Experimental Validation of IMC-PID-FOF Controllers on the Water Level Tank System

TASSADIT CHEKARI
L2CSP laboratory
University of Boumerdes
ALGERIA
t.chekari@univ-boumerdes.dz

RACHID MANSOURI
L2CSP laboratory
Mouloud Mammeri university
ALGERIA
rachid_mansouri_ummto@yahoo.fr

MAAMAR BETTAYEB
University of Sharjah
United Arab Emirates
CEIES, King Abdulaziz University
Saudi Arabia
maamar@sharjah.ac.ae

Abstract: In this paper, IMC-PID-FOF controllers are implemented on real time water level control of a coupled tank system. The 1DOF-IMC-PID-FOF controller is designed based on the IMC structure, the disturbance rejection is not considered in the controller design and the disturbance response has low performance. In the 2DOF-IMC-PID-FOF controller design, the disturbance rejection is considered, and solved separately from the set-point tracking problem. To do this, a complementary sensitivity function is defined and its time constant τ_t is a tuning parameter, used to adjust the speed of the disturbance response. In the experiment, set-point tracking and disturbance rejection tests are carried out to evaluate the performance of both 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF controllers.

Key–Words: Two Degrees Of Freedom (2DOF) control, IMC control, Bode's ideal transfer function, complementary sensitivity function, water level tank system.

1 Introduction

The PID controller is widely used in many control engineering applications, it consists of more than 90% of the industrial applications owing to its simplicity in design and implementation and generally good performance [1, 2]. The Internal Modern Control (IMC) structure is very much reported in the literature because it presents interesting advantages such as robustness to modeling error and process uncertainties and the reduction of the number of tuning parameters in the controller design method [3].

On the other hand, Fractional Order Control (FOC) which involves the introduction of the fractional order differentiator in the control laws received a great attention by the research community due to the advantages presented by this approach such as flexibility in design with more degrees of freedom to tune [11]. The early contributions in this field are attributed to Oustaloup [9] where he introduced the notion of CRONE control (the French acronym which means non-integer order robust control) and Podlubny [10] where he proposed a generalization of the PID controller namely FO-PID involving fractional order integrator and differentiator. Then, many other papers were published such as [12, 13, 14] and the reference therein. In [4], the authors proposed to design a fractional order controller based on IMC structure, the principle combines the robustness and controller design easiness in the IMC structure with the simple implementation of PID controller. The fractional property of the controller is imposed by the Bode's ideal transfer function chosen as reference model. In order to improve the disturbance rejection, a Two Degree Of freedom (2DOF) structure is used in [6, 8] to solve the set-point tracking and the disturbance rejection problems, separately. The principle is based on the equivalence between the 2DOF-IMC and conventional 2DOF structures.

Recently, some contributions are focused on the implementation and validation of the fractional order controllers on real time processes especially the coupled tank process [5]. This process can be configured with several manners to control the water level in different tanks. For this, monovariable such as multivariable controllers are implemented [15, 16, 17].

In this paper, we propose to implement the fractional order control scheme proposed in [4], the controller design method yields to IMC-PID-FOF controller which consists of two parts: PID cascaded with a Fractional Order Filter (FOF) hence the appointment of the IMC-PID-FOF controller. In this controller design method, the disturbance rejection is not considered. In this paper, the obtained controller is noted 1DOF-IMC-PID-FOF controller, it gives good step response and low disturbance response. To make the disturbance response faster, a 2DOF-IMC-PID-FOF controller, proposed in [6], is implemented. It consists of a set-point tracking controller determined by defining a Bode's ideal transfer function chosen as reference model similar to the method reported in [4] and the disturbance rejection controller determined by defining a complementary sensitivity function whose time constant τ_t is a tuning parameter. It is chosen small enough to reject quickly the effect of the disturbance on the process step response. The rest of the paper is organized as follows: Section (2) presents the main steps of 1DOF-IMC-PID-FOF controller and 2DOF-IMC-PID-FOF controller. The description and modeling of the water level tank system is given in Section (3) where the experimental results are discussed in Subsection (3.2). The paper ends with a conclusion.

2 Fractional Order Controllers Design

The main steps of the design method are presented for both 1DOF-IMC-PID-FOF controller and 2DOF-IMC-PID-FOF controller as reported, respectively in [4] and [6].

