Procedures of Power Distribution in Propulsion System of Torpedo-shaped Underwater Vehicle

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Abstract: - The paper concentrates on a problem of distributing control effort among multiple, redundant actuators in a propulsion system of a small torpedo-shaped unmanned underwater vehicle. The vehicle has no other actuators except thrusters so motion and positioning is realised only by change of their developed thrusts. The control allocation strategy used to map desired forces and torques to thrusts on all actuators is defined as an unconstrained model based optimisation problem. Such a solution is computationally efficiently but, in real conditions, when physical limitations are not taken into account it may be unrealized and cause temporarily loss of vehicle's controllability. To avoid a such situation a procedure of checking of capability of the propulsion system to produce demanded generalized forces is proposed to be introduced to the process of power distribution. The procedure, taking into account actuators constraints, allows to find such values of the generalized forces for which the distribution can be done correctly. To illustrate the proposed method a numerical example is given in the work.

Key-Words: - Over-actuated underwater vehicle, control system, thrust allocation, optimisation

1 Introduction

Unmanned underwater vehicles (UUV) have been a growing research area with strong support from both civilian and military applications. Main benefits of usage of them is to reduce risk to human life under dangerous environment and cost of exploration of deep seas.

A regarded vehicle is a small torpedo-shaped slowly swimming apparatus of four degrees of freedom DOF, being 1.4 m length, 0.36 m breadth, 0.36 m height and having a mass 45 kg or more (depending on configuration). It is controlled only by thrusters and supplied from an own source of energy. The vehicle is able to be equipped with sensors suitable for planned missions such as inspection of coastal and off-shore structures or hydrographical and biological surveys. Its main technical parameters are given in the Appendix 1.

The UUV is controlled both by a pilot located on a board of the floating or fixed platform and an onboard computerized control system, which allows to execute complex manoeuvres without constant human interventions. Basic modules of the control system are depicted in Fig. 1. The autopilot computes command signals (generalized forces) τ_d comparing desired vehicle's position and orientation η_d and linear and angular velocities \mathbf{v}_d with their current estimates $\mathbf{\eta} = [x, y, z, \phi, \theta, \psi]$ and $\mathbf{v} = [u, v, w, p, q, r]$. The thrust distribution module realizes a control allocation task distributing these commands into individual actuators in the propulsion system. An efficient control allocation generates control input **f** assuring that the vehicle's propulsion system develops the forces and torques determined by the autopilot.

Computation of a thrust vector **f** from the generalized forces τ_d is commonly treated as a model based optimisation problem that in the simplest form is unconstrained [1, 3, 4] and constraints put on maximum and minimum values of thrusts developed by the actuators are not taken into account. It may cause that required forces and torques acting on the vehicle's body will not be developed by the propulsion system due to a work of one or even more thrusters in saturation. Such a situation can make a contribution to deterioration of control and behaviour of the vehicle may differ from a required one significantly.

To cope with those difficulties a procedure of checking demanded generalized forces and if necessary decreasing their values is proposed to such values which the propulsion system can produce and unconstrained optimisation methods can be applied without any limitations.

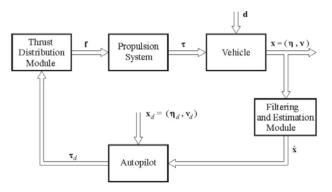


Fig. 1. General overview of UUV's control system.

The paper consists of the following five sections. In Section 2 a brief description of the problem of thrust allocation in a vehicle's propulsion system is presented. An algorithm of determination of feasible generalized forces is written in Section 3. In Section 4 a numerical example is provided. Conclusions are given in Section 5.

2 Description of Propulsion System

A propulsion system of the regarded UUV consists of five thrusters. Four of them, called roll axis thrusters, installed in the stern assure surge, pitch and yaw motions. The fifth one, called vertical axes thruster, located in a middle of the vehicle's body is responsible for heave motion. A general structure of the propulsion system shows Fig. 2.

