Application of Gradation Trajectories to a Yule–Nielsen Spectral Neugebauer Model

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Abstract: - The classic Yule–Nielsen spectral Neugebauer model (YNSN), as well as, its further modification the cellular Yule-Nielsen modified spectral Neugebauer model (CYNSN) are widely used to predict the spectral response of different printers especially when printing multispectral images. The models are engaged to define printer control values that produce appropriate spectral reflectances on a given medium. The Yule-Nielsen factor in the models empirically modeling the optical dot gain is obtained by different regression-based approaches and has no general definition algorithm. This is a kind of disadvantage. In the work, we compare the results of the modified CYNSN and the YNSN with approach of gradation trajectories (YNSNGT). The gradation trajectories are such a way of characterizing the printer, in which each tone value in the original layout is assigned a certain effective tone value so that the linear increment of the tone in the layout corresponds to a linear color increment on the print in the sense of dE_{00}. We have demonstrated that our YNSNGT approach provides comparable prediction accuracy with incomparably faster calculations.

Key-Words: - Color appearance, Gradation trajectories, Gradation surfaces, Printing art

1 Introduction

Since colorimetric color reproduction does not provide that breadth of possibilities and has less flexibility in terms of color management compared to spectral one, the latter is increasingly used to record full spectral color information about the source color of an object avoiding metamerism. This is of a particular importance for the print industry where the spectral printer characterization is the key topic for spectral printer modeling. By the approach, the relationship between the input device control values and the output reflectance spectrum is defined.

There are many different color prediction models in print. The empirical surface models take into account only superposition of ink halftones. The physically inspired models deal with a more detailed analysis of light-print interaction. The ink spreading models characterize the effective surface of a printed ink dot at a given nominal surface coverage compared to the effective surface coverage forming the physical dot gain [1].

The most recent spectral reflection models describe the spread-based light propagation probability and study the impact of different print factors influencing the range of printable colors. These models are able to predict the reflectance spectra as a function of ink surface coverage for 2–4 inks. Among them, the Yule–Nielsen spectral Neugebauer model (YNSN) is most commonly used [2–5]. The model is based on measurement of the Neugebauer primaries (the possible combinations of CMY/CMYK inks weighted by the area they take up on the substrate) and definition of relationship between them.

Due to its prevalence, this model has a very large number of variations, which have been growing from year to year. Among the existing modifications, the cellular Yule–Nielsen spectral Neugebauer (CYNSN) model is probably the most remarkable due to its accuracy and obviousness [6]. In the model, the reflection spectrum of the print color is predicted from the weighted sum of each primary colorant's one, and effects such as ink dot gain based on mechanical ink scattering and optical dot gain based on light scattering are modeled. At the same time, empirically selected dot gain curves and coefficient n values, respectively, are applied in the model.

The basis of this model is the division of the device color space into cellular regions to obtain a larger number of sub-models and primary colors, which significantly increases the accuracy of the spectrum prediction [2]. However, such a structure of

the model, especially with a large number of nodes, leads to the fact that its configuration is significantly time consuming and demanding to efforts to apply.

The models have been successfully applied to print color reproduction management [7–10]. Their major drawback is their demanding for computational resources, as m colorants require a solution of system of m^2 equations; therefore, they might be barely implicated into a real workflow.

The empirical approaches might be considered more promising. In previous works, the gradation trajectories as a three-dimensional treatment of tone reproduction curves (TRC) in CIE *Lab* space were introduced [11–13]. The initial information for plotting the gradation trajectories is not the optical density but the CIE *Lab* coordinates of a test patch. Actually, each colorant gradation scale might be corresponded to a certain gradation trajectory. We have confirmed the gradation trajectories' invariance for a given ink-substrate pare as a consequence of Grassmann's law. The use of gradation trajectories allows carrying the characterization at once in three different quantities: lightness L, color difference dE and color coordinates. A gradation trajectory is depicted in Fig.1.

