Evidence of dependence between volume, returns and volatility: A correlation of distances approach, using intraday data for all Ibovespa stocks

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Abstract: The relationships between volume, returns and volatility have been vastly explored in finance focusing developed markets. We present our approach to the Brazilian market, investigating the dynamics of sixty seven companies that are included in the portfolio of the Ibovespa index, the most influent index in South and Latin America. We utilize a strong statistical tool to measure association, the correlation of distances and measured volatility using squared returns. We find, for the entire sample, strong evidence of association between their returns and volatility. We also find significant association between volume and returns. We found only moderate interconnections between volume and lagged volatility, an indication of causality. Lastly, our results show some association of volume and returns.

Key-words: Correlation of distances, intraday, volatility, volume, returns, Ibovespa.

1 Introduction

It has been said in Wall Street “It takes volume to make prices move.” This paper examines the relationship between trading volume, returns and volatility (and their lags) using high frequency data in a 10 minutes interval, of all stocks comprising the Ibovespa index in Brazil; the largest stock market in Latin America. Trading volume, returns and volatility are major players in finance dynamics and even though their influences have been vastly studied in developed markets, there are not many evidences in developing countries, especially using intraday data and a somewhat new methodology to measure association, the distance correlation, at least new to finance.

There are a lot of takes on this topic, the relationships between volume, returns and volatility. Authors and market professionals dissent their perceptions around it. The arrival of information and the reaction to news causes investors to move markets; bulls or bears, one thing is for sure: investors, being home broker based or professional traders are heterogeneous and so are their interpretations of changes, news, and data. Even a run to the bank has multiple interpretations; another classic example is that even during a sell off, someone is buying. The set of analysis, presented here is consistent with the work of Copeland [1], the Sequential Information Arrival Hypothesis (SIAH) and predicts the causal relationship between volume and volatility. It assumes as we mentioned before, that investors react differently to new information. Hence the new price formation requires a few minutes perhaps, to balance itself out. Nevertheless, the SIAH suggests that market equilibrium is not instantaneous.

Another perspective is pointed out by Clark [2] and reviewed by Harris [3], the Mixture of Distribution Hypothesis (MDH) and assumes that there is a contemporaneous relationship between volatility and volume and not a casual or lead/lag type of association. As this work does, Darrat et al. [4] show evidence that high trading volume causes high return volatility as the SIAH proposes and the MDH opposes. In their study they studied 30 stocks in the Dow Jones Industrial Average (DJIA) utilizing intraday data.

Another interesting study is presented by Shalen [5] and later reviewed by Daigler and Wiley [6], that examine the volatility and volume relationship using volume data categorized by the type of investor/trader. Analyzing future markets, they suggest that the relationship is positive and driven by the general public and a group of traders who dwell away from the trading floor and do not have precise information on the order flow. Moreover, there is a distinction between floor traders (endangered species these days); clearing members who observe the order pattern and flow tend to decrease volatility. Uniformed floor traders increase volatility because they cannot differentiate liquidity demand from fundamental value change.

There is also an alternative view of the relationship between volume and volatility, which is that investors share common prior beliefs and even receiving the same information, they differ in the way they process and interpret this information. This phenomenon can be seen in the works of Harris and Raviv [7]. Our goal here does not go that far, we are not interested in the behavior of traders, analysts and the general investor; the focus is to show evidence of associations between volume, returns and volatility in its primal form. The idea that stock prices reflect all the known information for that particular time is debatable. This paper tries to investigate and shed some light in one of the multidimensional facets of price formation in the major Latin stock market, Brazil.

Predicting volatility is predicting future prices and understanding that behavior and the dynamics intrinsic to this relationship matter for traders, hedge fund managers, speculators and so on. Most of financial professionals and academics study the market because they believe that at some point, returns will not behave efficiently, as originally laid by Cootner [8] and later proposed by Fama [9] and reviewed many times such as Schwartz [10], Fama [11]. In other others, today most of the empirical studies in finance admit some sort of inefficiency in some markets at some point and do not behave as random walks Lo and MacKinlay [12]; and Lo, et al. [13]. In this work, we assume that markets behave inefficiently at some stage but the overall market is efficient in the long run, as proposed by Malkiel [14]. The role professionals and academics play is to early detect these deficiencies/inefficiencies and turn them into a strategy to make a profit or to minimize risk in a speculative or hedge position.

