A Study on Connectivity Concepts in Intuitionistic Fuzzy Graphs

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Abstract:- In this paper, we introduced some concepts of connectivity in an intuitionistic fuzzy graphs, also we study intuitionistic fuzzy cut vertices and intuitionistic fuzzy bridges in fuzzy graph. Connectivity in complete intuitionistic fuzzy graphs is also studied.

Key words:- fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy graphs, intuitionistic fuzzy cut vertex, intuitionistic fuzzy bridge.

1 Introduction

Zadeh in [1] introduced the concept of fuzzy sets in 1965. Fuzzy sets paved the way for new philosophical fuzzy logic thinking. This logic is used in the large number productions in electronics. Fuzzy sets and fuzzy logic theory have been applied widely in areas like database theory, robotics, expert systems, control theory, information theory, pattern recognition and nano-technology. In 1975, Rosenfeld [2], studied fuzzy graphs. Fuzzy graphs are useful to represent relationships which deals with uncertainty and its greatly different from graphs. Massa'deh et al [3, 4] studied more properties for fuzzy graph such as degree of vertices and isomorphism. Atanassov [5, 6] introduced the intuitionistic fuzzy set concepts, after that Karunambigai & Kalaivani [7] introduced the matrix representation of intuitionistic fuzzy graphs, while Mishra & Pal [8, 9] in 2013 and 2017 respectively discussed the product of interval valued intuitionistic fuzzy graph and regular interval valued, and in 2014 Yahya & Jahir [10] studied isomorphism on irregular fuzzy graph. Pathinathan & Rosline [11] gave the concept of vertex degree of cartesian product of intuitionistic fuzzy graph. In 2018, Sunny & Jose [12] introduced the notion of modular and homomorphic products on interval intuitionistic fuzzy graph. in 2020, Fallatah et. al.[13] and Alnaser et. al. [14] added new concepts which are intuitionistic fuzzy soft graph and bipolar intuitionistic fuzzy graphs. In this paper, we introduced some concepts of connectivity in an intuitionistic fuzzy graphs, also we studied intuitionistic fuzzy cut-vertices and intuitionistic fuzzy bridges in fuzzy graphs, on the other hand, some properties and concepts are added.

2 Preliminaries

In this section we review and recollect some concept for undirected graphs [2]. A graph is an ordered pair $G = (V, E)$, where V is the set of vertices of G and E is the set of edges of G, while a sub graph of $G = (V, E)$ is a graph $S = (W, F)$ such that $W \leq V$ and$F \leq E$. An undirected graphs that has no loops and not more than one edge between any two different vertices is a simple graph. A trivial graph is a simple graph with a single vertex. Two vertices i and j in an undirected graph G are said to be adjacent in G if (i, j) is an edge of G. $i\, j$ or $j\, i$ represented to edge

The set of all vertices adjacent to a vertex i in G is called the neighbor set of i and is denoted by $N(i)$. A $i_{o-}i_n$ path P in G is an alternating sequence of vertices and edges $i_0, e_1, i_1, e_2, \dots, e_n, i_n$ such that $i_k\ i_{k+1}$ is an edge for $i = 0,1, \dots, n-1$ The number of edges in the path is called the path length and P is called closed path or a cycle if $i_{o=}\, i_n$.

A graph G is said to be connected if there is a path joining any two vertices in G while, a graph G is said to be a tree if it is connected and acyclic. The connected components number in G is denoted by by $\omega(G)$. A vertex i of G is called cut vertex if $\omega(G-V) > \omega(G)$. Also, an edge e of G is said to be a cut edge if $\omega(G-e) > \omega(G)$.

A graph G is called complete graph if all the vertices in G are pairwise adjacent.

Definition 2.1: [1] a fuzzy set λ on G is just a map . $\lambda : G \to [0,1]$.

