Optimal Solving Large Scale Traveling Transportation Problems by Flower Pollination Algorithm

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Abstract: - The traveling transportation problem (TTP) is one of the classic algorithmic problems like the traveling salesman problem (TSP) in the field of computer science, operations research and logistics engineering. It has been classified as the NP-complete problems which can be effectively solved by metaheuristic searching techniques based on modern optimization context. Recently, the flower pollination algorithm (FPA) was developed and proposed as one of the most efficient population-based metaheuristic optimization searching techniques to solve continuous and combinatorial optimization problems. Moreover, the FPA's algorithm is not complex. In this paper, the FPA is applied to solve the large scale traveling transportation problems (LsTTP) consisting of more than 500 destinations. The FPA is tested against six standard LsTTP problems from literatures. Results obtained by the FPA will be compared with those obtained by the genetic algorithm (GA) and the particle swarm optimization (PSO). As results, the FPA can provide optimal solutions for all selected LsTTP problems superior to GA and PSO within shorter computational time.

Key-Words: - Large Scale Traveling Transportation Problem, Flower Pollination Algorithm, Metaheuristics

1 Introduction

After the traveling salesman problem (TSP) was defined in the 1800s by Hamilton and Kirkman [1] as the former, the traveling transportation problem (TTP) was launched as the later [1],[2]. The TTP is one of the classic algorithmic problems like the TSP in the field of computer science, operations research and logistics engineering. It aims to find an optimal tour for a traveling transportation wishing to visit each of a list of *n* cities exactly once and then return to the home city. Such optimal tour is defined to be a tour whose total distance is minimized. The TTP can be considered as one of the NP-complete combinatorial optimization problems [3]. By literature, many exact methods and algorithms have been launched to solve the TTP (TSP) problems such as branch-and-bound [4] and integer linear programming [5]. To-date, the metaheuristic optimization techniques have been applied to solve the TTP (TSP) problems, for example simulated annealing (SA) [6], tabu search (TS) [7], genetic algorithms (GA) [8], particle swarm optimization (PSO) [9] and cuckoo search (CS) [10]. Among those metaheuristic optimization techniques, the flower pollination algorithm (FPA) proposed by Yang in 2012 [11], is one of the most powerful population-based metaheuristic optimization techniques. The FPA algorithm mimics the behavior of pollination of flowering plant in nature. In the FPA algorithm, the Lévy flight distribution is used for efficiently movement by pollinators in order to generate the feasible solution within the particular search space. By literatures, the performance evaluation of the FPA against many standard test functions was proposed [11],[12]. In addition, the FPA was applied to solve many real-world optimization problems, for examples, pressure vessels design [11], disc break design [12], control system design [13-16] and model identification [17].

In this paper, The FPA is applied to solve the large scale traveling transportation problems (LsTTP) consisting of more than 500 destinations. Six standard LsTTP problems from literatures [18] are conducted to perform the effectiveness of the

FPA. Results of six LsTTP problems obtained by the FPA will be compared with those obtained by the genetic algorithm (GA) and the particle swarm optimization (PSO), respectively. This paper consists of five sections. The rest of the paper is arranged as follows. Problem formulation including TTP model and six selected LsTTP problems is illustrated in section 2. The FPA algorithm and FPA-based LsTTP solving approach are briefly described in section 3. Results and discussions are provided in section 4, while conclusions are summarized in section 5.

2 Problem Formulation

2.1 TTP Model

Mathematical speaking, the model of the TTP problems can be formulated based on the graph theory [2],[3].

