0.055	0.477*	0.325*	*dFuture_9*	-0.004	-0.054	-0.042	-0.071*
dWTI_10	-0.100*	-0.094*	-0.180	-0.012	*dBrent_10*	-0.032	-0.066	0.038*	0.063*
dFuture_10	0.152*	0.146*	0.288	0.084	*dFuture_10*	0.042	0.051	0.020	0.020

Note: * Indicates that the parameter is significant at a confidence level of at least 5%. These coefficients are estimated according to equation (1) and (2).

It is important to note that around 94.2% of observations belong to low regime and 5.8 % to high regime for both equations, but most part of these are concentrated on the last ten years (Figure 1), refocusing the argument that the market is becoming more volatile.

Figure 1: WTI and Brent log-returns behavior according to TVEC model
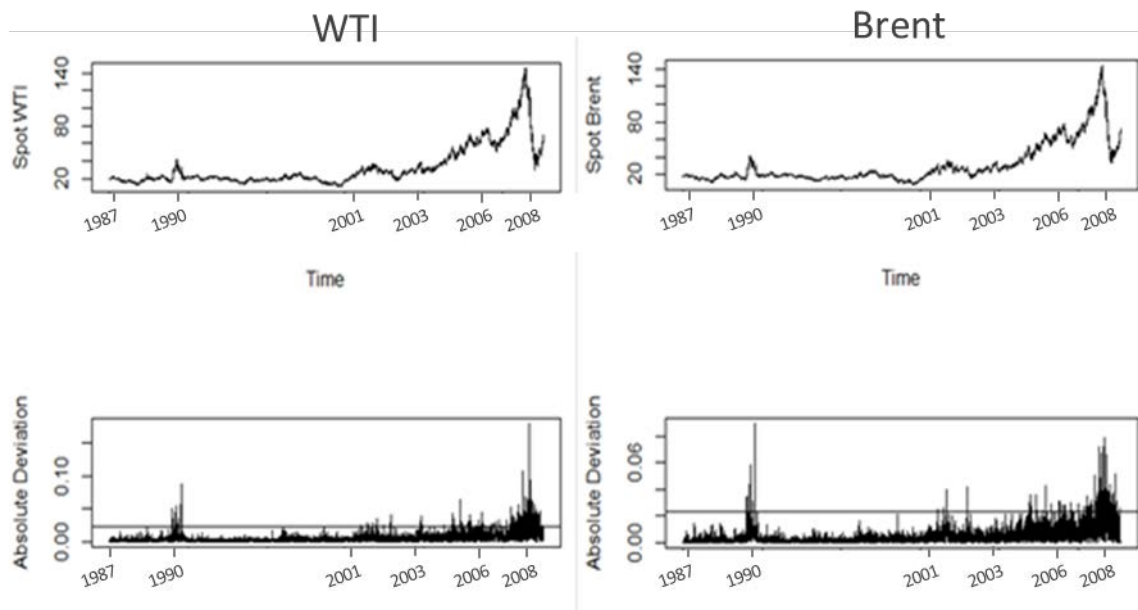
Note: Horizontal lines represent the Threshold line, splitting under high regime, above the line and low regime, below the line. Only the dates of major events are display

As we can see, TVEC model was capable to identify two different regimes. The first one is the regular regime, established when the deviation is small, meaning less volatility. This is the dominant situation, but in some moments the volatility is increased by "*good*" or "*bad*" news and we can observe the start of a new regime.

In August 1990 Iraq massively invaded Kuwait, in the next year, United States led a multinational force to defeat Iraq and the First Gulf War broke out. This war destroyed almost the entire crude oil extraction infrastructure in Iraq and Kuwait and created instability in this region. As we know, the Gulf region is very rich in crude oil resources, about 60% of United States' crude oil imports and 67% of the world oil came from there. This conflict generated political instability and market volatility became higher starting a new regime. This event was interpreted as a structural break by Salisu and Fasanya (2013) and Vivian and Wohar (2012).

In September 2001 the terrorist attack produced the same result and the war against terror had effects on oil market. It continued oscillating until the end of the second Gulf War in March 2003.

Next to 2006 we can observe more volatility again, caused by the uncertainty of Iraq's situation, its electoral process and increase in violence, the judgment of Saddan Hussein and conflicts between Israel and Palestine. This entire instability reflected on oil price. So, as we can see there are a lot of regime changes.

This characteristic became more important during the last four years; subprime and European crises increased the absolute deviation and consequently the volatility, making the difference between regimes become larger. The TVEC is capable to capture this volatility increase. During the 2008 financial crisis, it is easily observable that the volatility crossed the threshold line, reentering the high regime.