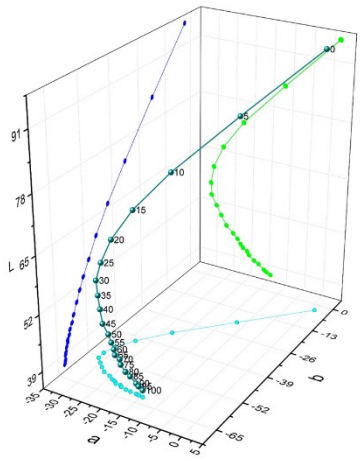

Fig. 1. A gradation trajectory with projections.

In this work, we are trying to apply the gradation trajectories approach to the classic Yule–Nielsen spectral Neugebauer model and compare the prediction results with the resent modified cellular Yule–Nielsen spectral Neugebauer model.

2 Models description

First, we will briefly introduce all the models applied.

2.1 Color difference as a metric

The CIE *Lab* color space expresses color as three numerical values: L for lightness, a for green-red, and b for blue-yellow color components. Thus, any color is uniquely described by a three-component vector in 3D *Lab* space whose metric is the color difference dE (or ΔE), which is defined at least by the root squared sum of color components differences. The presence of a metric in the sense of the method of measuring distances allows one to consider the color space a metric one. Up-to-date, there are thee common color difference equations exist: dE_{76}, dE_{94}, and dE_{00} [14–17] characterized by varying degrees of accuracy of accounting for various non-linearities of the real color space and its perception by man.

2.2 The classic Yule-Nielson Spectral Neugebauer model

For a common CMYK printing, the classic Yule-Nielson Spectral Neugebauer model can be described by the equation (1), where $R(\lambda)$ is spectral reflectance of given color at wavelength λ; $R_i(\lambda)$ is spectral reflectance of i^{th} primary color at wavelength R; w_i depends on the CMYK ink amount values of printed sample which can be described by Demichel equation [18]; n is the coefficient of the power function, known as the Yule-Nielson n-factor. The optimal n-factor is usually determined by the RRMSE (2) or any other error evaluation between predicted spectral reflectance and measured spectral reflectance of testing sample set.

$$R(\lambda) = \left(\sum_{i=1}^{16} w_i \times R_i(\lambda)^{1/n}\right)^n \qquad (1)$$

$$RRMSE = \sqrt{\frac{(R_1-R_2)\times(R_1-R_2)^T}{N}}, \qquad (2)$$

where R_1 and R_2 are the predicted and measured spectral reflectance (in vector row form) respectively; T is the operator of matrix transpose; N is the spectral reflectance dimensionality.

2.3 The cellular Yule-Nielson Spectral Neugebauer model

To further improve the spectral accuracy of the YNSN model more measurements than the Neugebauer primaries can be considered. For this reason, the control value space can be subdivided into base-size cells and the YNSN model can be applied in each particular cell utilizing the cell vertex as primaries in place of the Neugebauer primaries [19].

The cells are searched by various methods. Having determined the optimal cell size, a pigment recipe is derived using the inverse YNSN equation. The corresponding spectral reflectivity is predicted using a direct YNSN model. This algorithm is repeated for all base-size cells before the optimal cell

is found. That is why the volume of calculation of the traditional CYNSN model is so huge. As a rule, the smaller the cell size, the higher the accuracy of the model prediction, however, this rule is not true for all cases. One way or another, the cellular model requires a substantial number of calculations.

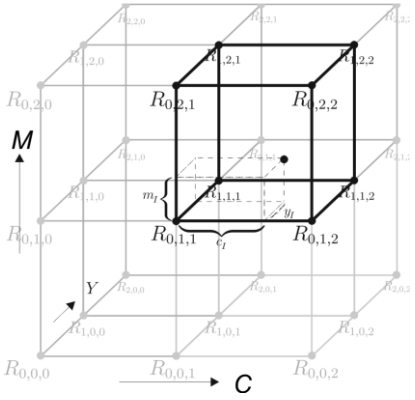

Fig. 2. CYNSN model for a CMY printer, cited by [19].

2.4 Gradation trajectories

The gradation trajectory is a locus of the points in the CIE *Lab* space, which coordinates correspond to the CIE *Lab* coordinates of the individual patches of the initial colorants arranged in ascending order of percentage of raster cells coverage in the layout from 0% (blank substrate) to 100% (full dye). TRC generally comprise no more than two dozen of fields, *i.e.* the gradation trajectory is, also, represented by a discrete set of points in CIE *Lab* space.