The analysis of volume and volatility has been done several times since the work found in Karpoff [15]; Hiemstra and Jones [16]; Chen, et al. [17], and more recently Huang and Wang [18]; Chuang et al. [19]. This work is inspired in the findings of Medeiros and Van Doornik [20] which analyze daily data of the Ibovespa utilizing GARCH, VAR models and Granger causality tests and find contemporaneous and casual relationship between returns and volume. Even though the topic is source to many studies, this paper brings a new methodology to analyze relationship and measure association the distance correlation, proposed by Székely, et al. [21]; along with the contemporaneous correlation. Moreover, the usage of intraday data in a developing country is another differentiation and edge. To the volatility we utilized the squared return. One more thing we try to prove is the casual relationship between volatility and volume, for that we estimate an association in time t and $t-1$, to examine if lagged volatility influences volume, in other words, if past innovations alter trading volume patterns, among other relationships better explained later.

Summarizing, this paper investigates the relationship between returns, volatility and volume as well as some lagged relationships, to demonstrate causality, of all Ibovespa stocks using intraday data. We utilized squared returns as proxy for volatility as in the works of Cumby, et al. [22]; Jorion [23], Figlewski [24], and more recently Ahoniemi and Lanne [25]. To measure association we utilize a new measure to finance, the distance correlation or correlation of distances. High frequency data in a 10 minutes interval of returns and volume is the basis, and all stocks that participate in the main index in Brazil, the Ibovespa are analyzed. In the end we will show strong evidence for some stocks that yes: “It takes volume to make prices move”, but the strongest evidence found is between returns and volatility, sturdy for all stocks investigated.

2 Literature Review

The basis for the analysis of volume and volatility was laid by Karpoff [15]; however Epps and Epps [26] have already studied the stochastic dependence of security price changes and transaction volumes, which postulate that the change in the logarithm price can be explained by a mixture of distributions, with transaction volume as the mixing variable.

Rogalsky [27] study the dependence of prices and volume, especially if they are causally related. Utilizing a cross-correlation approach with zero lags at 5% significance found strong evidence of that relationship. Suggesting that knowledge of volume behavior may improve price forecasts, over predictions based on the price alone. Tauchen and Pitts [28] analyze the price and volume variability relationship on speculative markets in two forms using daily data from the 90-day T-bills futures market. First, they derive the joint probability distribution of the price change and the trading volume over any interval of time within the trading day. Secondly, they determine how this joint distribution changes as more traders enter or exit the market.

Lamoureux and Lastrapes [29] utilize autoregressive conditional heteroskedasticity to model volatility and conclude that contemporaneous squared returns and volume are noisy predictors of future return and volatility. Medeiros and Van Doornik [20] studying the Brazilian market find significant evidence of contemporaneous relationship between volume and volatility, according to the cross-correlation analysis. It also finds that stock returns depend on trading volume; however the same does not happen the other way around.

The relationship between volume and volatility is not a consensus. Mestel, et al. [30] study the empirical relationship between stock returns, return volatility and trading volume on the Austrian stock market and found weak contemporaneous as well as dynamic associations between stock returns and trading volume, implying that it would be improbable to base forecasts in volume. On the other hand, they found strong contemporaneous relationship between volatility and volume. Chan and Fong [31] re-examine volatility and transactions by constructing the realized volatility measure from the sum of intraday squared returns, establish that the number of trades is the dominant factor behind the volatility-volume relation and that, beyond the trading volume or the number of trades, trade size adds very little explanatory power for realized volatility. Consistent with the theory of quadratic variation, realized volatility estimates are shown to be less noisy than standard volatility measures such as absolute returns used in previous studies.