According to Salisu and Fasanya (2013), the subprime crisis had such a strong impact on oil that it was able to generate a structural break in the series. It is possible to observe that other models, such as the ones based on structural breaks, are capable to capture crude oil volatility, pointing out for important moments such as the 1990 First Gulf War and the 2008 Subprime crisis. However, unlike the TVEC, these models neglect the important historical facts such as the 2001 Terrorist Attacks, the 2003 Second Gulf War and others, indicating that TVEC is more sensitive to volatility increase. According to Ljung-box test, residuals from the TVEC model have no autocorrelation, meaning that the data are independently distributed, and the proposed model was successful in estimating the mean.

Using the calculated mean, we estimate the Copula-DCC-GARCH (Table 6 and Table 7). Results indicate that price volatility is conditional to past information, as evidenced by statistical significance of the parameters). Volatility seems to be more dependent on previous periods, for both variables, several lags were considered significant. As we can see, joint distribution is very significant, fitting perfectly to data, proving that TVEC-Copula-DCC-GARCH presents a very good fit for oil's volatility.

Table 6: TVEC Copula-DCC-GARCH (5,10) estimated coefficients of WTI / Future volatility model.

Parameter	Estimate	Std. Error	t value	Pr(>|t|)
WTI / Future volatility model				
ω^{WTI}	0.000	0.000	4.664	0.000
α_1^{WTI}	0.067	0.015	4.243	0.000
α_2^{WTI}	0.078	0.021	3.711	0.000
α_3^{WTI}	0.041	0.016	2.515	0.011
α_5^{WTI}	0.047	0.019	2.483	0.013
β_1^{WTI}	0.236	0.068	3.445	0.000
β_{10}^{WTI}	0.262	0.073	3.562	0.000
v^{WTI}	0.934	0.016	56.707	0.000
γ^{WTI}	6.781	0.601	11.271	0.000
ω^{Future}	0.000	0.000	3.695	0.000
α_1^{Future}	0.044	0.014	2.991	0.002
α_2^{Future}	0.047	0.012	3.709	0.000
α_5^{Future}	0.007	0.003	2.218	0.026
β_1^{Future}	0.770	0.038	20.17	0.000
β_{10}^{Future}	0.106	0.035	3.048	0.002
v^{Brent}	0.933	0.015	59.352	0.000
γ^{Brent}	7.688	0.771	9.967	0.000
$Joint_{dcca1}$	0.072	0.009	7.737	0.000
$Joint_{dccb1}$	0.926	0.009	95.425	0.000
$Joint_{mshape}$	4.778	0.227	20.997	0.000

Note: The non-significant parameters where omitted.

Table 7: TVEC Copula-DCC-GARCH (5,10) estimated coefficients of Brent / Future volatility model.

Parameter	Brent / Future volatility model			
	Estimate	Std. Error	t value	Pr(>|t|)
ω^{Brent}	0.000	0.000	2.539	0.011
α_1^{Brent}	0.078	0.111	0.702	0.048
β_1^{Brent}	0.910	0.102	0.000	0.000
$dBrent_{skew}$	0.981	0.026	37.388	0.000
$dBrent_{shape}$	6.156	2.857	2.154	0.031
ω^{Future}	0.000	0.000	3.581	0.000
α_1^{Future}	0.046	0.014	3.260	0.001
α_2^{Future}	0.048	0.026	1.819	0.068
α_5^{Future}	0.007	0.003	1.894	0.058
β_1^{Future}	0.765	0.034	21.942	0.000
β_{10}^{Future}	0.114	0.021	5.261	0.000
ν^{Brent}	0.934	0.015	59.749	0.000
γ^{Future}	7.375	0.727	10.135	0.000
$Joint_{dcca1}$	0.020	0.004	4.592	0.000
$Joint_{dccb1}$	0.960	0.011	86.879	0.000
$Joint_{mshape}$	6.655	0.592	11.228	0.000

Note: The non-significant parameters where omitted.

TVEC-Copula-DCC-GARCH presents a superior fit to estimate crude oil volatility, adapting very well to the leptokurtic and skewed distribution.

In order to provide an empirical test, we estimated the Conditional Volatility for WTI and Brent spot and future prices and it was calculated the optimal hedge ratios using out-sample data (Figure 2 and Figure 3 respectively).

Analyzing the volatility, we can perceive that returns show turbulence clusters. This pattern reflects the fact that the variability of log-returns is conditional to past occurrences, as captured by TVEC-Copula-DCC-GARCH model. This volatility clusters occur according to identified sector crisis and reinforce the argument that the static estimative is not adequate, and the dynamic approach is the one that fits best the situation.

Portfolio weights change according to volatility oscillation, adapting to the period. In high regime the dynamic estimate proves to be more efficient than the static. Optimal hedge ratio is similar for WTI and Brent, an expected result considering the correlation matrix values.