Color characteristics (hue, saturation and brightness), as well as, the color coordinates, might be assumed as a continuous function of the percentage of raster cell filling. In other words, we can expect a nearly continuous change of color characteristics when filling percentage of a raster cell changes continuously.

We assume that the tone part in the layout is denoted as a t parameter in the range from 0 (paper) to 1 (full dye). Therefore, the trajectories might be analytically described by polynomials of nth degree in the parametric equation (3) since color changes continuously in accordance with continuous increment of t.

$$\begin{gathered} t \in [0;1] \\ a = a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \\ b = b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \\ L = (L_0 - L_\infty) \times \exp(-L_3t^3 - L_2t^2 - L_1t) + L_\infty \end{gathered} \tag{3}$$

where L_i, a_i, b_i, $i = (1..4)$ are some numerical coefficients, (L_0, a_0, b_0,) are CIE *Lab* coordinates of the unprinted substrate, L_∞ is visual brightness of a hypothetical continuous ink layer of infinite thickness.

The major idea of the gradation trajectories approach implementation is the following. Gradation trajectory's arc length is defined as an integral in the dE units. The arc is split into n segments of equal length in order to match them with the same tone increment. The proposed method of gradation trajectories, on the contrary to common color prediction models, is quite simple in description, does not require cumbersome computations, and, thus, can be successfully applied in practice.

The formulas (3) for a and b coordinates, in fact, is the expansion of the coordinate functions in a Taylor series near the point corresponding to the substrate to be printed. The L coordinate is a decreasing function on a segment bounded from above (paper is the brightest object) and from below ("ink in a tin" is the darkest object).

Since we assume the functions (3) to be continuous on the segment, then, according to the Weierstrass approximation theorem [20], their analytical form could be given by a polynomial of a certain extent. In words, any continuous function on a closed and bounded interval can be uniformly approximated on that interval by polynomials to any degree of accuracy. In our case, the polynomial of the fourth degree is applied.

The analytical description of the gradation trajectory allows the possibility of calculation of such space curve features as the curvature (k) and torsion (τ) [21, p.49]. In our case, the curvature is described by the equation (4).

$$\begin{cases} r = (a(t), b(t), L(t)) \\ k = \frac{|r' \times r''|}{|r'|^3} \end{cases} \tag{4}$$

Curvature of a gradation trajectory may act as a powerful tool regulating, as for instance, the amount of inks applied. The maximum of curvature corresponds to the region of the sharpest bending of the gradation trajectory, *i.e.* the sharpest change in color tone for a color channel and the sharpest drop in brightness for the other. These are the points that are chosen to limit the maximum ink supply for the specified colorants. This is especially important for inkjet printing systems, which are developing extremely fast up to date.

Application of gradation trajectories to the YNSN model (YNSNGT) might be done as following.

3 Building a YNSNGT model

As already noted, the main advantage of the cellular model is the high accuracy of spectral data prediction. Such, it is known that the average prediction error of a monochromatic (for a given

wavelength) reflection coefficient is about half a percent (more than $5*10^{-3}$). The disadvantages of this model include the following.

The selection of the *n*-factor is carried out empirically, *i.e.* often, this is a simple enumeration of the values in a certain range. Further, we are faced with the fundamental impossibility of predicting if the recipe of at least one of the components of the CMYK coincides with the recipe of the nodal (vertex) point of the tuning sample, since in this case the sample cannot be unambiguously assigned to any cell of the model.

Further, the number of samples for building a model is growing to an extent equal to the number of base colorants used. Such, CYNSN with a 20% tone increment (5 intervals, 6 nodal points for each primary color) for the CMY printer requires $6^3 = 216$ tuning patches, whilst CMYK printing requires, already, $6^4 = 1296$ patches. For more complex color combinations, either the model is split into combined sub-colors/sub-models, or the number of nodal points is reduced. Such, for the CMYKOG printer with the same number of nodal points, for each color, the number of configuration patches should already be $6^6 = 46656$. This approximately corresponds to 2 m^2 of printed scales for measuring with an automatic spectrophotometer (iOne iSis or similar). The total number of tuning patches *N* can be estimated by the equation (5), in which g is the number of grid nodes and recipes per colorant, c is the number of primary colorants.

$$N = g^c \tag{5}$$

This work serves to overcome these shortcomings. In this paper, we attempt to use the proposed concept of gradation trajectories in the basic YNSN model. Thus, we fundamentally refuse the cellular character of the model in the hope of describing, using gradation trajectories, in a fairly accurate way, the change in tone over the entire range from 0 to full dye.

