Application of Lévy-Flight Firefly Algorithm to Multiobjective PIDA Controller Design Optimization for AVR System

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Abstract: - In 2008, the firefly algorithm (FA) was firstly proposed as one of the most powerful population-based metaheuristic optimization techniques for solving continuous and combinatorial optimization problems. However, many real-world engineering problems are typically formulated as the multiobjective optimization problems with complex constraints. In this paper, the Lévy-flight firefly algorithm (LFA) is applied to simultaneously minimize two particular objective functions, i.e. rise time and maximum overshoot, in order to obtain the optimal PIDA controllers for the automatic voltage regulator (AVR) system. As results, it was found that the LFA can provide the optimal PIDA controllers according to the predefined objective and constraint functions. Moreover, the LFA can perform the optimal Pareto front containing the optimal PIDA controllers for the AVR system.

Key-Words: - Lévy-Flight Firefly Algorithm, PIDA Controller, AVR System, Muitiobjective Optimization

1 Introduction

Metaheuristic optimization techniques have become acceptable worldwide as one of the most efficient intelligent tools applied to various complex and NP-hard real-world problems [1-4]. Many metaheuristic algorithms have been consecutively developed and launched. They can be classified into two main categories, i.e. trajectory-based and population-based metaheuristic algorithms. The former has strong intensification (or exploitation) property, while the later has strong diversification (or exploration) characteristics. [1-4]. Among those, the firefly algorithm (FA) is one of the most efficient population-based metaheuristic algorithms. The FA was originated by Yang in 2008 [4],[5] based on the flashing behavior of fireflies. Since its first appearance in 2008, in the last few years, the FA has been applied to almost every area of sciences and engineering, including power systems [6], image processing [7], antenna design [8], civil engineering [9], robotics [10], semantic web [11], chemistry [12], meteorology [13], wireless sensor networks [14], control engineering [15] and so forth.

The first version of the FA initiated by Yang in 2008 [4],[5] used the normal distribution, whereas the last version of the FA modified by Yang in 2010 employed the Lévy-flight distribution to randomly generate new solutions [16]. It is named the Lévy-flight firefly algorithm (LFA). From the preliminary study of Yang [16], the LFA was tested against several nonlinear and multimodal standard test functions. Results obtained by the LFA outperformed those by traditional algorithms including genetic algorithms (GA) and particle swarm optimization (PSO). The state-of-the-art and its applications of the LFA have been reviewed and reported [17],[18].

For our previous work, the FA was applied to design an optimal PIDA controller for the automatic voltage regulator (AVR) system via single-objective function [19]. In this paper, the LFA is then conducted to design an optimal PIDA controller for the AVR system based on muitiobjective optimization. The rise time and the maximum overshoot of the time-domain responses, which conflict to each other, are set as two particular objective functions to be minimized. Set of the optimal PIDA controllers will be obtained by the LFA to formulate the Pareto front and perform trade-off characteristics according to muitiobjective optimization context. This paper consists of five sections. After an introduction is proposed in section 1, the rest of the paper is arranged as follows. The LFA algorithms are briefly described in section 2. Problem formulation consisting of a concept of muitiobjective optimization with Pareto optimality and LFA-based PIDA design framework for the AVR system is provided in section 3. Results and discussions are given in section 4. Conclusions are followed in section 5.

2 Lévy-Flight Firefly Algorithm

Proposed by Yang in 2008, the FA was formulated based on the flashing behavior of fireflies [4],[5]. The flashing light of fireflies is produced by a process of bioluminescence to attract mating partners for communication and to attract potential prey. The FA's algorithm is developed from three idealized rules:

1) Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;

2) The attractiveness is proportional to the brightness, and they both decrease as their distance increases. Thus for any two flashing fireflies, the less brighter one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly; and

3) The brightness of a firefly is determined by the landscape of the objective function.