Figure 3: WTI / Future conditional estimated volatility, correlation and dynamic optimal hedge ratio.
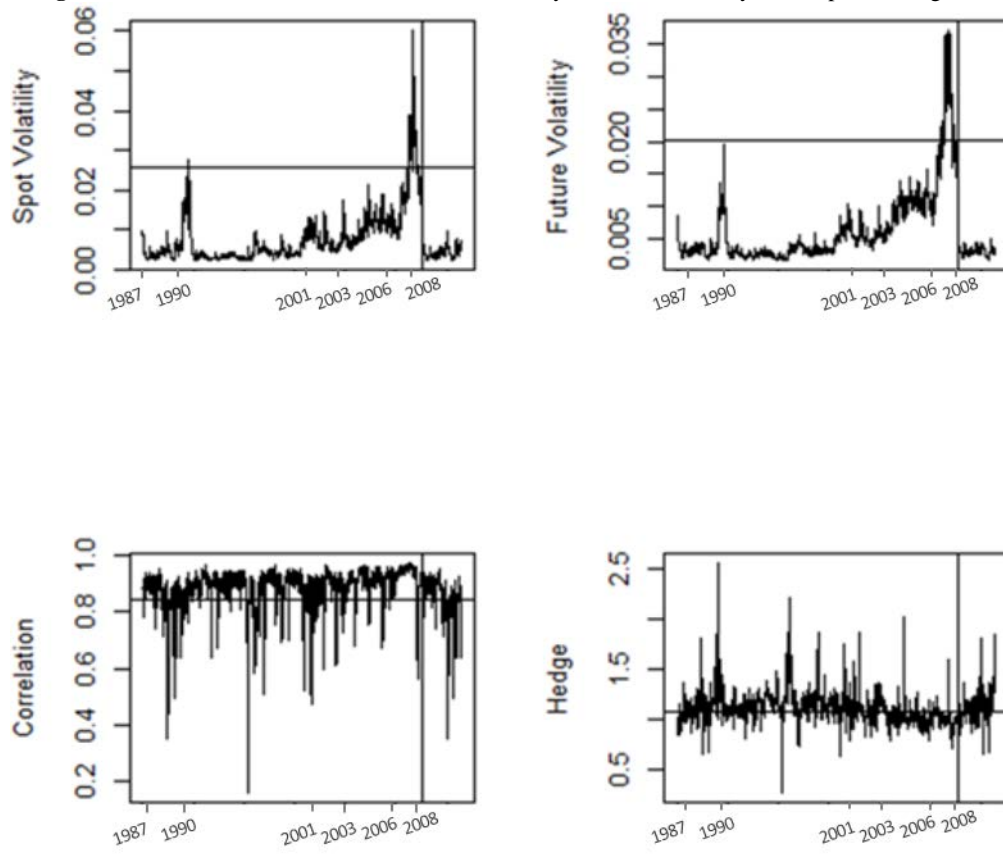
Note: Vertical lines represent the division between estimation and prediction and the horizontal ones represent the static estimative. Only the dates of major events are display