Let $G = (V, E)$ be a complete undirected graph with vertices V, $|V| = n$, where n is the number of cities, and edges E with edge length c_{ij} for the-ij city (i, j). This work focus on the symmetric TTP case in which $c_{ij} = c_{ji}$, for all cities (i, j). As the constrained optimization problem, the TTP problem is defined for minimization as stated in (1) – (5). J in (1) is the objective function standing for the total distant for traveling in which J will be minimized by the FPA. Constraint (2) ensures that each city is entered from only one other city, while constraint (3) ensures that each city is only departed to on other city. Constraint (4) eliminates subtours. This enforces that there is only a single tour covering all cities, and not two or more disjointed tours (subtour) that only collectively cover all cities. The final constraint in (5) is a binary constraint, where $x_{ij} = 1$ if edge (i, j) is in the solution and $x_{ij} = 0$, otherwise.

$$\textbf{Min} \quad f(\cdot) = \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{\substack{j \in V \\ j \neq i}} x_{ij} = 1, \quad i \in V \tag{2}$$

$$\sum_{\substack{i \in V \\ i \neq j}} x_{ij} = 1, \quad j \in V \tag{3}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \le |S| - 1, \; \forall S \subset V \tag{4}$$

$$x_{ij} = 0 \text{ or } 1, \quad i, j \in V \tag{5}$$

2.2 Selected LsTTP Problems

In this work, the LsTTP consisting more than 500 destinations [2-3],[19] are considered. Six standard LsTTP problems suggested by literatures [18] are selected and summarized in Table 1. The destination locations of LsTTP#1 (Att532) are plotted in Fig. 1 to demonstrate their locations as an example.

Table 1 Six standard LsTTP problems [18].

Entries	Names	Number of Cities	Optimal Solutions (Km.)	Comments
LsTTP#1	Att532	532	92,794	Padberg/Rinaldi
LsTTP#2	Gr666	666	294,358	Groetschel
LsTTP#3	Rat783	783	8,806	Pulleyblank
LsTTP#4	U1060	1,060	224,094	Drilling problem
LsTTP#5	D1291	1,291	50,801	Drilling problem
LsTTP#6	Nrw1379	1,379	56,638	Nordrhein/Westfalen

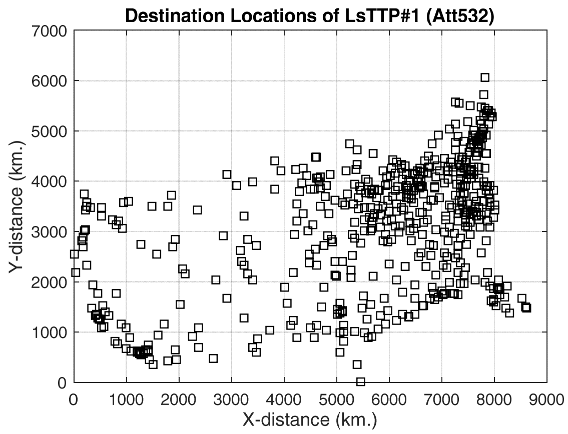


Fig. 1 Destination locations of LsTTP#1 (Att532).

3 FPA-Based LsTTP Solving

3.1 FPA Algorithms

The objective of the flower pollination in nature is the survival of the fittest and optimal reproduction of flowering plants. Pollination in flowering plants can be divided into two forms, i.e. self-pollination (or local pollination) by using abiotic pollinators (about 10-20% of flower pollination) and cross-pollination (or global pollination) by using biotic pollinators (about 80-90% of flower pollination) [20]. Self-pollination is the fertilization of one flower from pollen of the same flower or different flowers of the same plant, while cross-pollination occurs when pollen grains are moved to a flower from another plant [21],[22].

From characteristics of flower pollination, the FPA algorithm proposed by Yang [11] is based on four particular rules as follows:

- Biotic and cross-pollination are global pollination process via Lévy flight (Rule-1).

- Abiotic and self-pollination are local pollination process with random walk (Rule-2).
- Pollinators such as insects can develop flower constancy, which is equivalent to a reproduction probability that is proportional to the similarity of two flowers involved (Rule-3).
- Local pollination and global pollination can be controlled by a switch probability $p \in [0, 1]$ (Rule-4).