The contemporaneous correlation also has been largely used in all fields of sciences. In finance, Kahya [32] investigate the effects of non-overlapping trading hours on the correlations and cross-serial correlations of stock prices in non-contemporaneous markets. Diebold and Mariano [33] utilize this measurement to analyze forecasting methods. Bekaert and Hoerova [34] include this correlation in their model to assess variance in the US S&P500 options prices.

Gebka [35] study the co-movements of index returns, volatility and volume for eight Asian stock markets and the United States. They do not find returns spillovers, and spillovers in absolute returns to be strong in both directions, and spillovers in volatility to go from east to west. Volume depends on shocks in domestic and foreign returns, so does volatility.

One innovative aspect of this work is the application of distance correlation or correlation of distances (dCor) to measure association between two random vectors in an unequal dimension, This measure is derived from a number of statistical methods such as distance variance, distance standard deviation and distance covariance and its applications is firstly presented in the work of Székely, et al. [21] “Measuring and testing dependence by correlation of distances”. At the time we write this paper, we do not see any previous application of this measure in finance. In other fields of science, the method can be seen in several works such as Kong, et al. [36] e Ramalhinho-Lourenço, et al. [37].

3 Data

Intraday data of returns and volume in a 10 minutes interval is the base for the dataset. The trading days chosen span from January to March 2013, in a total of 1870 observations per stock. The source of this data was the Thompson Reuters Eikon Software® which is extremely reliable and one of the most trustworthy trading software in the market today.

As common in finance, returns were calculated in a logarithmic base, to make the time series stationary and what was compared to extract association were the index returns in a logarithmic base and the stock returns also in a logarithmic base in the same time

Whenever intraday data is in usage some major drawbacks come to life, for an example when compiling 10 minutes interval of less liquid stocks, there are times that that particular stock have not been traded in a few of the 10 minutes along the entire trading day, but the index was. So what is seen is a long record of index data but not so long of the less liquid stocks. To correct and unsettle that, all blank spaces were filled with the last traded value and zero volume. In this way, all stocks have equal number of observations and more importantly, the stock is set at its real price in time, and appropriated volume level, considering the same 10 minutes interval. The volatility considered is the squared return, for every 10-minute tick return as in the work of Ahoniemi and Lanne [25].

3.1 The Ibovespa

The Ibovespa is the main indicator of the Brazilian stock market's average performance. The Ibovespa reflects the variation of the main stock market in Brazil and its most traded stocks. There have been no methodological changes to the index since its inception in 1968, when it has been attributed a base value of 100 points as of a hypothetical investment. The participation of each stock in the portfolio has a straight relation with its representativity in the cash market, measured in terms of number of trades and financial value, adjusted to the sample size. From time to time less traded stocks give place to others that obtained greater numbers in a set time frame. In this work, the stocks that pertain to the index in the beginning of the sample size will be evaluated, if there were changes in the period, these changes will not affect the results of this work.

The index main objective is to be an average indicator of the market performance. For that purpose, its composition aims at reflecting as close as possible the real configuration of the cash market operations on BM&FBOVESPA. In terms of liquidity stocks that integrate its theoretical portfolio represent more than 80% of the number of trades. In terms of market capitalization, the issuing companies of the stocks that compose the BM&FBOVESPA Index theoretical portfolio are responsible, in average, for approximately 70% of the sum of all BM&FBOVESPA overall market cap.

Of a total of 71 stocks, 67 were investigated, four stocks were left out because inconsistent data (VAGR3, HYPE3, BRPR3 and BBDC3). All codes and names as well as their percentage in the index can be seen in Annex 1.

4 Methodology

In this section we lay out the relationships we wish to investigate, as well as the methods used to get there. As mentioned before the econometrics will be based on correlation of distances, and squared returns as a proxy for volatility. We investigate five types of association of three variables: risk (volatility), returns and volume. We will also lag behind 1 step some variables to investigate causality. The relationships that include volatility will be analyzed by both methods.

The associations we investigate are: a) Volume and volatility; b) volume and returns; c) volume and lagged returns; d) volume and lagged volatility; e) returns and volatility; f) returns and lagged volatility; g) returns and lagged volume; h) volatility and lagged returns; i) volatility and lagged volume.