Definition 2.2 :[2] A map $\vartheta: Gx\ G \to [0,1]$ is said to be fuzzy relation on λ if $\vartheta(i,j) \leq \min\{\mu_{(i)}, \mu_{(j)}\}\ \forall\ i,j \in G$ on the other hand , a fuzzy relation ϑ is called reflexive if $\vartheta(i,j) = \lambda_{(i)} \forall\ i\ \in G$, while ϑ is called symmetric if $\vartheta(i,j) = \vartheta(j,i)\ \forall\ i,j\ \in G$.

Definition 2.3: [2] a fuzzy graph is a path $G = (\delta, \lambda)$ such that δ is a fuzzy subset on a set V and λ is a fuzzy relation on δ .Since V is non empty and finite, λ is symmetric and reflexive. Therefore, if $G = (\delta, \lambda)$is a fuzzy graph , then $\delta: V \to [0,1]$ and $\lambda: V \times V \to [0,1]$ such that $\lambda(i,j) \leq \min\{\delta_{(i)}, \delta_{(j)}\} \forall\ i,j \in V$.

Remark 2.4: [1] we represent the underlying graph of $G = (\delta, \lambda)$by $G^+ = (\delta^+, \lambda^+)$such that $\delta^+ = \{i \in V; \delta_{(i)} > 0\}\ and\ \lambda^+ = \{(i,j) \epsilon V \times V;\ \lambda(i,j) > 0\}$, $G = (\delta, \lambda)$ is a trivial fuzzy graph , if $G^+ = (\delta^+, \lambda^+)$is trivial.

3 Connectivity in intuitionistic fuzzy graph

Definition 3.1: [7] an intuitionistic fuzzy graph $G = (V, E)$ such that $V = \{i_1, i_2, \dots, i_n\}$ where $\lambda_1: V \to [0,1]$ and $\varphi_1: V \to [0,1]$ represent the membership degree of non-membership of $i_k \in V$ respectively, also $0 \leq \lambda_1(i_k) + \varphi_1(i_k) \leq 1$ for every $i_k \in V (k = 1, \dots\dots, n)$

1) $\varphi_1 \leq V \times V$such that $\lambda_2: V \times V \to [0,1]$and $\varphi_2: V \times V \to [0,1]$such that $\lambda_2(i_{k+} i_n) \leq \min\{\lambda_1(i_k), \lambda_1(i_n)\}$ and $\varphi_2(i_{k+} i_n) \leq \max\{\varphi_1(i_k), \varphi_1(i_n)\}$ and $0 \leq \lambda_2(i_k, i_n) + \varphi_2(i_k, i_n) \leq 1$ for every $(i_k, i_n) \in E$

Definition 3.2: [7] the underlying crisp graph of an intuitionistic fuzzy graph $G = (\delta, \lambda)$is the graph $G = (V`, E`)$such that $V` = \{i \in V; \gamma_\delta(i) > 0\ or\ \mu_\delta(i) > 0\}$ and $` = \{(i,j); \gamma_\lambda(i,j) > 0\ \ or\ \ \gamma_\lambda(i,j) > 0\}$

Definition 3.3: an intuitionistic fuzzy graph $G = (\delta, \lambda)$is connected if the underlying crisp graph is connected.

Definition 3.4: A partial an intuitionistic fuzzy subgraph of an intuitionistic fuzzy graph G is an intuitionistic fuzzy graph $\delta = (\delta`, \lambda`)$such that $\gamma_{\delta`}(i_k) \leq \gamma_\delta(i_k)$ and $\varphi_{\delta`}(i_k)\ \geq\ \varphi_\delta(i_k) \forall\ i_k\ \in$

V and $\gamma_{\lambda`}(i_k, i_n) < \gamma_{\lambda}(i_k, i_n)$ and $\gamma_{\lambda`}(i_k, i_n) > \gamma_{\lambda}(i_k, i_n)$ for every $i\ j \in V$.