In FPA algorithm, a solution $\boldsymbol{x}_i$ is equivalent to a flower and/or a pollen gamete. For global pollination, flower pollens are carried by pollinators. With Lévy flight, pollens can travel over a long distance. Therefore, Rule-1 and flower constancy in Rule-3 can be expressed in (6), where $\boldsymbol{g}^*$ is the current best solution found among all solutions at the current generation/iteration t, and L stands for the Lévy flight that can be approximated by (7), where $\Gamma(\lambda)$ is the standard gamma function.

For local pollination, Rule-2 and Rule-3 can be represented by (8), where $\boldsymbol{x}_j$ and $\boldsymbol{x}_k$ are pollens from the different flowers of the same plant species, while ε stands for random walk by using uniform distribution in [0,1] as stated in (9). Once setting a = 0 and b =1 in (9), it is called a standard uniform distribution. Flower pollination activities can occur at all scales, both local and global pollination. In this case, a switch probability or proximity probability p in Rule-4 is used to switch between common global pollination to intensive local pollination. The FPA algorithm can be summarized by the pseudo code as shown in Fig. 2.

```
- Objective function f(x), x = (x1, x2,…, xd)
- Initialize a population of n flowers/pollen gametes
  with random solutions
- Find the best solution g* in the initial population
- Define a switch probability p ∈ [0, 1]
while (t < MaxGeneration)
    for i = 1 : n (all n flowers in the population)
        if rand < p,
            - Draw a step vector L via Lévy flight in (7)
            - Activate global pollination in (6)
        else
            - Draw ε from a uniform distribution in [0, 1]
            - Randomly choose j and k among all the solutions
            - Invoke local pollination in (8)
        end if
        - Evaluate new solutions
        - If new solutions are better, update them in the population
        - Update t
    end for
    - Find the current best solution g*
end while
```

Fig. 2 Pseudo code of FPA algorithms.

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + L(\boldsymbol{x}_i^t - \boldsymbol{g}^*) \tag{6}$$

$$L \approx \frac{\lambda\Gamma(\lambda)\sin(\pi\lambda/2)}{\pi}\frac{1}{s^{1+\lambda}}, \quad (s >> s_0 > 0) \tag{7}$$

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \varepsilon(\boldsymbol{x}_j^t - \boldsymbol{x}_k^t) \tag{8}$$

$$\varepsilon(\rho) = \begin{cases} 1/(b-a), & a \le \rho \le b \\ 0, & \rho < a \text{ or } \rho > b \end{cases} \tag{9}$$

From Yang's research reports [11], the number of pollens n = 25, proximity probability p = 0.8 and λ = 1.5 works better for most applications and are recommended parameter set based on preliminary parametric studies.

3.2 FPA-Based LsTTP Solving

The FPA algorithm shown in Fig. 2 will be adapted in order to solve LsTTP problems as follows:

Step-0 Initialize the objective function $f(\cdot)$ in (1) and constraint functions in (2) – (5). Randomly generate a population of n flowers. Fine the best solution $\boldsymbol{g}^*$ among initial population. Define a switch probability p = 0.8 (or 80%). Set MaxGen as the termination criteria (TC) and Gen = 1 as a generation counter.

Step-1 If Gen ≤ MaxGen, go to Step-2. Otherwise go to Step-4.

Step-2 If `rand` < p, draw a step vector L via Lévy flight in (7) and activate global pollination in (6) to generate a new solution $\boldsymbol{x}$. Otherwise draw a uniform distribution $\varepsilon \in [0, 1]$ in (9). Randomly select j and k among all solutions. Invoke local pollination in (8) to generate a new solution $\boldsymbol{x}$.

Step-3 If $f(\boldsymbol{x}) < f(\boldsymbol{g}^*)$, update solution $\boldsymbol{g}^* = \boldsymbol{x}$ and update Gen = Gen+1. Otherwise update Gen = Gen+1. Go to Step-1 to proceed next generation.

Step-4 Report the best solution found and stop the search process.