In the charts (-1) indicates a lagged variable and in all our study they have been lagged just one-step backwards, one ten minutes tick. The reason is to determine causality.

4.1 The econometrics

This part, we scrutinize the principles of the two econometric methods used, the correlation of distances and the squared returns. To understand distance correlation, firstly one needs the concepts below:

4.1.1 Distance covariance

To begin it is needed to understand the definition of sample distance covariance. Let $(X_k, Y_k), k = 1, 2, \ldots, n$ be a statistical sample from a pair of real value vector valued random variables (X,Y). First we compute all pairwise distances:

$$a_{j,k} = \left\| X_j - X_k \right\|, j, k = 1, 2, \ldots, n, \quad (1)$$

$$b_{j,k} = \left\| Y_j - Y_K \right\|, j, k = 1, 2, \ldots, n, \quad (2)$$

where the parenthesis denote de Euclidean norm. That is, compute the n by n distance matrices $a_{j,k}$ and $b_{j,k}$.Then take all doubly centered distances $A_{j,k} = a_{j,k} - \overline{a}_{j.} + \overline{a}_{.k} + \overline{a}_{..}$,and $B_{j,k} = b_{j,k} - \overline{b}_{j.} + \overline{b}_{.k} + \overline{b}_{..}$. Where $\overline{a}_{j.}$ is the j-th row mean, $\overline{a}_{.k}$ is the k-th column mean, and $\overline{a}_{..}$, is the grand mean of the distance matrix of the X sample. The notation is similar for the b values. In the matrices of centered distances $\left(A_{j,k}\right)$ and $\left(B_{j,k}\right)$ all rows and all columns sum to zero. The squared sample distance covariance (3) is simply the arithmetic average of the products $A_{j,k}, B_{j,k}$:

$$dCov_n^2\left(X,Y\right)=\frac{1}{n^2}\sum_{j,k=1}^{n}A_{j,k}B_{j,k} \quad (3)$$

Similarly, the distance variance (4) and distance standard deviation (5).

$$dVar_n^{(X)}=dCov_n^2\left(X,X\right)=\frac{1}{n^2}\sum_{k,l}A_{k,l}^2\ , \quad (4)$$

$$dStd_n^{(X)}=\sqrt{dCov_n^2\left(X,X\right)=\frac{1}{n^2}\sum_{k,l}A_{k,l}^2}\ . \quad (5)$$

Finally, the notation for distance correlation (6), similar to the regular correlation; however it utilizes the distance variance and distance covariance.

$$dCor\left(X,Y\right)=\frac{dCov\left(X,Y\right)}{\sqrt{dVar\left(X\right)dVar\left(Y\right)}}\ . \quad (6)$$

4.1.2 Squared returns

Various measures have been suggested as proxy variables for volatility in financial markets. Although these days GARCH models have taken over as the most popular, intraday data is tough. Due to jumps in volatility at the opening and heteroskedasticity patterns throughout the trading time, most of the GARCH models have been disregarded to model intraday data. Nevertheless, the most traditional is to use squared returns as proxy, and it have been done several times in finance such as in Pagan and Schwedt [38], Ahoniemi and Lanne [25], Reschenhofer [39] and many more.

It follows the notation: $h=\left(\log A_t-\log A_{t-1}\right)^2$. Where A is the return and t is time. It is simple and effective.

5 Results

In a nutshell, Table 1 tells it all. The highest level of association for an individual stock is found in the binary return and volatility. Moreover, this association displayed the higher mean of all associations studied, and even the minimum is considered extremely high for financial datasets. Although the other types of relationships did not display a significant mean, some stocks within the sample have. The same goes to volume versus volatility, volume versus returns and volume and lagged volatility. We considered that anything over 0.30 is significant.

Table 1. Statistics of dCorrelations per variable pair of all Ibovespa Stocks.