In this paper, gradation trajectories are such a way of characterizing the printer, in which each tone value in the original layout is assigned a certain effective tone value so that the linear increment of the tone in the layout corresponds to a linear color increment on the print in the sense of dE_{00}.

Color prediction models are usually used in relation to inkjet printing, however, in this work we use an electrophotographic printer as a model printing device, since our main goal is to illustrate the fundamental possibility of using gradation trajectories in color prediction issues.

When calculating the Neugebauer primaries, the initial data are not the actual value of the formulation, but some "effective" values, which are taken from the one-dimensional LUT table formed by the gradation trajectory, and can be described by the formal dependence (6) where t_{fact} is factually printed tone fraction (in color quanta, which are the tone fraction expressed in bits, such that 0 represents a blank substrate and 255 is equal to full dye), f is a gradation trajectory operator, t_{eff} is the effective tone fraction, based on which the primary elements of the Neugebauer mosaic are calculated.

$$t_{eff} = f(t_{fact}) \tag{6}$$

4 Experiment

Fort the experiment, we use the following equipment and materials: 4-colorants (CMYK) Konica Minolta Bizhub Pro C6000L is used as a printing system, the printing substrate is the Moorim Neo Star Matt paper 140 g/m^2, the test gradation scale images are synthesized using ChartGenerator in the ProfileMaker's MeasureTool, the measuring is made by the iOne iSis automatic spectrophotometer.

First, we build a CYNSN model. The test chart consists of $6^4 = 1296$ patches that corresponds to 6 nodal points for each primary color (see Fig.2). The nodal patches recipes are selected form the following sequence: {0.0, 0.2, 0.4, 0.6, 0.8, 1.0} parts of tone for each CMYK colorant. Together with the test chart, the tuning one is added and printed. It consists of inter-nodal values of sequence {0.1, 0.3, 0.5, 0.7, 0.9} parts of tone for each CMYK colorant.

The Yule-Nielsen *n*-factor is selected form the sequential search of substitution into the equation (1) from the initial range [0.5–5] with step of 1/20 of the range width. During the computation, we define the affiliation of each patch from the tuning set to a particular cell from the test set. The recipes and spectra of both the nodal and inter-nodal points are the source of data for the equation (1).

For each spectrum from the tuning chart, the prediction error (2) is computed. Based on the obtained values, the dependence RRMSE.mean=f(*n*) is build and analyzed. RRMSE.mean is the mean of all RRMSE values of the tuning set (errors between predicted and factual spectra). At the next iteration, the range of *n* is shrank near the minimum of RRMSE.mean=f(*n*). After a couple of iterations, the dependence takes the form depicted in Fig.3 where *n*-factor might be defined with 0.05 accuracy. The computation time is about 200 sec per iteration.

The quality of prediction is also evaluated by building the RRMSE distribution histogram and

dependences RRMSE *vs* sum of tone, which indicates whether lights of shadows are predicted better. We also build the dependence dE_{00} *vs* sum of tone with the same purpose.

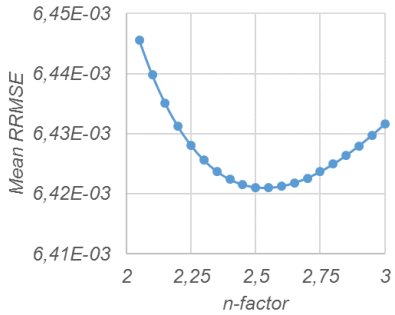

Fig. 3. CYNSN model: n-factor definition.