2.1 1DOF-IMC-PID-FOF Controller

The controller design method proposed in [4] is based on the equivalence between the IMC and the unity feedback structures. In the IMC structure of Figure (1(a)), $g(s)$ represents the process to be controlled, $g_m(s)$ is the process model and $c_{imc}(s)$ is the IMC controller. $c(s)$ in Figure (1(b)) represents the controller to be determined.

The main steps of the 1DOF-IMC-PID-FOF controller design are [3, 4]:

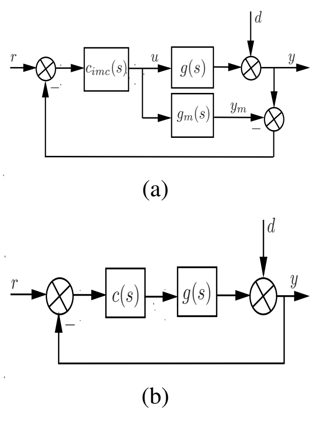


Figure 1: IMC and conventional feedback control structures

- Step 1: $g_m(s)$ is factorized as:

$$g_m(s) = g_m^-(s) g_m^+(s) \tag{1}$$

$g_m^-(s)$ is the invertible part and $g_m^+(s)$ contains time delays and right half plane zeros with $g_m^+(0) = 1$

- Step 2: IMC controller is defined as:

$$c_{imc}(s) = \frac{1}{g_m^-(s)} f(s) \tag{2}$$

The reference model $f(s)$ is the closed-loop Bode's ideal transfer function given by [4]:

$$f(s) = \frac{1}{1 + \tau_c s^{\alpha+1}}, \qquad 0 < \alpha < 1 \tag{3}$$

τ_c and α are respectively the time constant and fractional order.
The Bode's ideal transfer function exhibits interesting properties such as constant phase margin and robustness to process gain variations [12].

- Step 3: The equivalence between the two structures of Figure (1) gives:

$$c(s) = \frac{c_{imc}(s)}{1 - g_m(s) c_{imc}(s)} \tag{4}$$

It is shown in [4] that $c(s)$ can be put in the form of a PID controller cascaded with a fractional order filter.

In the 1DOF-IMC-PID-FOF controller design method, the disturbance rejection problem is not taken into account and the obtained disturbance response has low performance. To overcome this problem, the 2DOF structure is used to design 2DOF-IMC-PID-FOF controller where the set-point tracking and the disturbance rejection problems are solved separately.

2.2 2DOF-IMC-PID-FO-Controller Design

A fractional order controller design method is proposed in [6], the principle is based on the 2DOF structure to separate the set-point tracking problem from the disturbance rejection one. The 2DOF-IMC structure used and the conventional 2DOF control scheme are shown, respectively, in Figures (2(a)) and (2(b)).
In the 2DOF-IMC structure (Figure (2(a))), $g(s)$ represents the process to be controlled, $g_d(s)$ is the disturbance transfer function, $g^+(s)$ is the non invertible part of $g(s)$, $f(s)$ is the reference model given by

Equation (3). $c_f(s)$ is the disturbance rejection controller and $c_s(s)$ represents the set-point tracking controller. In the conventional 2DOF control scheme, the corresponding disturbance rejection controller is $c_1(s)$ and the corresponding set-point tracking controller is $c_2(s)$.

The 2DOF-IMC-PID controller design procedure re-

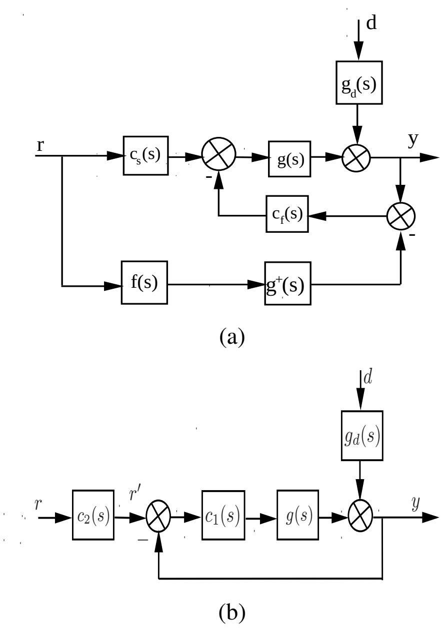


Figure 2: 2DOF-IMC and conventional 2DOF control schemes

quires the following steps:

- Step 1:
  The set-point tracking controller $c_s(s)$ is calculated according to Equations (1) to (3).
- Step 2:
  The disturbance rejection controller $c_f(s)$ is defined as:
  $$c_f(s) = \frac{1}{g(s)} \frac{t(s)}{1 - t(s)} \tag{5}$$
  where $t(s)$ is the complementary sensitivity function defined as:
  $$t(s) = \frac{g^+(s)}{1 + \tau_t s} \tag{6}$$
  The time constant τ_t is a tuning parameter ($\tau_t < \tau_c$), it is chosen small enough to reject quickly the effect of the disturbance.
  Although the choice of the fractional order complementary sensitivity function gives more degrees of freedom to tune, it is shown in [6, 7] that the integer order complementary sensitivity function of Equation (6) gives better disturbance response
- Step 3:
  The equivalence between the two structures of Figure (2) yields:
  $$c_1(s) = c_f(s), \quad c_2(s) = \frac{c_s(s)}{c_f(s)} + g^+(s)f(s) \tag{7}$$
  $f(s)$ is defined by Equation (3)

3 Application

Both 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF controllers are implemented on real time coupled tank setup [5] to control the water level on a single tank.

3.1 Process Description and Modeling

The experimental setup shown in Figure (3), consists of four primary tanks and a reservoir tank. It is linked to a computer with a connector box [5, 18]. Out of the four tanks, only the first tank is used in the present work and is represented in Figure (4).

Taking the mass balance and using the Bernoulli's

Figure 3: Coupled tank process linked to the computer

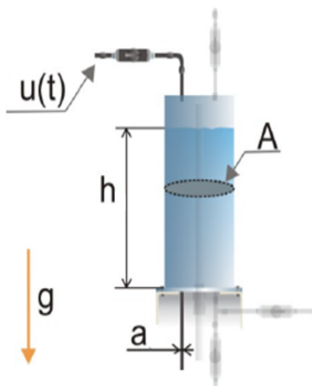


Figure 4: Single tank phenomenological model

law, the simplest nonlinear model of the single tank relating the water level h with the voltage u applied to the pump is:

$$A\frac{dh}{dt} = \mu u(t) - a\sqrt{2gh(t)} \quad (8)$$

h represents the water level in the tank.
g is the gravitation.
The process parameters values and their definitions are given in Table (1).
For real implementation of the two controllers 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF, the nonlinear model (8) is first linearized around a working point (u_0, h_0) and the linearized model obtained is:

Table 1: Parameters of the coupled tank process

Symbol and Value	Description
$\mu = 2.2 \times 10^{-3}\, m/v.s$	Constant relating the control voltage with the water flow from the pump
$a = 50.265 \times 10^{-6}\, m^2$	Tank outlet area
$A = 0.01389\, m^2$	Cross sectional area of the tanks

$$\frac{d}{dt}(h - h_0) = \frac{\mu}{A}(u - u_0) - \frac{ag}{A\sqrt{2gh_0}}(h - h_0) \quad (9)$$

Equation (9) is rewritten as:

$$\frac{d}{dt}(\Delta h(t)) = \frac{\mu}{A}\Delta u(t) - \frac{ag}{A\sqrt{2gh_0}}\Delta h(t) \quad (10)$$

Taking Laplace Transform of Equation (9), the transfer function of the single tank is:

$$\frac{\Delta h(s)}{\Delta u(s)} = \frac{\mu}{s + \frac{ag}{A\sqrt{2gh_0}}} \quad (11)$$

In our study, the chosen working point is: $u_0 = 2.77$ et $h_0 = 7.55$ and the numerical expression of the model is:

$$\frac{\Delta h(s)}{\Delta u(s)} = \frac{10.44}{47.44s + 1} \quad (12)$$

Model (12) is used to design both 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF controllers.

3.2 Fractional Order Controllers Implementation

Based on the numerical model of Equation (12), using Equations (1) to (4) and for the values $\tau_c = 10$ and $\alpha = 0.1$ chosen to reduce the control effort and the overshoot of the water level step response, the 1DOF-IMC-PID-FOF controller determined for controlling the water level in the tank is:

$$c(s) = \frac{1}{s^{1.1}} 0.4544 \left(1 + \frac{1}{47.44s}\right) \quad (13)$$

In the 2DOF control scheme and according to the steps described in the subsection (2.2), two controllers are determined to handle the set-point tracking and the disturbance rejection problems, separately. For the same values of the parameters τ_c and α and for $\tau_t = 8$, the controllers obtained for set-point tracking and disturbance rejection are, respectively:

$$c_2(s) = \frac{1 + 8s}{1 + 10s^{1.1}} \quad (14)$$

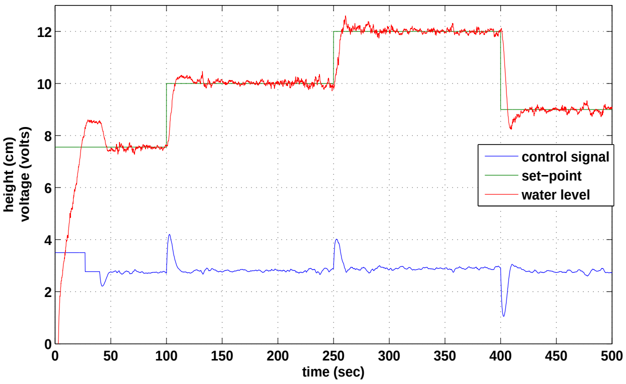
$$c_1(s) = 0.5682 \left(1 + \frac{1}{47.44s}\right) \quad (15)$$

Before both 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF controllers start, the tank water level has to be brought to the chosen working point in which the two controllers are determined ($u_0 = 2.77$ et $h_0 = 7.55$).
The water level is measured in centimeters (cm) and the voltage (control signal) in volts (V).
The fractional integrals are implemented in Software Matlab using the Oustaloup continuous approximation method in the frequency range $[10^{-6}, 10^{0}]$ rad/sec using 10 cells.
In the current work, the sampling time for carrying out the experiment is 0.1
In the experimental manipulations, the set-point tracking test consists in tracking the following step changes:

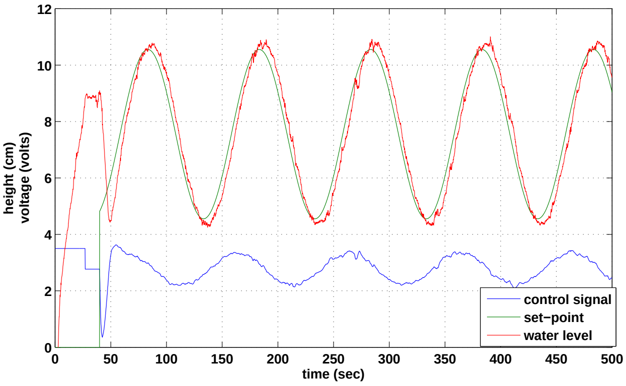
- From the working point $h_0 = 7.55$ cm, the water level h reaches 10 cm at 100 s, then rises 2 cm. Finally, it falls 3 cm at 400 s.

- The water level h should track the desired sinusoïdal signal, the amplitude is 3 cm and the period is 100 s, around the working point $h_0 = 7.55$ cm.

The experimental results obtained, when the 1DOF-IMC-PID-FOF controller and 2DOF-IMC-PID-FOF controller are implemented, are shown respectively, in Figures (5) and (6). From Figure (5(a)), it is observed that the 1DOF-IMC-PID-FOF controller ensures set-point tracking. As an example, from the working point 7.55 cm, the water level h is brought to different level values. When a desired sinusoïdal signal is imposed, the controller ensures the tracking with the presence

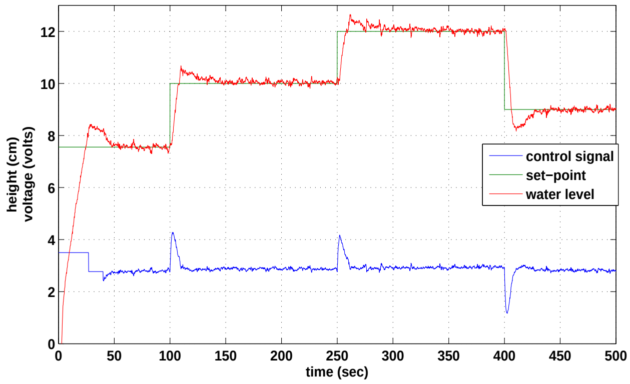


(a)

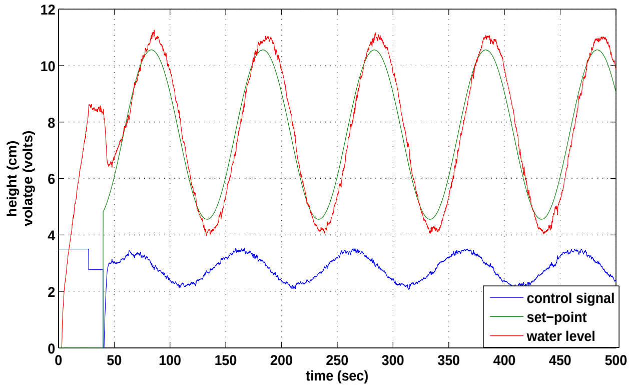


(b)

Figure 5: Water level and control signal evolution when 1DOF-IMC-PID-FOF controller is implemented



(a)



(b)

Figure 6: Water level and control signal evolution when 2DOF-IMC-PID-FOF controller is implemented

of error between the desired variable reference and the water level evolution as shown in Figure (5(b)).