In FA, there are two important issues: the variation of light intensity and formulation of the attractiveness. The attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function. Along the distance r, the light intensity I varies according to the inverse square law $I(r) = I_s/r^2$, where I_s is the intensity at the source. For a given medium with a fixed light absorption coefficient, the light intensity I varies with the distance r as stated in (1), where I_0 is the original light intensity.

$$I = I_0 e^{-\gamma r} \tag{1}$$

$$\beta = \beta_0 e^{-\gamma r^2} \tag{2}$$

$$r_{ij} = \left\| \boldsymbol{x}_i - \boldsymbol{x}_j \right\| = \sqrt{\sum_{k=1}^{d} (\boldsymbol{x}_{i,k} - \boldsymbol{x}_{j,k})^2} \tag{3}$$

The attractiveness of a firefly observed by adjacent fireflies is proportional to the light intensity. This can define the variation of attractiveness β with the distance r as expressed in (2), where β_0 is the attractiveness at r = 0. From parametric studies, β_0 = 1 is suggested for most applications [4],[5]. The scaling factor γ in (1) and (2) is defined as the light absorption coefficient. In addition in (1) and (2), the distance r_{ij} between any two fireflies i and j at their locations $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ can be calculated by the Cartesian distance as expressed in (3), where $\boldsymbol{x}_{i,k}$ is the k^{th} component of the spatial coordinate $\boldsymbol{x}_i$ of i^{th} firefly.

For an original FA, the movement of a firefly i is attracted to another more attractive (brighter) firefly j is determined by (4), where α_t is the randomization parameter, and $\boldsymbol{\varepsilon}_i$ is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at time t [5]. In addition, α_t can be controlled during iterations as stated in (5), where α_0 is the initial randomness scaling factor, and δ is a cooling factor.

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \beta_0 e^{-\gamma r_{ij}^2} (\boldsymbol{x}_j^t - \boldsymbol{x}_i^t) + \alpha_t \boldsymbol{\varepsilon}_i^t \tag{4}$$

$$\alpha_t = \alpha_0 \delta^t, \quad (0 < \delta < 1) \tag{5}$$

For the Lévy-flight firefly algorithm (LFA) proposed by Yang in 2010 [16], the movement of a firefly i is attracted to another more attractive (brighter) firefly j is determined by (6), where the second term is due to the attraction while the third term is randomization via Lévy flights with α being the randomization parameter. The product $\oplus$ means entrywise multiplications. The `sign[rand-1/2]` where `rand` $\in$ [0, 1] essentially provides a random sign or direction while the random step length is drawn from a Lévy distribution having an infinite variance with an infinite mean. From (6), a symbol Lévy(λ) represents the Lévy distribution as expressed in (7). The step length s can be calculated by (8), where u and v stand for normal distribution as stated in (9). Standard deviations of u and v are also expressed in (10). The LFA algorithms can be represented by the flow diagram as shown in Fig. 1.

$$\begin{aligned} \boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \beta_0 e^{-\gamma r_{ij}^2} (\boldsymbol{x}_j^t - \boldsymbol{x}_i^t) \\ + \alpha \mathtt{sign}\left[\mathtt{rand} - \frac{1}{2}\right] \oplus \text{Lévy}(\lambda) \end{aligned} \tag{6}$$

$$\text{Lévy} \approx u = t^{-\lambda}, \quad (1 < \lambda \leq 3) \tag{7}$$

$$s = \frac{u}{|v|^{1/\beta}} \tag{8}$$

$$u \approx N(0.\,\sigma_u^2), \quad v \approx N(0.\,\sigma_v^2) \tag{9}$$

$$\sigma_u = \left\{ \frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma[(1+\beta)/2]\beta 2^{(\beta-1)/2}} \right\}^{1/\beta}, \quad \sigma_v = 1 \tag{10}$$