Second, we build a YNSNGT model. The initial information is the spectra of Neugebauer primaries and gradation trajectories of the CMYK colorants. To select the Yule-Nielson factor (n), the same set of tuning patches as in the previous part of the experiment is used. At the first stage, the recipes of the tuning patches are transformed by means of gradation trajectories (Fig. 4), *i.e.* each actual fraction of the CMYK tone is associated with an "effective" fraction.

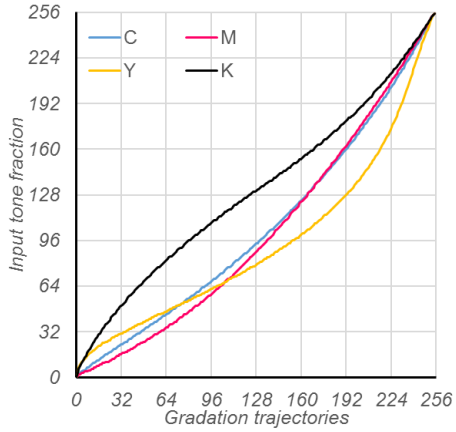

Fig. 4. YNSNGT model: gradation tracectories.

The action of gradation trajectories is more conveniently explained by the following example. Suppose a patch has 0.7C in its actual recipe. The gradation trajectory shows that a value of 0.7 corresponds to C_{eff} = 0.568. This means that feeding 0.7C to the input of the printing system, we actually obtain 0.568 part of the maximum possible Cyan in terms of the color difference dE_{00} (7).

$$\frac{dE_{00}\,(paper;\ 0.7C)}{dE_{00}\,(paper;\ 1.0C)} = 0.568 \qquad (7)$$

Thus, the elements of the Neugebauer mosaic for each patch in the tuning set are calculated using the Demichel equations not from actual, but from effective patch recipes. The search for the Yule-Nielson factor is carried out according to the same algorithms as in the previous stage of the experiment.

5 Results and discussion

We noted that definition of n-factor in the CYNSN model is depicted in Fig.3. As it may be seen, the lowest mean RRMSE ($6.42*10^{-3}$) corresponds to n-factor of 2.55. On contrary, the YNSNGT model shows an unexpected n-factor value less than 1 (to be precise, 0.36 corresponded to RRMSE of 0.024), see Fig. 5. If we take the inverse value of n-factor, we obtain the picture similar to the cellular model (compare Fig. 6 and Fig. 3).

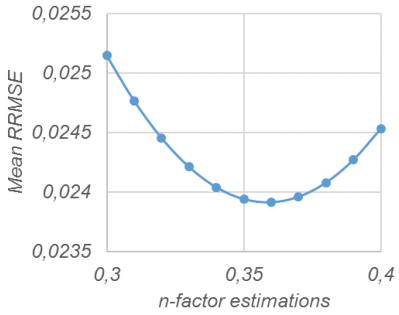

Fig. 5. YNSNGT model: n-factor estimations.

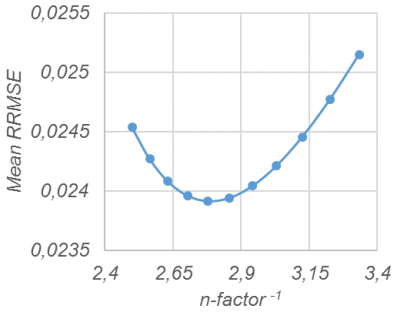

Fig. 6. YNSNGT model: inverse n-factor behaviour.

There is no doubt that the CYNSN model currently has the accuracy of the prediction of spectral information comparable to the accuracy of printing and measurements. For this reason, we consider this model as a benchmark, the achievement of which allows us to evaluate the accuracy of the newly proposed models.

Based on the analysis of the prediction error of both spectral (Fig. 7 a, c and Fig. 8 a, c) and color (Fig. 7 b, d and Fig. 8 b, d) information, it should be noted that YNSNGT in this form is inferior in accuracy to the cellular model. In particular, the mean spectral prediction error is 4 times greater, and the color prediction error is about 5 times greater.

Despite the higher prediction error, in the model based on gradation trajectories, the cloud of errors lies more closely. In addition, the use of the YNSNGT model reduces the calculation time by about 20 times (maximum 10 seconds when using the same computing power). Thus, this approach can be applied in real workflows.