Figure (6(a)) shows that the 2DOF-IMC-PID-FOF controller tracks appropriately different set-point reference variations. As an example, at 250 s, the water level rises to reach 12 cm. In Figure (6(b)) and for the same sinusoïdal reference, 2DOF-IMC-PID-FOF controller ensures also the tracking of the variable reference with a slight larger error compared to that obtained when the 1DOF-IMC-PID-FOF controller is implemented.

Figure (7) shows the water level evolution and the control effort provided by both the 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF controllers when the water level in the tank is brought to $h_0 = 10$ cm. This working point is different from that in which the two controllers are determined. This is, for the same step changes as the set-point tracking test.

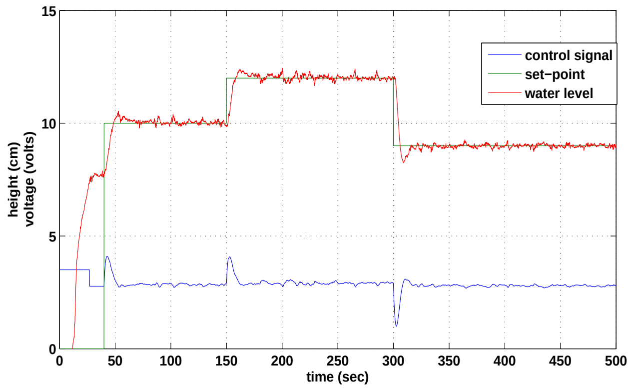
From the results of Figure (7), it is observed that both controllers ensure appropriately the set-point tracking. To evaluate the robustness of the two fractional order controllers with respect to the disturbances, the following test of the disturbance rejection is carried out:

- The water level reaches the working point $h_0 = 7.55$ cm and rises to reach 11 cm at 100 s. Then, 50 cl of extra water is added in the tank at time 150 s.

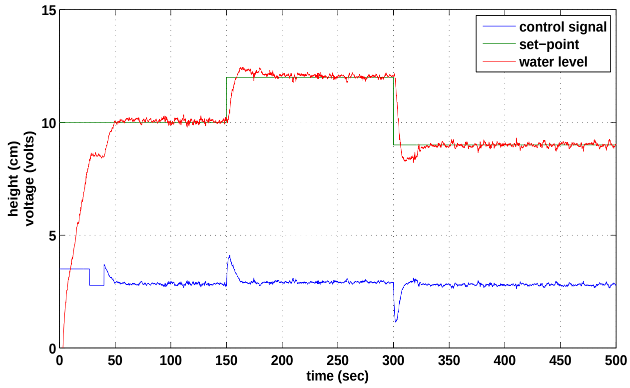
- At time 240 s, the additional outflow valve (MV4) is half opened, then it is completely opened at 300 s.

The water level evolution and the control effort provided by both 1DOF-IMC-PID-FOF and 2DOF-IMC-PID-FOF controllers are recorded and shown in Figure (8).

From Figure (8), it is observed that both controllers compensate the effect of the extra water added on the water level evolution. However, this effect is reduced (quickly rejected) when the 2DOF-IMC-PID-FOF controller is implemented as shown in blue curve at time 150 s. The control effort provided by the



(a)



(b)

Figure 7: Water level and control signal evolutions when h is brought to another working point ($h_0 = 10cm$)

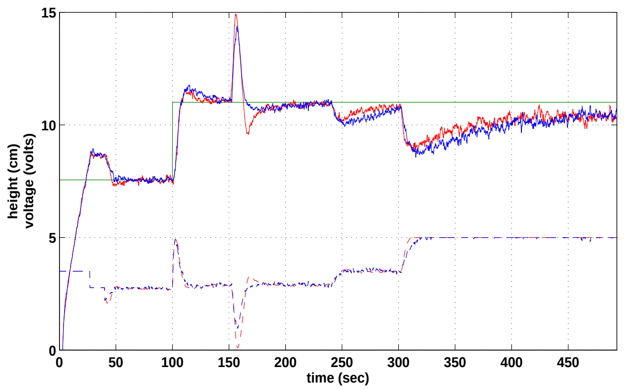


Figure 8: Water level h and control signal u evolutions when IMC-PID-FOF and 2DOF-IMC-PID-FOF are implemented: 1DOF-IMC-PID-FOF (–), 2DOF-IMC-PID-FOF, (–), desired reference (–)

2DOF-IMC-PID-FOF controller at this time is reduced compared to that provided by the 1DOF-IMC-PID-FOF controller.