	Mean	Minimum	Maximum	Standard Deviation
Volume vs Volatility	0.16	0.05	0.46	0.10
Volume vs Return	0.12	0.04	0.35	0.07
Volume vs Return(-1)	0.08	0.03	0.27	0.04
Volume vs Volatility(-1)	0.11	0.03	0.34	0.06
Return vs Volatility	0.59	0.51	0.61	0.02
Return vs Volatility(-1)	0.11	0.06	0.15	0.02
Return vs Volume(-1)	0.07	0.04	0.19	0.02
Volatility vs Return(-1)	0.10	0.05	0.14	0.02
Volatility vs Volume(-1)	0.09	0.03	0.28	0.04

Fig. 1 shows four relationships and arranged the strongest of the four, by descending order for an easier notation. We determined that any dCorrelation over 0.30 would be significant and this figure displays three associations with overcome that mark: volume and volatility, volume and return and volume and lagged volatility. Some stocks showed an extremely high relationship in the pair volume and volatility. Needless to say that in most of the firms did not present any association at all, between the variables proposed by this research. So we will only bring up any association coefficient over 0.30 and for volume and volatility. LLXL3 (0.46), OGXP3 (0.39), GFSA3 (0.36), CSNA3 (0.36), MMX3 (0.36), BISA3 (0.34), ELET3 (0.36) and RSID3 (0.30). Volume and returns evidenced only two companies with high coefficients, OGXP3 (0.35) and LLX3 (0.30). Whereas volume and lagged volatility brings only one firm: OGXP3 (0. 34).

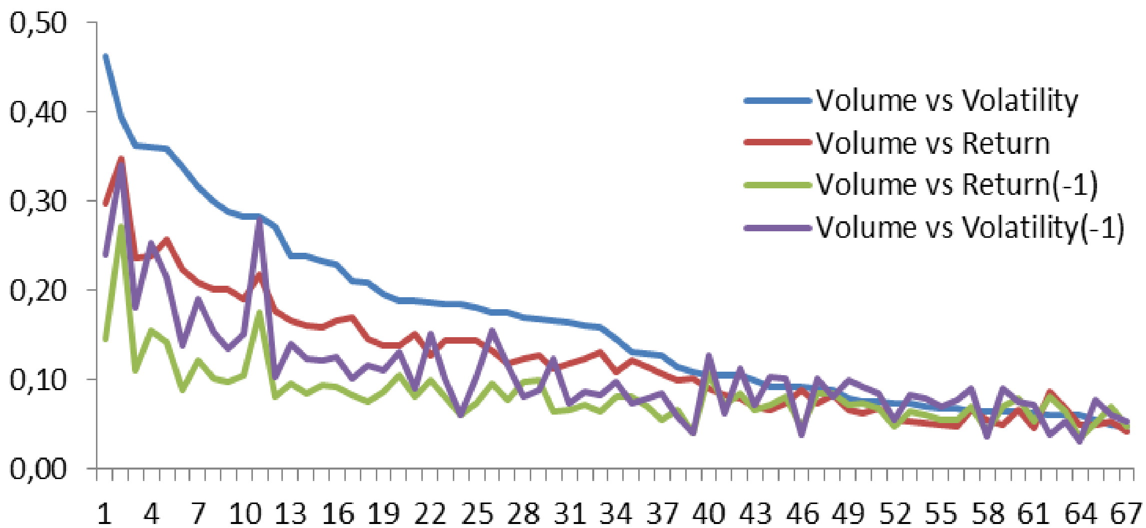

Fig. 1. Ibovespa stocks and their variables coefficient of dCorrelations by descending order of volume and volatility and volume and returns, lagged returns and lagged volatility.

Fig. 2 displays the most striking results this research produced, an across the board extremely strong relationship between volatility and returns, for all 67 stocks investigated. It also shows a moderate association between a few stocks when it comes to volume and returns and volume and lagged volatility. So basically every firm displays a strong relationship between volatility and returns, and the companies that topped the list with a coefficient of (0.61) are: CPFL6, CCRO3, CIEL3, HGTX3, BTOW3, DTEX3, TRPL4 and NATU3. Runner ups display only a small difference from the winners, LREN3, CTIP3, SUZB5, LIGT3, PCAR4, LLXL3, CRUZ3, CYRE3, PDGR3, BRML3, UGPA3, BISA3, CESP 6, CSAN3, ALL3 and CPFE3 with (0,60) dCorrelation. And as per the other pairs: returns and lagged volatility and returns and lagged volume, no stock came across with high coefficients and the overall mean did not present any interesting findings.