4 Results and Discussions

For solving LsTTP problems, the FPA algorithms were coded by MATLAB version 2017b (License No.#40637337) run on Intel(R) Core(TM) i5-3470 CPU@3.60GHz, 4.0GB-RAM. 50 trials are conducted to find the best solution for each LsTTP problem. For a fair comparison, searching parameters of GA and PSO are set based on those of FPA according to recommendations of Yang [11] as follows:

For GA :

- number of population (offspring) = 25
- crossover = 0.8 (80%)
- mutation = 0.2 (20%)
- TC : MaxGen = 100,000,

For PSO :

- number of population (particles) = 25
- cognitive learning rate = 2.0
- social learning rate = 2.0
- inertia weight $\theta_{\min}$ = 0.4 and $\theta_{\max}$ = 0.9
- TC : MaxGen = 100,000,

For FPA [11] :

- number of flowers n = 25
- switch probability p = 0.8 (80%)
- TC : MaxGen = 100,000.

As results, the optimal solutions and the search times consumed are summarized in Table 2 and Fig. 3. It was found in average that the FPA can provide the superior solutions to PSO and GA, respectively. Moreover, from Fig. 3, it can be observed that the FPA spend less time consumed than the PSO and GA, respectively. Fig. 4 depicts the convergent rates over 50 trials of the LsTTP#1 (Att532) problem preceded by the FPA for the first 5,000 generation as an example. From Fig. 4, it was found that the FPA shows very robust performance for providing the optimal solution. The optimal solutions of the LsTTP#1 (Att532) obtained by the GA, PSO and FPA are depicted in Figs. 5 – 7, respectively.

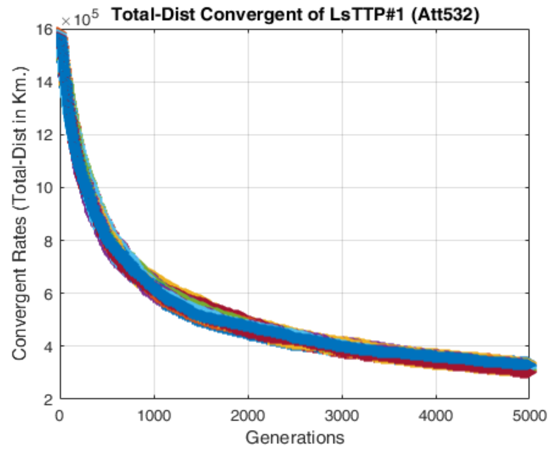


Fig. 4 Convergent rates of LsTTP#1 by FPA.

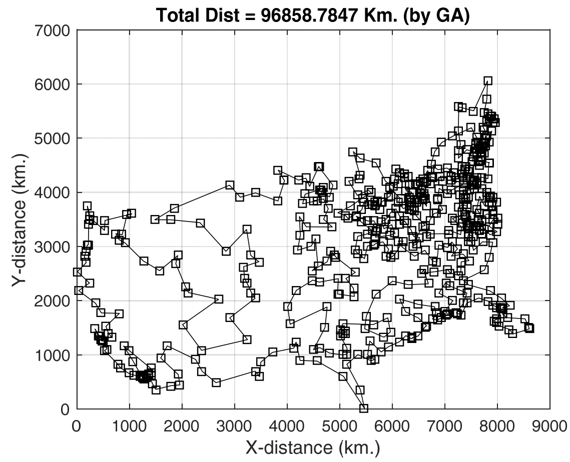
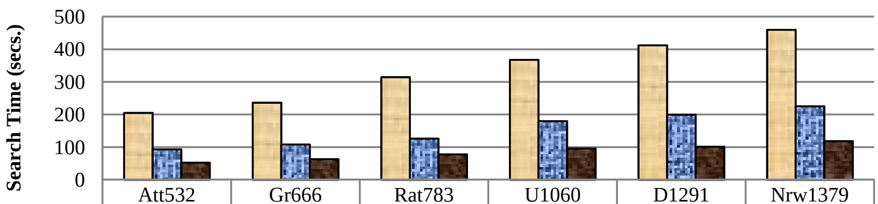


Fig. 5 Optimal tour of LsTTP#1 by GA.