Definition 3.5: an intuitionistic fuzzy subgraph of G is an intuitionistic fuzzy graph $\delta = (\delta`, \lambda`)$ such that $\gamma_{\delta`}(i_k) < \gamma_{\delta}(i_k)$ and $\varphi_{\delta`}(i_k) = \varphi_{\delta}(i_k) \forall\, i_k$ Also, in the vertex set of δ and $\gamma_{\lambda`}(i_k, i_n) = \gamma_{\lambda}(i_k, i_n)$ and $\varphi_{\lambda`}(i_k, i_n) = \varphi_{\lambda}(i_k, i_n) \forall\, i_k i_n$ edge in δ.

Example 3.6: G_1 fig 3.6.1 is an intuitionistic fuzzy graph δ_1 in fig 3.6.2 is a partial an intuitionistic fuzzy subgraph and δ_2 in fig 3.6.3 is an intuitionistic fuzzy subgraph G_1

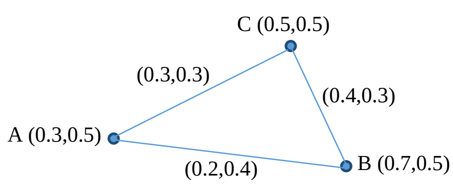

Fig. 3.6.1

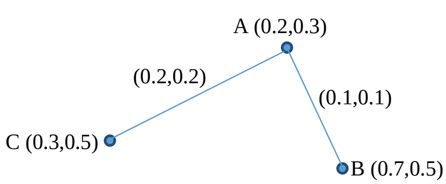

Fig. 3.6.2

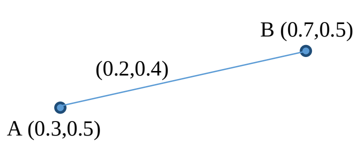

Fig. 3.6.3

Definition 3.7: an intuitionistic fuzzy graph G is called strong if $\mu_{\lambda 2kn} = \min\{\mu_{\lambda 1k}, \mu_{\lambda 1n}\}$ and $\varphi_{\lambda 2kn} = \max\{\varphi_{\lambda 1k}, \varphi_{\lambda 1n}\}$ for every edge $i_k, i_n \in E`$.

Definition 3.8: an intuitionistic fuzzy graph G is called complete if $\lambda_{2kn} = \min\{\lambda_{1k}, \lambda_{1n}\}$ and $\varphi_{2kn} = \max\{\varphi_{1k}, \varphi_{1n}\} \forall\, i_k, i_n \in V$.

Definition 3.9: A path P in an intuitionistic fuzzy graph G is a sequence of distinct vertices $i_1, i_2, \dots, i_n$ such that either one of the following axioms is hold

1) $\lambda_{2kn} > 0$ and $\varphi_{2kn} = 0$ for some k, n

2) $\lambda_{2kn} = 0$ and $\varphi_{2kn} > 0$ for some k, k

Example 3.10: an intuitionistic fuzzy graph in fig. 3.10.1 is not a path

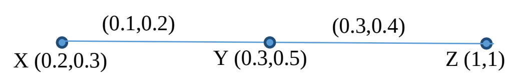

Fig. 3.10.1

Definition 3.11: a sequence of vertices $i_1, i_2, \dots, i_k$ not necessarily distinct is said to be walk or $k-$walk if at least one of $\lambda_{2kn}(k+1)$ is different from zero for $k = 1, \dots, n-1$

Corollary 3.12: an intuitionistic fuzzy graph is connected iff every pair of vertices is joined by a $k-$path.

Proof: Straightforward.

4 An Intuitionistic Fuzzy Cut Vertices And Bridges

Definition 4.1: If graph G is an intuitionistic fuzzy graph with intuitionistic function, γ1and γ2. A vertex $i \in V$ is called an intuitionistic fuzzy cut vertex if there exist two vertices $i, j \in V; i \neq j = x$, such that $G_{G-n}(i,j) < G_G(i,j) and\ L_{G-n}(i,j) > L_G(i,j)$ where $G(i,j)$ represent the gain of $i\ and\ j$ and $L(i,j)$ represent the loss of $i\ and\ j$.

Theorem 4.2: A vertex i in an intuitionistic fuzzy graph G is an intuitionistic fuzzy cut vertex if and only if i is a vertex in every $\max(k-m)$ gain path and i in is in every $\min(k-m)$ loss path for some k, m in V.