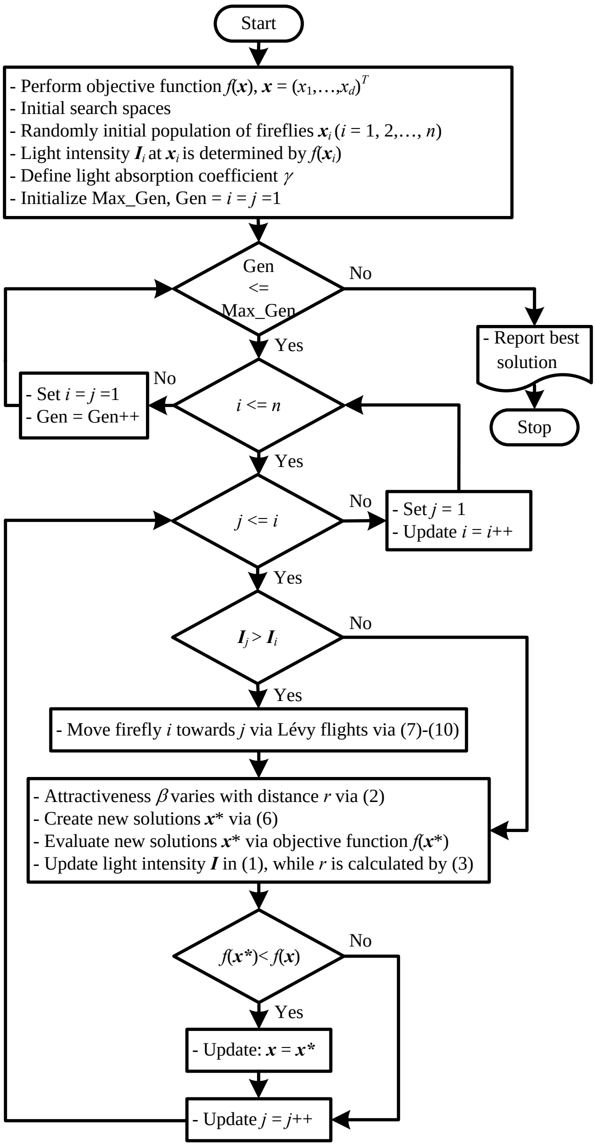

Fig. 1 Flow diagram of LFA algorithms.

3 Problem Formulation

In this section, the problem formulation is presented. It is divided into two parts, that is, a concept of muitiobjective optimization and the LFA-based PIDA design framework for the AVR system.

3.1 Multiobjective Optimization

Based on optimization context [1],[2],[20], multiobjective optimization problems can be formulated as expressed in (11), where $\boldsymbol{f}(\boldsymbol{x})$ is the multiobjective function consisting of $f_1(\boldsymbol{x}),\ldots, f_n(\boldsymbol{x})$, $n \geq 2$, $g_j(\boldsymbol{x})$, $j = 1, 2,\ldots,m$, is the inequality constraints and $h_k(\boldsymbol{x})$, $k = 1, 2,\ldots,p$, is the equality constraints. The optimal solutions, $\boldsymbol{x}^*$, are ones can make $\boldsymbol{f}(\boldsymbol{x})$ minimum and make both $g_j(\boldsymbol{x})$ and $h_k(\boldsymbol{x})$ satisfied. Regarded to the Pareto optimality [21-23], a solution vector, $\boldsymbol{u} = (u_1,\ldots,u_n)^T \in S$, is said to dominate another solution vector $\boldsymbol{v} = (v_1,\ldots,v_n)^T$, denoted by $\boldsymbol{u} \prec \boldsymbol{v}$, if and only if $u_i \leq v_i$ for $\forall i \in \{1,\ldots,n\}$ and for $\exists i \in \{1,\ldots,n\}$: $u_i \leq v_i$. This implies that no component of $\boldsymbol{v}$ is smaller than the corresponding component of $\boldsymbol{u}$, and at least one component of $\boldsymbol{u}$ is strictly smaller stated in (12).

$$\left.\begin{aligned} &\textbf{Min } \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),\ldots, f_n(\boldsymbol{x})\} \\ &\textit{subject to} \quad g_j(\boldsymbol{x}) \leq 0, \quad j = 1,\ldots,m \\ &\qquad\qquad\quad h_k(\boldsymbol{x}) = 0, \quad k = 1,\ldots,p \end{aligned}\right\} \tag{11}$$

$$\forall i \in \{1,\ldots,n\} : u_i \leq v_i \wedge \exists i \in \{1,\ldots,n\} : u_i < v_i \tag{12}$$