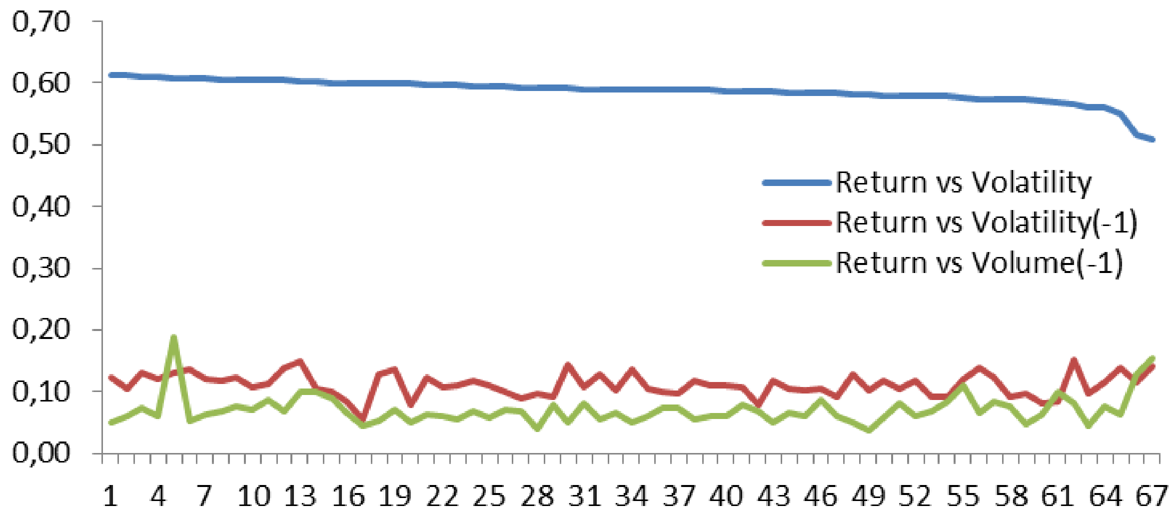

Fig. 2. Ibovespa stocks and their variables dCorrelations by descending order of return and volatility and return and lagged volume and volatility

At last, Fig. 3 evidences no strong association between volatility and lagged returns and lagged volume. Although some spikes can be seen, none of them takes over the 0.30 mark, set to be relevant, so they cannot be classified as significant.