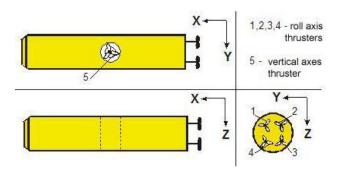


Fig. 2. Structure of the UUV propulsion system,

The vertical axis thruster produces propelling force Z in the normal axis which is equal to a developed thrust force.

All roll axis thrusters are the same type and mounted in a vertical plane, symmetrically in relation to a symmetry center of the transverse section. If all of them give the same thrusts then produced moments compensate and resultant propelling force X causes movement along the

longitudinal axis. In contrary apart of translation also rotations about transversal and vertical axes, caused by not equal zero resultant propelling moments (torques) M and N, are realized. The produced force X and torques M and N are a linear combination of thrusts developed by the roll axis thrusters. Hence, from an operating point of view, the control system should have a procedure of power distribution among the roll axis thrusters and it is done in the trust distribution module (see Fig. 1).

Lets denote:

 $\mathbf{\tau}_d = [\tau_{d1}, \tau_{d2}, \tau_{d3}]^T = [X_d, M_d, N_d]^T$ - a vector of demanded generalized forces (force in the longitudinal axis and torques about the transversal and vertical axes consequently),

 $\mathbf{f} = [f_1, f_2, ..., f_4]^T$ - a thrust vector produced by the roll axis thrusters,

 f_i – thrust of the i^{th} thruster,

and assume that components of the both vectors are bounded:

$$\tau_{di}^2 - \left(\tau_i^{\max}\right)^2 \le 0 \qquad \text{for} \quad i = \overline{1,3} \qquad (1)$$

$$f_j^2 - (f_j^{\max})^2 \le 0$$
 for $j = \overline{1,4}$ (2)

Values of τ_i^{max} and f_j^{max} depend on the actuator's construction and their configuration in the propulsion system.

The relationship between the forces and torques acting on the vehicle's body, and thrusts developed by the thrusters is a complicated function which depends on density of water, vehicle's velocity, actuators' diameters and revolutions, etc. A detailed analysis of thruster dynamics can be found e.g. in [8, 9, 10].

As shown in [2, 5, 6] a dependence between the vector of generalized force $\mathbf{\tau}_d$ and the thrust vector **f** can be expressed by a linear model in a form:

$$\boldsymbol{\tau}_d = \mathbf{T}\mathbf{f} \tag{3}$$

where **T** is so called thruster configuration matrix. For the regarded roll axis thrusters the matrix **T** takes the following form:

$$\mathbf{T} = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ d_1 & d_2 & -d_3 & -d_4 \\ d_1 & -d_2 & d_3 & -d_4 \end{bmatrix}$$
(4)

where d_i is a distance of the *i*th thruster from a symmetry centre of the transverse section.

The thrust allocation problem, i.e. computation **f** from τ_d , is usually formulated as the least-squares optimisation problem and described in the following form [3, 4, 5]:

$$\mathbf{f} = \mathbf{T}^* \boldsymbol{\tau}_d \tag{5}$$

where:

$$\mathbf{T}^* = \mathbf{T}^T \left(\mathbf{T} \mathbf{T}^T \right)^{-1} \tag{6}$$

This method of thrust allocation allows to find a minimum-norm solution but it should be noted that (5) belongs to unconstrained optimisation problems, it means there are no bounds on elements of the vector \mathbf{f} , so the obtained components f_i may not satisfy (2) and then generation of the desired vector $\mathbf{\tau}_d$ by the propulsion system is not possible. In a such case the new vector of generalized forces meeting the condition (2) must be determined.

It is proposed to be realised in thrust distribution module in two stages (see Fig. 3). In the first one a capacity of the propulsion system to develop the demanded generalized forces τ_d is checked and a vector of feasible commands τ'_d determined, (i.e. such values of forces and torques which the propulsion system can produce). In the second step the real allocation of thrusts among the actuators is carried out on the basis of τ'_d . A method of evaluation of the vector of feasible generalized forces $\tau'_d = [\tau'_{d1}, \tau'_{d2}, \tau'_{d3}]^T$ is presented in the next section.