A solution $\boldsymbol{x}^* \in S$ is called a non-dominated solution if no solution can be found that dominates it. In other words, a solution $\boldsymbol{x}^* \in S$ is Pareto optimal if for every $\boldsymbol{x} \in S$, $\boldsymbol{f}(\boldsymbol{x}) \in F$ does not dominate $\boldsymbol{f}(\boldsymbol{x}^*) \in F$, that is $\boldsymbol{f}(\boldsymbol{x}^*) \prec \boldsymbol{f}(\boldsymbol{x})$. For a given multiobjective optimization problem, the Pareto optimal set is defined as P^* stated in (13). The Pareto front PF^* of a given multiobjective optimization problem can be defined as the image of the Pareto optimal set P^* expressed in (14).

$$P^* = \{\boldsymbol{x} \in F \mid \exists \boldsymbol{x}^* \in F : \boldsymbol{f}(\boldsymbol{x}^*) \prec \boldsymbol{f}(\boldsymbol{x}) \tag{13}$$

$$PF^* = \{\boldsymbol{s} \in S \mid \exists \boldsymbol{s}^* \in S : \boldsymbol{s}^* \prec \boldsymbol{s} \tag{14}$$

3.2 LFA-Based PIDA Design Framework

The LFA-based PIDA controller design framework for the AVR system is represented in Fig. 2. The AVR is commonly used in the generator excitation system of hydro and thermal power plants. The main role of the AVR is to regulate generator voltage and control the reactive power flow at a specified level. In this work, a simple AVR consists of four main components, i.e. amplifier, exciter, generator, and sensor, respectively as shown in Fig. 2, where E is the error voltage between the referent input voltage $V_{ref}(s)$ and sensor voltage V_B, while U , V_R and V_F are the controlled, amplified, and excited voltage signals, and $V_o(s)$ is the output voltage. Four main components of the AVR are linearized and modeled by transfer functions [24],[25] as visualized in Fig. 2. From [24],[25], the amplifier gain model K_A is in the range of 10 to 400, while the amplifier time constant τ_A is very small ranging

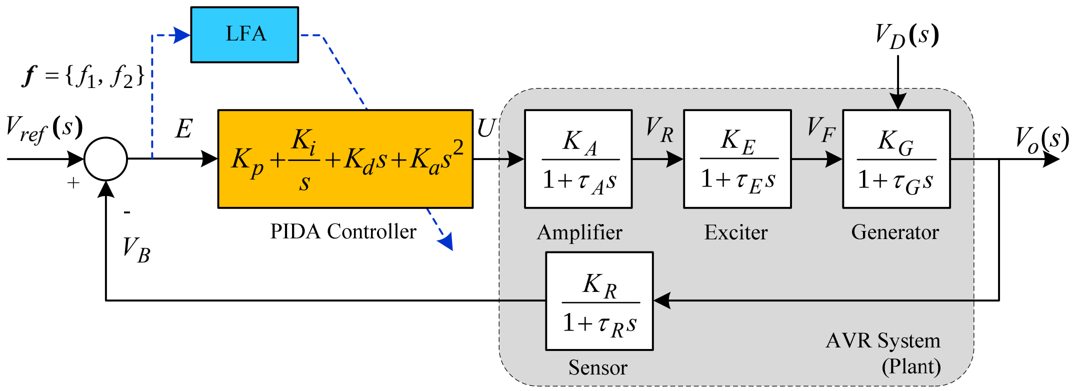

Fig. 2 LFA-based PIDA controller design for AVR system.

from 0.02 to 0.1 sec. For an exciter, a gain K_E is in the range of 1 to 400 and a time constant τ_E is from 0.25 to 1.0 sec. For a generator, a gain K_G may vary from 0.7 to 1.0, while a time constant τ_G is varied from 1.0 to 2.0 sec. Finally, a sensor gain K_R is very small ranging from 0.1 to 1.0, and its time constant τ_R is varied from 0.001 to 0.06 sec. Models of four main components will be used as a system plant in the control loop.