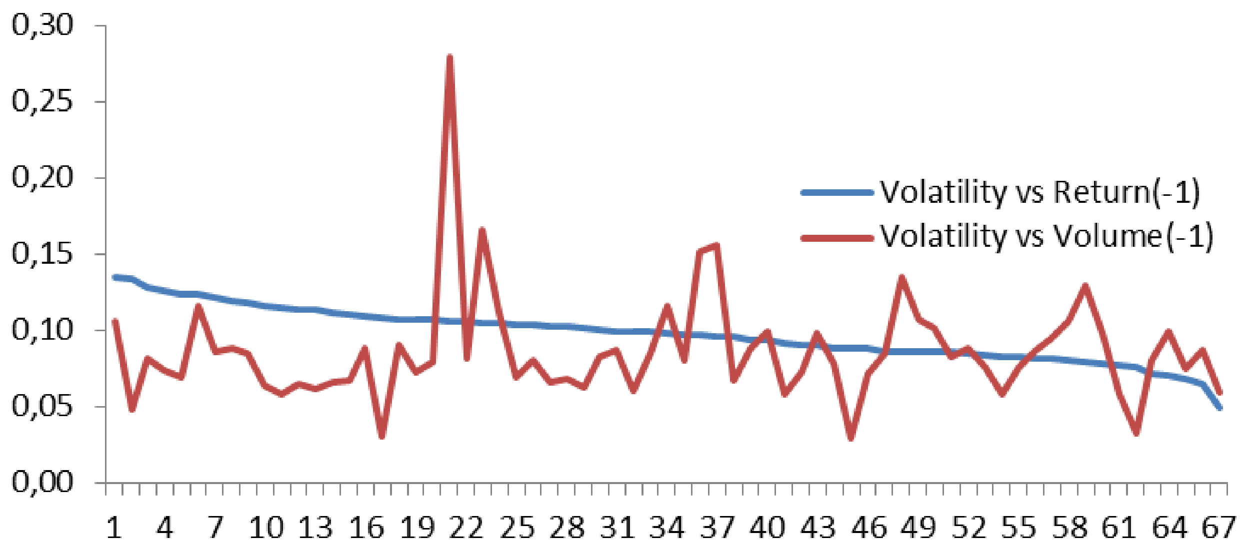

Fig. 3. Ibovespa stocks and dCorrelation between volatility, lagged returns and lagged volume.

6 Conclusions

Our findings are consistent with similar works carried out in developed markets such as Hiemstra and Jones [16], Darrat, et al. [4] and Naka and Oral [40]. Fierce relationships are hard to find in finance, however we find, for several stocks, strong evidence of association in some of the relationships tested.

The strongest of all pairs of associations, is returns and volatility, that is robust for all the stocks in our sample, but the overall mean is weak for all the rest. Although some other relationships produced a high degree of correlation for a hand full of stocks, the great majority did not display any relevant results in any associations at all.

Even knowing the shortcomings, we also used GJR-GARCH to model volatility and applied the same methodology. GARCH models are not ideal for intraday datasets such as in this research, but we wanted a base line for comparison. It produced weaker associations but many stocks that topped the lists with higher degrees of association, are common whichever volatility proxy used.

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Annex 1 – Ibovespa portfolio composition, codes, company names, type of stock, theoretical quantity and participation (%).