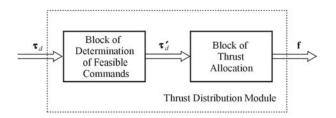


Fig. 3. A block diagram of the thrust distribution.

3 Algorithm of Determination of Feasible Propelling Force and Torques

Let us denote:

1. τ_1^{\max} , τ_2^{\max} and τ_3^{\max} - maximum values of propelling force X and torques M and N developed by the propulsion system:

$$\begin{split} \tau_{1}^{\max} &= \sum_{i=1}^{4} \left| f_{i}^{\max} \right| \\ \tau_{2}^{\max} &= \sum_{i=1}^{4} \left| f_{i}^{\max} d_{i} \right| , \\ \tau_{3}^{\max} &= \sum_{i=1}^{4} \left| f_{i}^{\max} d_{i} \right| \end{split}$$

- 2. O an origin of the Cartesian coordinate system,
- 3. P a point in the 3-dimensional space with coordinates $(\tau_{d1}, \tau_{d2}, \tau_{d3})$,
- 4. \overrightarrow{OP} a position vector of the point P.

Evaluation of capacity of the propulsion system to generation of the desired propelling forces and moment τ_d requires taking into consideration both limitations (1) and (2) simultaneously.

The first one indicates that the vector $\mathbf{\tau}_{d}$ is produced only if the position vector \overrightarrow{OP} is entirely contained in a cubicoid having vertexes in points: $(\tau_{1}^{\max}, \tau_{2}^{\max}, \tau_{3}^{\max}), (\tau_{1}^{\max}, \tau_{2}^{\max}, -\tau_{3}^{\max}), (\tau_{1}^{\max}, -\tau_{2}^{\max}, \tau_{3}^{\max}),$ $(\tau_{1}^{\max}, -\tau_{2}^{\max}, -\tau_{3}^{\max}), (-\tau_{1}^{\max}, \tau_{2}^{\max}, \tau_{3}^{\max}), (-\tau_{1}^{\max}, \tau_{2}^{\max}, -\tau_{3}^{\max}),$ $(-\tau_{1}^{\max}, -\tau_{2}^{\max}, \tau_{3}^{\max}), (-\tau_{1}^{\max}, -\tau_{2}^{\max}, -\tau_{3}^{\max})$ (see Fig. 4).

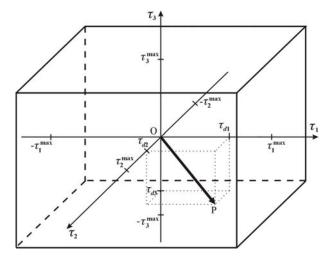


Fig. 4. View of the cubicoid and the position of vector \overrightarrow{OP} .

Since the components of the vector $\mathbf{\tau}_d$ are a linear combination of thrusts developed by all roll axis thrusters propellers then fulfilling only the condition (2) is not a guarantee their generation. E.g. if any component of the vector $\mathbf{\tau}_d$ corresponds an assignment $\tau_{di} = \tau_i^{\text{max}}$, then entirely power of the propulsion system is used to its production and the rest of the components must be equal to zero.

Therefore evaluation of capacity of the propulsion system to generation of the vector $\boldsymbol{\tau}_d$ requires taking into account also the inequality (1).