Referring to the control loop in Fig. 2, the PIDA controller receives the error signal, $E(s)$, and produces the control signal, $U(s)$, to control the output response, $C(s)$, referring to the referent input, $R(s)$, and regulate the output response, $C(s)$, from the external disturbance signal, $D(s)$. The s-domain transfer function of the PIDA controller $G_c(s)$ is stated in (15), where K_p, K_i, K_d and K_a are proportional, integral, derivative and accelerated gains, respectively.

$$\left.\begin{aligned} G_c(s) &= K_p + \frac{K_i}{s} + K_d s + K_a s^2 \\ &= \frac{K_a s^3 + K_d s^2 + K_p s + K_i}{s} \end{aligned}\right\} \quad (15)$$

$$\left.\begin{aligned} &\textbf{Min } \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x})\}, \\ &f_1(\boldsymbol{x}) = t_r, \\ &f_2(\boldsymbol{x}) = M_p \end{aligned}\right\} \quad (16)$$

$$\left.\begin{aligned} \textit{Subject to} \quad & t_r \le 0.3 \text{ sec.}, \quad M_p \le 10\,\%, \\ & t_s \le 1.5 \text{ sec.}, \quad E_{ss} \le 0.01\,\%, \\ & 0 \le K_p \le 1, \quad 0 \le K_i \le 1, \\ & 0 \le K_d \le 0.3, \quad 0 \le K_a \le 0.01 \end{aligned}\right\} \quad (17)$$

In the time-domain response of a controlled system, rise time (t_r) and maximum percent overshoot (M_p) are usually conflict to each other. Therefore, two particular objective functions, i.e. $f_1(\boldsymbol{x}) = t_r$ and $f_2(\boldsymbol{x}) = M_p$ are then set as stated in (16) to be minimized by the LFA in order to obtain the optimal PIDA parameters, i.e. K_p, K_i, K_d and K_a, for the AVR system, corresponding to their constraints and search spaces as given in (17).

4 Results and Discussions

To design optimal PIDA controllers for the AVR system by the LFA based on multiobjective optimization context, the LFA algorithms were coded by MATLAB version 2017b (License No.#40637337) run on Intel(R) Core(TM) i5-3470 CPU@3.60GHz, 4.0GB-RAM. Search parameters of the LFA are set according to Yang's recommendations [16], i.e. the numbers of fireflies n = 30, α_0 = 0.25, β_0 = 1, λ = 1.50 and γ = 1. The maximum generation $MaxGen$ = 100 is then set as the termination criteria (TC) in each trial. 50 trials are conducted to find a set of the optimal PIDA controllers for the AVR system. In this work, the parameters of the AVR system are set according to [24],[25] as follows: K_A = 10, τ_A = 0.1 sec., K_E = 1.0, τ_E = 0.4 sec., K_G = 1.0, τ_G = 1.0 sec., K_R = 1.0 and τ_R = 0.01 sec.

After the searching process of the LFA over 50 trials stopped, 50 optimal PIDA controllers are successfully obtained and summarized in Table 1 with their corresponding responses where t_s is settling time and E_{ss} is steady-state error. As non-dominated solutions, 50 sets of obtained PIDA controllers are plotted in Fig. 3 to formulate the Pareto front and perform trade-off characteristics between $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$. Tracking (or command following) responses of the AVR system with PIDA controllers are depicted in Fig. 4, while regulating (or disturbance rejection) responses of the AVR system with PIDA controllers are plotted in Fig. 5. From obtained results, it was found that the optimal PIDA controller's parameters obtained by the LFA for the AVR system and their corresponding responses are very satisfactory according to the design constraints defined in (17).

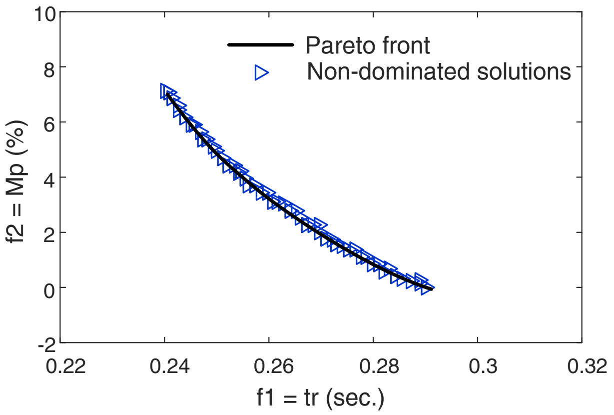

Fig. 3 Pareto front of AVR system with PIDA controllers.