Code	Stock	Type	Theoretical Quant.	Part. (%)	Code	Stock	Type	Theoretical Quant.	Part. (%)
ALLL3	ALL AMER LAT	ON NM	4.128.629.524.957	0.781	HGTX3	CIA HERING	ON ED NM	149.692.304.098	1.090
AMBV4	AMBEV	PN	1.087.358.918.177	1.622	HYPE3	HYPERMARCAS	ON ED NM	4.530.248.226.493	1.298
BBAS3	BRASIL	ON NM	6.411.452.068.051	2.864	ITSA4	ITAUSA	PN EB N1	15.377.764.525.495	2.519
BBDC3	BRADESCO	ON EJ N1	1.047.620.841.292	0.649	ITUB4	ITAUUNIBANCO	PN ED N1	724.741.448.351	4.402
BBDC4	BRADESCO	PN EJ N1	5.839.504.764.485	3.468	JBSS3	JBS	ON ED NM	7.638.127.628.505	0.852
BISA3	BROOKFIELD	ON NM	18.716.371.971.127	0.745	KLBN4	KLABIN S/A	PN N1	3.043.059.590.579	0.743
BRAP4	BRADESPAR	PN EDJ N1	1.704.250.642.088	0.762	LAME4	LOJAS AMERIC	PN	2.795.556.402.165	0.865
BRFS3	BRF FOODS	ON NM	1.578.958.445.611	1.383	LIGT3	LIGHT S/A	ON ED NM	1.150.626.563.301	0.410
BRKM5	BRASKEM	PNA N1	238.684.890.063	0.733	LLXL3	LLX LOG	ON NM	16.631.015.114.838	0.578
BRML3	BR MALLS PAR	ON ED NM	3.307.585.725.395	1.395	LREN3	LOJAS RENNER	ON NM	694.872.564.665	0.927
BRPR3	BR PROPERT	ON ED NM	1.803.850.868.459	0.689	MMXM3	MMX MINER	ON NM	26.606.550.275.198	0.993
BTOW3	B2W VAREJO	ON NM	1.601.296.661.934	0.348	MRFG3	MARFRIG	ON NM	5.438.752.201.609	0.683
BVMF3	BMFBOVESPA	ON NM	10.944.296.755.683	2.840	MRVE3	MRV	ON NM	10.481.483.792.585	1.562
CCRO3	CCR SA	ON NM	4.406.996.039.123	1.562	NATU3	NATURA	ON NM	122.088.411.742	1.100
CESP6	CESP	PNB ED N1	1.426.178.635.514	0.553	OGXP3	OGX PETROLEO	ON NM	1,534.6119396256	5.061
CIEL3	CIELO	ON EB NM	1.708.157.992.821	1.666	OIBR3	OI	ON N1	2.813.810.379.021	0.269
CMIG4	CEMIG	PN EDB N1	4.131.135.187.173	1.590	OIBR4	OI	PN N1	15.452.778.646.535	1.281
CPFE3	CPFL ENERGIA	ON NM	1.231.426.737.497	0.483	PCAR4	P.ACUCAR-CBD	PN ED N1	367.293.741.991	0.737
CPLE6	COPEL	PNB ED N1	655.396.228.563	0.421	PDGR3	PDG REALT	ON NM	6.341.423.711.523	2.617
CRUZ3	SOUZA CRUZ	ON	1.359.383.419.761	0.738	PETR3	PETROBRAS	ON EJ	7.713.597.950.835	2.641
CSAN3	COSAN	ON NM	900.076.813.104	0.768	PETR4	PETROBRAS	PN EJ	22.219.368.941.141	8.009
CSNA3	SID NACIONAL	ON	11.272.759.365.722	1.579	RENT3	LOCALIZA	ON EDB NM	1.142.871.150.192	0.683
CTIP3	CETIP	ON NM	187.402.056.549	0.834	RSID3	ROSSI RESID	ON NM	16.374.370.791.956	1.012
CYRE3	CYRELA REALT	ON ED NM	4.158.017.413.484	1.319	SANB11	SANTANDER BR	UNT N2	410.903.521.492	1.055
DASA3	DASA	ON NM	3.449.666.411.434	0.671	SBSP3	SABESP	ON NM	1.200.913.818.032	0.611
DTEX3	DURATEX	ON NM	2.030.625.157.907	0.567	SUZB5	SUZANO PAPEL	PNA ED N1	6.527.840.011.669	0.855
ELET3	ELETROBRAS	ON EJ N1	6.793.299.123.247	0.617	TIMP3	TIM PART S/A	ON NM	8.114.501.960.693	1.214
ELET6	ELETROBRAS	PNB EJ N1	4.205.919.734.287	0.644	TRPL4	TRAN PAULIST	PN N1	411.617.538.361	0.254
ELPL4	ELETROPAULO	PN N2	4.531.542.946.239	0.645	UGPA3	ULTRAPAR	ON NM	751.556.613.105	0.727
EMBR3	EMBRAER	ON NM	2.227.019.181.119	0.715	USIM3	USIMINAS	ON N1	1.543.688.548.148	0.267
ENBR3	ENERGIAS BR	ON NM	2.977.988.109.271	0.655	USIM5	USIMINAS	PNA N1	12.170.934.299.396	2.064
FIBR3	FIBRIA	ON NM	1.960.500.891.795	0.756	VAGR3	V-AGRO	ON NM	36.586.722.966.432	0.257
GFSA3	GAFISA	ON NM	19.376.227.793.427	1.411	VALE3	VALE	ON N1	4.371.774.782.878	2.663
GGBR4	GERDAU	PN N1	8.563.632.471.661	2.395	VALE5	VALE	PNA N1	14.793.477.511.384	8.585
GOAU4	GERDAU MET	PN N1	1.696.693.486.111	0.597	VIVT4	TELEF BRASIL	PN	956.511.146.749	0.933
GOLL4	GOL	PN N2	3.319.192.281.658	0.717	Total	**TOTAL**		**6,336.65257692868**	**100.000**

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