Analysis of values that elements of the vector $\boldsymbol{\tau}_d$ may take under limitations (1) and (2) leads to the following conclusion: τ_{d1} , τ_{d2} and τ_{d3} can be produced by the propulsion system if and only if the position vector \overrightarrow{OP} is entirely contained in a trisoctahedron with vertexes in points: $(\tau_1^{\max}, 0, 0)$, $(0, \tau_2^{\max}, 0)$, $(0, 0, \tau_3^{\max})$, $(-\tau_1^{\max}, 0, 0)$, $(0, -\tau_2^{\max}, 0)$, $(0, 0, -\tau_3^{\max})$ (see Fig. 5). This situation proceeds if the following inequality holds:

$$\frac{\left|\tau_{d1}\right|}{\tau_{1}^{\max}} + \frac{\left|\tau_{d2}\right|}{\tau_{2}^{\max}} + \frac{\left|\tau_{d3}\right|}{\tau_{3}^{\max}} \le 1$$
(7)

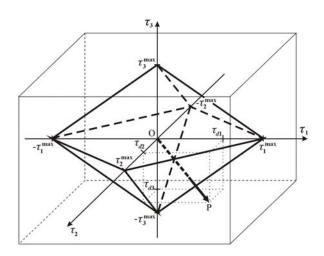


Fig. 5. View of the trisoctahedron and the position of vector \overrightarrow{OP} .

If (7) is false then the point $P = (\tau_{d1}, \tau_{d2}, \tau_{d3})$ lies outside of the octahedron and the vector $\boldsymbol{\tau}_d$ cannot be generated. It means that the vector of feasible commands $\boldsymbol{\tau'}_d = [\tau'_{d1}, \tau'_{d2}, \tau'_{d3}]^T$ must be determined. Its elements, under assumption that a reciprocal ratios of corresponding themselves components of the vectors $\boldsymbol{\tau}_d$ and $\boldsymbol{\tau'}_d$ are preserved:

$$\frac{\tau'_{d1}}{\tau'_{d2}} = \frac{\tau_{d1}}{\tau_{d2}} \quad \text{and} \quad \frac{\tau'_{d1}}{\tau'_{d3}} = \frac{\tau_{d1}}{\tau_{d3}}$$
(8)

can be calculated by means of the following equations:

$$\tau'_{d1} = sign(\tau_{d1}) \left(\frac{1}{\tau_{1}^{\max}} + \frac{1}{\tau_{2}^{\max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_{3}^{\max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1}$$

$$\tau'_{d2} = sign(\tau_{d2}) \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right|$$

$$\tau'_{d3} = sign(\tau_{d3}) \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right|$$

(9)

Basis on the above considerations an algorithm of evaluation of the vector $\boldsymbol{\tau}_d$ and determination of $\boldsymbol{\tau}'_d$ has been designed (see Fig. 6). Input data to the algorithm are quantities τ_1^{max} , τ_2^{max} , τ_3^{max} and the vector $\boldsymbol{\tau}_d$. The vector of feasible commands $\boldsymbol{\tau}'_d$ is computed according to (9).

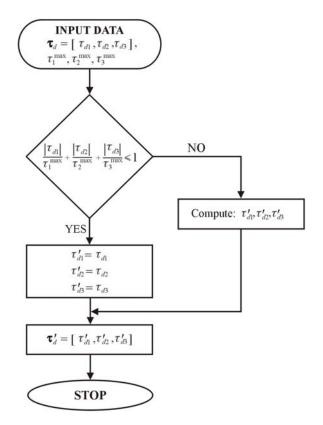


Fig. 6. A block diagram of the algorithm of determination of feasible commands.

A proof of (8).

Assume that $\tau_{di} \ge 0$ for $i = \overline{1,3}$. Such approach allows the analysis to restrict into a subspace limited by positive semi-axes of the coordinate system (see Fig. 7).

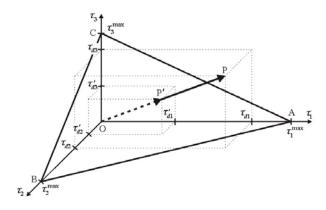


Fig. 7. A view of position vectors \overrightarrow{OP} and $\overrightarrow{OP'}$ for positive semi-axes of the Cartesian coordinate system.