It is shown from the simulation results in [6, 7, 8] that the process disturbance response can be improved when the 2DOF structure is used by reducing the tuning parameter τ_t (time constant of the complementary sensitivity function). This can be confirmed by the recorded experimental results for different values of the parameter τ_t shown in Figure (9).

From Figure (9), it is observed that the disturbance

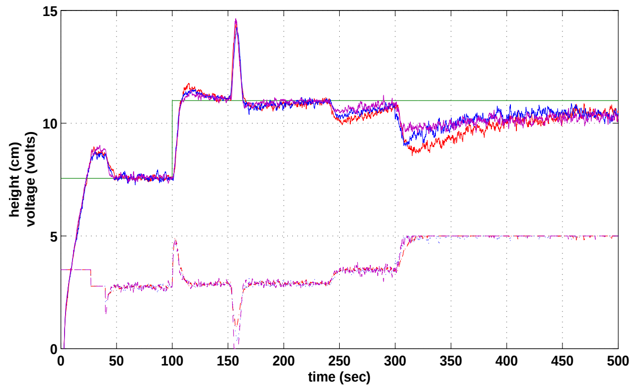


Figure 9: Water level h and control signal u evolutions when 2DOF-IMC-PID-FOF controller is implemented for different values of the parameter τ_t: reference (–), $\tau_t = 8$ (–), $\tau_t = 6$ (–), $\tau_t = 4$ (–)

response of the water level becomes faster when τ_t is smaller as can be seen when the outflow valve is half and completely opened at times, respectively, 240 s and 300 s. This can be clearly seen at the zoomed Figure (10).

In our study, the value $\tau_t = 6$ is chosen sufficient to

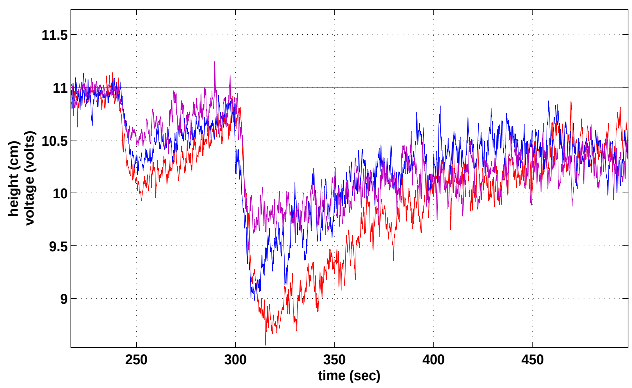


Figure 10: Water level h and control signal u evolutions when 2DOF-IMC-PID-FOF controller is implemented for different values of the parameter τ_t: reference (–), $\tau_t = 8$ (–), $\tau_t = 6$ (–), $\tau_t = 4$ (–)

satisfy the trade-off between disturbance rejection and noise attenuation [6, 8, 7].

4 Conclusion

In this work, fractional order controllers are implemented on a real time coupled tank system to control the water level in single tank. The 1DOF-IMC-PID-FOF controller is designed based on the equivalence between the IMC and the unity feedback structures and choosing the Bode's ideal transfer function as reference model. On the other hand, the 2DOF-IMC-PID-FOF controller is determined on the basis of a 2DOF structure. Therefore, the set-point tracking problem and the disturbance rejection one are considered, separately. With this control scheme, the disturbance rejection can be improved by adjusting the tuning parameter τ_t which is a time constant of the complementary sensitivity function.

The experimental results obtained have shown that both 1DOF-IMC-PID-FOF controller and 2DOF-IMC-PID-FOF controller ensure the set-point tracking for different step variations and sinusoïdal reference. They also ensure the disturbance rejection. However, the disturbance response of the water level control becomes faster when the 2DOF-IMC-PID-FOF controller is implemented. This is due to the use of the tuning parameter τ_t to adjust the the speed of the disturbance response. As future works, the coupled tank can be configured as a multivariable process and the water level is controlled in more than one tank. For this, fractional order multi-loop control schemes can be implemented.

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