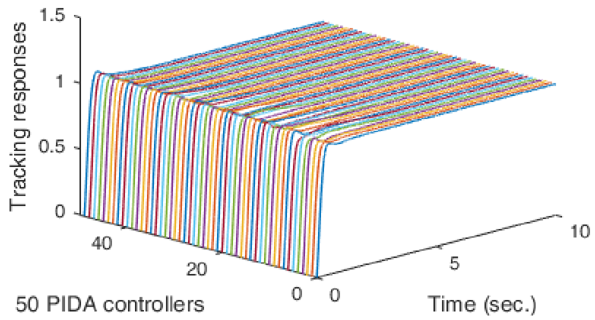

Fig. 4 Tracking responses of AVR system with PIDA controllers.

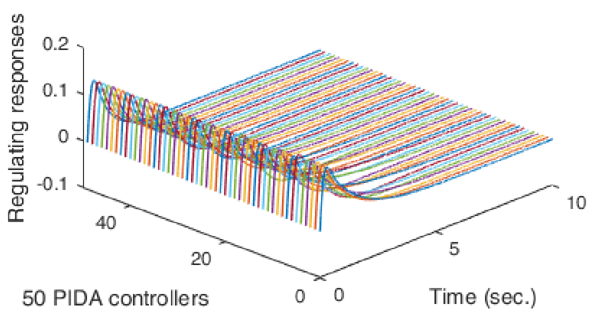

Fig. 5 Regulating responses of AVR system with PIDA controllers.

5 Conclusions

In this paper, the application of Lévy-flight firefly algorithm (LFA) to multiobjective PIDA controller design optimization for the AVR system has been proposed. Two particular objective functions, $f_1(\boldsymbol{x})$ = rise time and $f_2(\boldsymbol{x})$ =maximum overshoot, have been set to be minimized by searching for the appropriate values of the PIDA controllers by the LFA. As results, the LFA could provide the optimal PIDA controllers according to the predefined objective and their constraint functions. 50 sets of optimal PIDA controllers as non-dominated solutions have been performed the Pareto front between $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$, optimally. Tracking and regulating responses of the AVR system have been successfully achieved by the PIDA controllers designed by the LFA.

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Table 1: 50 PIDA controllers obtained by LFA for AVR system and their corresponding responses.