Let A = $(\tau_1^{\text{max}}, 0, 0)$, B = $(0, \tau_2^{\text{max}}, 0)$, C = $(0, 0, \tau_3^{\text{max}})$, P = $(\tau_{d1}, \tau_{d2}, \tau_{d3})$ and P'= $(\tau'_{d1}, \tau'_{d2}, \tau'_{d3})$ be points in the three dimensional space. An equation of a plane including the points A, B and C has a form:

$$\frac{\tau_1}{\tau_1^{\max}} + \frac{\tau_2}{\tau_2^{\max}} + \frac{\tau_3}{\tau_3^{\max}} = 1$$
(10)

Let us assume that the point $P' = (\tau'_{d1}, \tau'_{d2}, \tau'_{d3})$ is a common point of a line containing the position vector \overrightarrow{OP} and the plane defined by (10). Substituting the coordinates of the point P' into (10) and taking into account the requirements (8) the following set of equations is given:

$$\begin{cases} \frac{\tau'_{d1}}{\tau_1^{\max}} + \frac{\tau'_{d2}}{\tau_2^{\max}} + \frac{\tau'_{d3}}{\tau_3^{\max}} = 1\\ \frac{\tau'_{d1}}{\tau'_{d2}} = \frac{\tau_{d1}}{\tau_{d2}}\\ \frac{\tau'_{d1}}{\tau'_{d3}} = \frac{\tau_{d1}}{\tau_{d3}} \end{cases}$$
(11)

Hence, solving (11) the following expressions for calculation of τ'_{d_1} , τ'_{d_2} and τ'_{d_3} are obtained:

$$\tau'_{d1} = \left(\frac{1}{\tau_1^{\max}} + \frac{1}{\tau_2^{\max}}\frac{\tau_{d2}}{\tau_{d1}} + \frac{1}{\tau_3^{\max}}\frac{\tau_{d3}}{\tau_{d1}}\right)^{-1}$$

$$\tau'_{d2} = \frac{\tau_{d2}}{\tau_{d1}}\tau'_{d1}$$

$$\tau'_{d3} = \frac{\tau_{d3}}{\tau_{d1}}\tau'_{d1}$$
 (12)

Finally, transformation of the above dependences to the entirely Cartesian coordinate system yields:

$$\tau'_{d1} = sign(\tau_{d1}) \left(\frac{1}{\tau_{1}^{\max}} + \frac{1}{\tau_{2}^{\max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_{3}^{\max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1}$$

$$\tau'_{d2} = sign(\tau_{d2}) \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right|$$

$$\tau'_{d3} = sign(\tau_{d3}) \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right|$$
(13)

End of prove.

4 Numerical Example

Computations are done for the following data of the propulsion system of the UUV [7]:

$$f_i^{\text{max}} = 75,0 \text{ N}$$
 for $i = \overline{1,4}$,
 $d_i = 0.09 \text{ m}$

hence:

$$\tau_1^{\max} = \sum_{j=1}^{4} \left| f_j^{\max} \right| = 4 \cdot 75.0 = 300.0 \text{ N}$$

$$\tau_2^{\max} = \sum_{i=1}^{4} \left| f_i^{\max} d_i \right| = 4 \cdot 75.0 \cdot 0.09 = 27.0 \text{ Nm}$$

$$\tau_3^{\max} = \sum_{i=1}^{4} \left| f_i^{\max} d_i \right| = 4 \cdot 75.0 \cdot 0.09 = 27.0 \text{ Nm}$$

Let assume that

$$\mathbf{\tau}_d = \begin{bmatrix} 100.0 & -18.0 & 9.0 \end{bmatrix}^T$$

STEP 1

Check the inequality (7):

$$\begin{aligned} \left|\tau_{d1}\right| &\leq -\frac{\tau_{1}^{\max}}{\tau_{2}^{\max}} \left|\tau_{d2}\right| - \frac{\tau_{1}^{\max}}{\tau_{3}^{\max}} \left|\tau_{d3}\right| + \tau_{1}^{\max} \\ \left|100.0\right| &\leq -\frac{300.0}{27.0} \left|-18.0\right| - \frac{300.0}{27.0} \left|9.0\right| + 300.0 \\ 100.0 &\leq 0.0 \end{aligned}$$

The dependence (6) is false.