Table 2 Optimal solutions of LsTTP problems obtained by GA, PSO and FPA.

Entries	Names	Optimal Solutions (Km.)	GA (Km.)	PSO (Km.)	FPA (Km.)
LsTTP#1	Att532	92,794	96,858.78	94,490.88	**92,797.62**
LsTTP#2	Gr666	294,358	312,754.15	301,218.84	**295,018.46**
LsTTP#3	Rat783	8,806	9,548.63	9,014.56	**8,810.71**
LsTTP#4	U1060	224,094	238,474.95	234,454.38	**225,174.58**
LsTTP#5	D1291	50,801	53,748.28	51,464.87	**50,866.23**
LsTTP#6	Nrw1379	56,638	60,101.47	58,415.83	**56,767.02**



	Att532	Gr666	Rat783	U1060	D1291	Nrw1379
GA	205.17	236.36	314.85	367.74	412.39	459.89
PSO	93.36	108.42	126.28	179.91	199.57	225.34
FPA	52.59	63.45	78.13	96.37	101.42	118.54

Fig. 3 Search time consumed for LsTTP problems by GA, PSO and FPA.

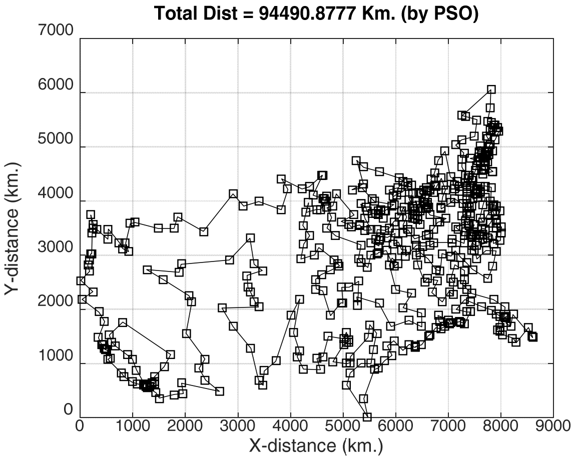


Fig. 6 Optimal tour of LsTTP#1 by PSO.

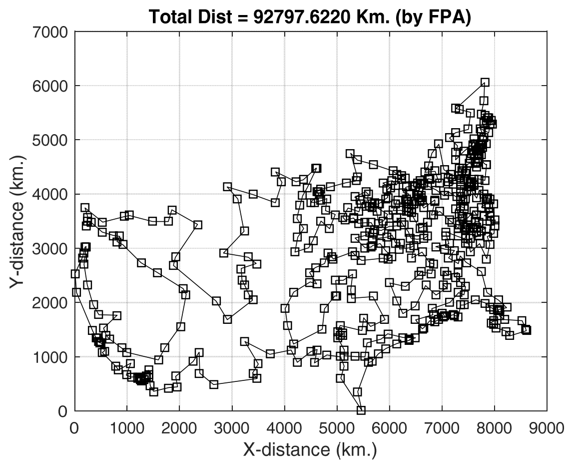


Fig. 7 Optimal tour of LsTTP#1 by FPA.

5 Conclusions

Solving the LsTTP problems by the FPA has been proposed in this paper. As one of the population-based metaheuristic optimization search techniques, the FPA has been applied to solve six selected standard LsTTP problems from literatures based on the modern optimization context. Results obtained by the FPA have been compared with those obtained by GA and PSO. As experimental results, the FPA has shown very robust performance for providing the optimal solution. With a fair comparison, it was found that for all selected LsTTP problems the FPA can provide the optimal solutions superior to PSO and GA, respectively. In addition, the FPA spent less computational time consumed than the PSO and GA, significantly. This can be concluded that the FPA is one of the most effective metaheuristic optimization techniques alternatively used for solving LsTTP problems, effectively.

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