PIDA No#	PIDA controller's parameters				Responses			
	K_p	K_i	K_d	K_a	$f_1(\boldsymbol{x})$ t_r (sec.)	$f_2(\boldsymbol{x})$ M_p (%)	t_s (sec.)	E_{ss} (%)
1.	0.846024	0.539313	0.299987	0.009998	0.270021	0.000000	0.410000	0.000001
2.	0.838297	0.602389	0.299990	0.009999	0.269876	0.300204	0.400000	0.000000
3.	0.806808	0.613666	0.299988	0.008085	0.268747	0.636751	0.390000	0.000000
4.	0.844065	0.533188	0.300000	0.006392	0.266984	1.307865	1.020000	0.000002
5.	0.779164	0.658077	0.299988	0.009934	0.264957	1.409115	0.450000	0.000001
6.	0.877813	0.660406	0.299984	0.010000	0.263248	1.668555	0.380000	0.000000
7.	0.719457	0.700331	0.299986	0.008362	0.261240	2.520076	2.190000	0.000002
8.	0.964169	0.528245	0.299995	0.009999	0.260210	2.794901	0.580000	0.000052
9.	0.940636	0.653474	0.299996	0.010000	0.260028	3.204826	0.650000	0.000000
10.	0.897606	0.780465	0.299991	0.009999	0.260014	3.253317	0.770000	0.000000
11.	0.891220	0.820574	0.299986	0.009999	0.260005	3.475652	1.440000	0.000000
12.	0.989782	0.541024	0.299983	0.010000	0.259917	3.555930	0.620000	0.000054
13.	0.953455	0.664484	0.300000	0.009999	0.259840	3.618364	0.670000	0.000000
14.	0.966625	0.510907	0.299981	0.008149	0.259701	3.644928	1.200000	0.000078
15.	0.943977	0.723377	0.299990	0.009999	0.259693	3.880744	0.720000	0.000000
16.	0.926938	0.648208	0.299990	0.007540	0.259581	3.939943	0.630000	0.000000
17.	0.999988	0.562378	0.299988	0.010000	0.259496	3.981911	0.640000	0.000039
18.	0.969642	0.648106	0.299997	0.009776	0.259324	3.988423	0.670000	0.000001
19.	0.813201	0.907979	0.299997	0.009997	0.259140	4.017212	2.030000	0.000001
20.	0.948408	0.638904	0.299986	0.008184	0.259093	4.114728	0.640000	0.000001
21.	0.969620	0.671554	0.299984	0.009725	0.259024	4.202277	0.690000	0.000000
22.	0.999965	0.602366	0.299991	0.009999	0.259011	4.293520	0.670000	0.000015
23.	0.938238	0.815793	0.299991	0.009999	0.259005	4.544896	0.920000	0.000000
24.	0.999975	0.640605	0.299991	0.010000	0.258896	4.593256	0.690000	0.000005
25.	0.999997	0.641186	0.299990	0.010000	0.258546	4.598262	0.690000	0.000005
26.	0.999999	0.641186	0.299990	0.010000	0.258310	4.598427	0.690000	0.000005
27.	0.999964	0.642427	0.299989	0.010000	0.258294	4.607199	0.690000	0.000005
28.	0.958590	0.783677	0.299994	0.010000	0.258015	4.752795	0.820000	0.000000
29.	0.935873	0.861061	0.299987	0.010000	0.257887	4.893529	1.340000	0.000000
30.	0.998341	0.687555	0.299985	0.010000	0.257624	4.928583	0.730000	0.000001
31.	1.000000	0.683109	0.299986	0.010000	0.257420	4.933428	0.720000	0.000001
32.	0.999979	0.696564	0.299994	0.010000	0.257228	5.041405	0.730000	0.000000
33.	0.999949	0.704418	0.299994	0.009999	0.257085	5.104554	0.740000	0.000000
34.	0.957756	0.829153	0.300000	0.009999	0.256602	5.126159	0.950000	0.000000
35.	0.929096	0.904153	0.299986	0.010000	0.256469	5.130600	1.540000	0.000000
36.	0.941326	0.877485	0.299987	0.010000	0.256203	5.167980	1.380000	0.000000
37.	0.999983	0.715371	0.299987	0.010000	0.255872	5.194031	0.750000	0.000000
38.	0.999990	0.739741	0.299987	0.010000	0.255536	5.391459	0.780000	0.000000
39.	0.981187	0.819827	0.299998	0.009978	0.255201	5.609652	0.890000	0.000000
40.	0.999988	0.789630	0.299985	0.010000	0.254854	5.798575	0.830000	0.000000
41.	0.999997	0.801844	0.299987	0.010000	0.254544	5.900466	0.850000	0.000000
42.	0.996214	0.815113	0.299985	0.010000	0.254020	5.921130	0.880000	0.000000
43.	0.999457	0.806589	0.299991	0.009999	0.253696	5.927196	0.860000	0.000000
44.	0.992112	0.844931	0.299989	0.009998	0.253207	6.072431	0.950000	0.000000
45.	0.999966	0.823675	0.299990	0.009999	0.253034	6.081243	0.890000	0.000000
46.	0.999902	0.842957	0.299983	0.010000	0.252669	6.240252	0.930000	0.000000
47.	0.999957	0.844477	0.299997	0.009998	0.252207	6.254190	0.930000	0.000000
48.	0.999971	0.832714	0.299992	0.008906	0.251943	6.629873	0.870000	0.000000
49.	0.999989	0.908276	0.299994	0.009999	0.251479	6.790362	1.160000	0.000000
50.	0.999474	0.947491	0.299989	0.010000	0.250112	7.113321	1.310000	0.000000

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