It means that the point P=(100.0, -18.0, 9.0) lies outside of the trioctahedron having the following vertexes: (300, 0, 0), (0, 27, 0), (0, 0, 27), (-300, 0, 0), (0, -27, 0), (0, 0, -27).

Go to STEP 2.

STEP 2

Calculate the components of the vector $\mathbf{\tau}'_d$:

$$\begin{aligned} \tau'_{d1} &= sign(\tau_{d1}) \left(\frac{1}{\tau_{1}^{\max}} + \frac{1}{\tau_{2}^{\max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_{3}^{\max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1} = \\ &= sign(100.0) \left(\frac{1}{300.0} + \frac{1}{27.0} \left| \frac{-18.0}{100.0} \right| + \frac{1}{27.0} \left| \frac{9.0}{100.0} \right| \right)^{-1} \\ &= 75.0 \\ \tau'_{d2} &= sign(\tau_{d2}) \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right| = \\ &= sign(-18.0) \left| \frac{-18.0}{100.0} 75.0 \right| = -13.5 \\ \tau'_{d3} &= sign(\tau_{d3}) \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right| = \\ &= sign(9.0) \left| \frac{9.0}{100.0} 75.0 \right| = 6.75 \end{aligned}$$

STEP 3

Substitute the components of τ'_d as new values of τ_d :

$$\tau_{d1} = 75.0$$

 $\tau_{d2} = -13.5$
 $\tau_{d3} = 6.75$

To check the correctness of the calculations the ratio of the corresponding components of the vectors $\mathbf{\tau}_d$ and $\mathbf{\tau}'_d$ is computed according to (8):

$$\frac{\tau_{d1}}{\tau_{d2}} = \frac{100.0}{-18.0} = -5.6 \qquad \frac{\tau'_{d1}}{\tau'_{d2}} = \frac{75.0}{-13.5} = -5.6$$
$$\frac{\tau_{d1}}{\tau_{d3}} = \frac{100.0}{9.0} = 11.1 \qquad \frac{\tau'_{d1}}{\tau'_{d3}} = \frac{75.0}{6.75} = 11.1$$

Obtained quantities indicate that the ratio is preserved. It confirms the correctness of the procedure of determination of admissible generalized forces.

5 Conclusions

The paper presents a method of determination of feasible control commands for the small torpedoshaped underwater vehicle. For the UUV moving in 3 DOF it is necessary to distribute the generalized control forces $\tau \in \Re^{p}$ to 4 propellers in terms of the thrust vector $\mathbf{f} \in \Re^n$. A control allocation strategy is defined as the unconstrained model based problem. optimisation However in real applications, physical limitations (e.g. saturations) must be taken into account and hence a constrained optimisation problem must be solved. To cope with those difficulties a procedure of checking demanded control inputs was worked out. It allows to decrease and find such values of components of vector generalized forces τ that the the unconstrained optimisation methods can be used without any restrictions. A main advantage of the proposed method is its simplicity

The developed procedure of determine of feasible control inputs is of general character and can be successfully applied to all types of the underwater vehicles.

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Appendix 1

Technical specifications of the UUV.

The regarded unmanned underwater vehicle, called "Sea Anemone", was designed and built for the Polish Navy.

A. External dimensions:

- 1. length -1.40 m
- 2. width -0.36
- 3. height -0.36 m
- B. Mass 45.0 kg
- C. Buoyancy -1.0 N to 2.0 N
- D. Operating depth -200 m
- E. Maximum speed -3 m/s
- F. Range 500 m
- G. Propulsion:
 - roll axis thrusters four thrusters, 3 blade screw thrusters, electrically driven, each 50 W power;
 - vertical axes thruster single thruster, electrically driven 3 blade screw propeller in tunnel, 50 W power
- H. Mission duration time -30 minutes
- I. Energy source

J. Control

 lithium ion accumulator battery
 remote, computer

aided, using single optical fibre of 2000 m length