Proof: Suppose that G is an intuitionistic fuzzy graph with intuitionistic function, $\mu\_(1)$ & μ_2 and let X be an intuitionistic fuzzy cut vertex, then there exist some vertices $i, j\ in\ G$ such that $i \neq j \neq X$ & $G\_(G - X)\ (i,j) < G\_C\ (i,j)$ & $L\_(G-X)\ (i,j) > L\_G\ (i,j)$.

If we removed X from G removes all $\max(i-j)$ gain paths and if we removed X from G removes all $\min(i-j)$ loss paths, then X is in every $\max(i-j)$ and in every $\min(i-j)$.

Let X is in any $\max(i-j)$ and in any $\min(i-j)$.Thus, if we removed X from G then we remove all $\max(i-j)$ and all $\min(i-j)$.Therefore, the gain will decrease and the loss will increase between $i\ and\ j$, Hence $G-(G-X)\ (i,j) < G\_G\ (i,j)$ & $L\_(G-X)\ (i,j) > L\_G\ (i,j)$ and we get X is an intuitionistic fuzzy cut vertex.

Corollary 4.3: If G is an intuitionistic fuzzy graph. A vertex X is cut vertex of gain if and only if X is in every $\max(i-j)$ gain path, where $i \neq j \neq X$ and is a cut vertex of loss if and only if X is in every $\min(z-w)$ loss path, where $z \neq w \neq X$.

Proof: Straightforward.

Corollary 4.4: An edge $e \in E$ of an intuitionistic fuzzy graph G is intuitionistic fuzzy bridge if and only if it is in every $\max(i-j)$ gain path and every $\min(i-j)$ loss path.

Proof: Straightforward.

Theory 4.3: An edge $i\,j$ is an intuitionistic fuzzy bridge if and only if $G_{G-i\,j}\,(i,j) \leq \mu_\gamma\,(i\,j)$ and $L_{G-i\,j}(i,j) > \vartheta_{2\gamma}\,(i\,j)$.

Proof: Suppose that G is an intuitionistic fuzzy graph and $i\,j$ is an edge in G where $G_{G-i\,j}\,(i,j) \leq \mu_{2\gamma}\,(i\,j)$ and $L_{G-i\,j}\,(i,j) > \vartheta_{2\gamma}\,(i,j)$, Hence $\mu_{2\gamma}\,(i,j) < G\,(i\,,j)$ & $\vartheta_{2\gamma}\,(i,j) > L\,(i,j)$ then we get $G_{G-i\,j}\,(i,j) < G_G\,(i,j)$ & $L_{G-i\,j}\,(i,j) > L_G\,(i,j)$, if follow $i\,j$ is an intuitionistic fuzzy bridge.

Suppose that $i\,j$ is an intuitionistic fuzzy bridge, by corollary 4.4 there exist a vertex pair $v, w \in V$; $i\,j$ is present on every $\max(v-w)$ gain path and on every $\min(v-w)$ loss path. Now, let $G_{G-i\,j}\,(i,j) \geq \mu_{2\gamma}\,(i\,j)$, thus $G_{G-i\,j}\,(i,j) = G_G\,(i,j)$, then there is a $\max(i-j)$ gain path in G(say Q) such as different from $i\,j$. If R is a $\max(v-w)$ gain path in G, put $i\,j$ in R by Q to obtain an $v-w$ walk, since this walk contains an $v-w$ path, the path gain is greater than or equal to $G_G(u,v)$ which is not possible hence $G_{G-i\,j}\,(i,j) < \mu_{2\gamma}\,(i\,j)$

5 Conclusion

In this paper, connectivity intuitionistic fuzzy graph, intuitionistic fuzzy cut-vertices and intuitionistic fuzzy bridges and some properties of these concepts are discussed. Our future plan is to extend our research to some other properties of path, trees in intuitionistic fuzzy graphs. Also we will study this concept to generalize some important ideas on many papers such that (see [15, 16, 17,18,19]).

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