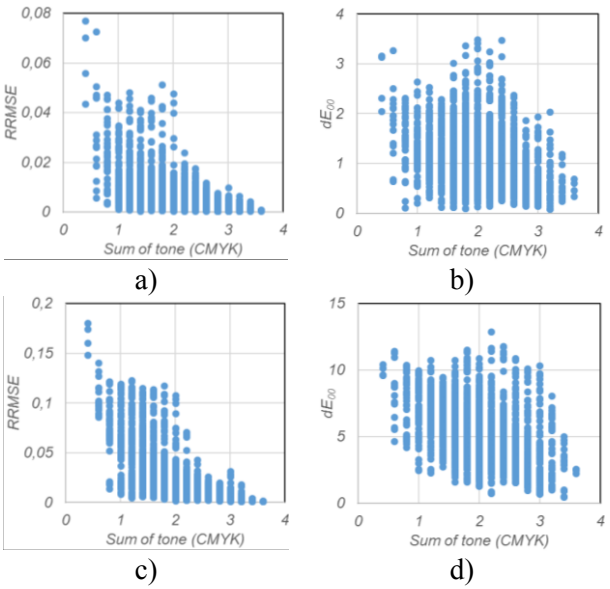

Fig. 7. Models errors distributions *vs* sum of tone: a) CYNSN RRMSE, b) CYNSN dE_{00}, c) YNSNGT RRMSE, d) YNSNGT dE_{00}

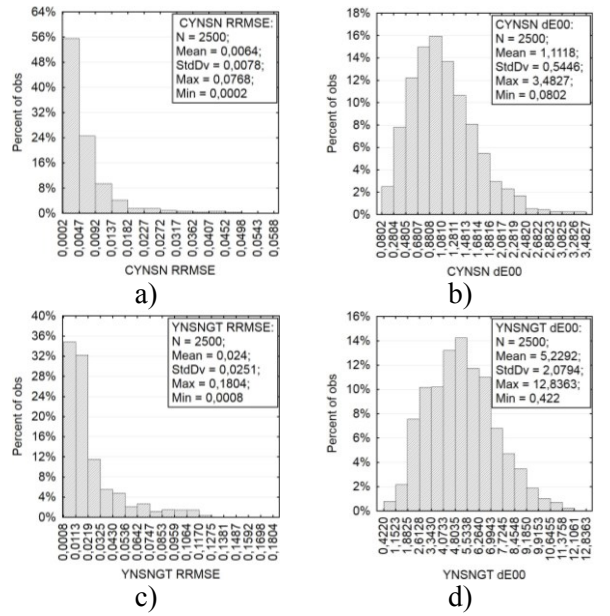

Fig. 8. Models errors distributions: a) CYNSN RRMSE, b) CYNSN dE_{00}, c) YNSNGT RRMSE, d) YNSNGT dE_{00}

6 Conclusion

In this work, we apply the gradation trajectories approach to the classic Yule–Nielsen spectral Neugebauer model and compare the prediction results with the widely used cellular Yule–Nielsen spectral Neugebauer model. As a model printing device, we use a common 4-colors (CMYK) electrophotographic printer. Certainly, the results can be easily extrapolated to inkjet printers.

The gradation trajectories are three-dimensional treatment of TRC in the CIE *Lab* space, they are continuous curves bounded on a given interval and are a particular way of characterizing a printer. Their basic feature is that each tone value in the initial layout is assigned a certain ("effective") tone value so that the linear increment of the tone in the layout corresponds to a linear color increment (in the sense of dE_{00}) on the print. The gradation trajectories are not affected by the method of rasterization and measurement techniques, *i.e.* they are color space invariants.

Despite the fact that accuracy of the CYNSN model is still higher than one of YNSNGT we may assume they both satisfactory for application in computations. However, the gradation trajectory approach is rather simpler and far faster than the cellular method.

The gradation trajectories approach is very flexible. In this work, for the formation of gradation trajectories, we used the value of color difference dE_{00}. However, optical density, saturation, etc., can be, also, utilized in absolutely the same way.

Further studies will be devoted to the search for a criterion on the basis of which it is possible to construct a gradation trajectory optimal for a given model.

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