Figure 4: Brent / Future conditional estimated volatility, correlation and dynamic optimal hedge ratio
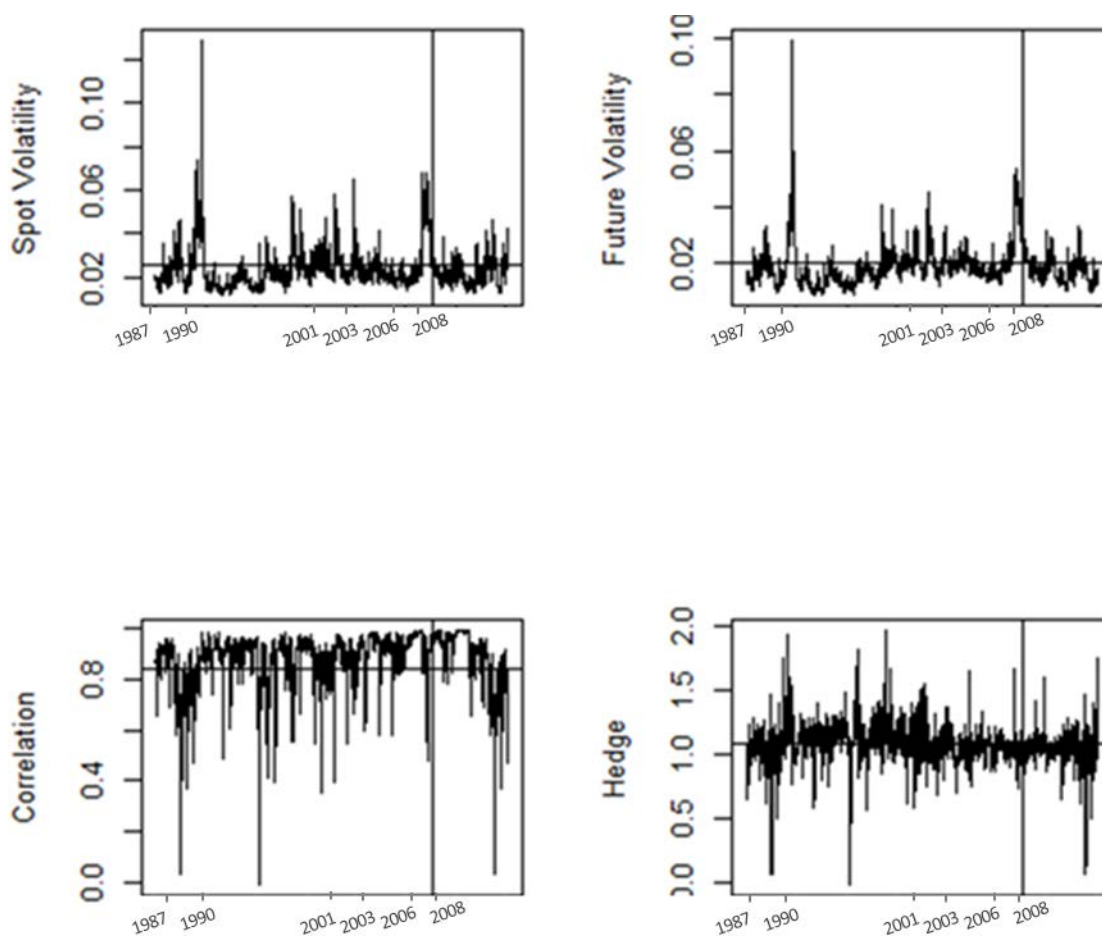
Note: Vertical lines represent the division between estimation and prediction and the horizontal ones represent the static estimative. Only the dates of major events are display.

Models have a good predictive power, especially considering the turbulent period designated to the predictions. Since 2009, successive crises had affected the market, and the predictions adjusted well to them.

Table 8 provides a comparison between CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK models tested by Chang, McAleer and Tansuchat (2011). It is possible to observe that these models had an efficiency rate around 80% for WTI and 56% for Brent. It is important to note that in Chang, McAleer and Tansuchat (2011), the periods analyzed were from 4 November 1997 to 4 November 2009, thus, the European crisis was not considered and predictions were not tested for posterior period.

Table 8: Comparison among CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK models tested by Chang, McAleer and Tansuchat (2011) and TVEC-Copula-DCC-GARCH model tested in current study.

Model	Optimal portfolio weights		Average OHR		Variance of portfolios		Hedge effectiveness	
	WTI	Brent	WTI	Brent	WTI	Brent	WTI	Brent
CCC*	0.350	0.383	0.955	0.840	1.349	2.682	80.857	56.724
VARMA-GARCH*	0.351	0.377	0.956	0.846	1.373	2.706	80.513	56.346
DCC*	0.478	0.366	0.923	0.824	1.342	2.663	80.942	57.045
BEKK*	0.571	0.355	0.922	0.827	1.417	2.710	79.886	56.294
Diagonal BEKK*	0.501	0.351	0.941	0.843	1.340	2.655	80.983	56.294
Copula-DCC-GRCH	0.467	0.391	0.935	0.856	1.421	2.876	85.956	65.234

Note: * Denotes models tested by Chang, McAleer and Tansuchat (2011), and ** denote the model tested in current study. The Variance Portfolios values are multiplied by 10^{-4}

Using TVEC model to estimate the ean, and Copula-DCC-Garch to analyze the variance, we were capable to obtain a variance reduction, meaning a hedge effectiveness around 89% for both. Thus, using this approach we had a gain around 9% for WTI and 33% for Brent compared to the models tested by Chang, McAleer and Tansuchat (2011) in a more turbulent period.

Furthermore, the proposed prediction adjusted very well to the series behavior, demonstrating a good adaptability to crisis periods, like European crisis. This characteristic is the most important for a hedge operation, considering its purpose of reducing risk, one of the main elements in times of crises.

5 Conclusions

To model crude oil volatility has been a challenge, especially considering the characteristics of the returns of this asset. Crude oil is subjected to political and economic instability, which provide recurrent regime changes. TVEC model proves to be efficient in capturing this instability, at the same time Copula functions respect the skewed and leptokurtic distribution of the returns. This paper shows that TVEC-Copula-DCC-

GARCH presents a superior fit to model crude oil volatility. The out-sample hedge test shows that this model presents significant empirical results. Although the estimation and tests were conducted in a more turbulent period than the tests with CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK, models models from previous studies, our results evidence that TVEC-Copula-DCC-GARCH is capable to provide more efficient hedge reasons due to its superior fit.

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