# One Kind of Distributed Localization Method for Sensor Networks*

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*Abstract:*- In order to solve the problem of localization of the acoustic source by using sensor networks, one kind of distributed location scheme was proposed, here. Firstly, the sensor networks reference node and network topologies were defined. Then, the pulse-coupled clock synchronization algorithm was introduced to synchronize all node clocks, and besides that, the time difference of arrival of the source sound signals to all the nodes was measured on the basis of the clock synchronization. Finally, the position of the sound source was located by sensor networks equipped with distributed localization algorithm. In case of completeness, the four different forms of configuration, classical wedge, quad, symmetric wedge and circle, were taken for examples to verify the suggested idea. And each kind of network distribution topological configuration form was computed repeatedly for 1000 times, under the linear iterative algorithm.

*Key-Words*:- localization, acoustic source, pulse-coupled clock synchronization, time difference of arrival, wireless sensor networks, linear iterative algorithm

## 1 Introduction

With the rapid development of technologies of modern signal processing, microelectronics, wireless communication and computer network, the demand for wireless mobile communication was gradually strengthened. And traditional wireless mobile communication networks, which were based on central control and relied on pre-built network infrastructures, can no longer satisfy the requirements in certain special situations, such as battlefield operations, post-disaster rescue, and field detection, and et al.

In the wireless sensor networks positioning, the first important step is that each node needs to be performed clock synchronization, which requires an accurate time stamp to ensure the accurate positioning. Recently, the clock synchronization algorithm of several sensor networks has been designed. And the already available clock synchronization algorithm can be, RBS (Reference Broadcast Synchronization)[1] or GTSP (Gradient Time Synchronization Protocol)[2] and or the other ones. The pulse-coupled algorithm, which was in application of bio-oscillators, was studied in [3] and the clock synchronization algorithm for wireless sensor networks was introduced in [4]. The scalability of pulse-coupled synchronization for wireless sensor networks was introduced in [5-6], where the optimized phase response function was employed to accelerate the pulse coupling.

The second important step is positioning. Usually, the simplest localization method of wireless sensor networks is to equip the GPS receiver for each node. However, because of the limitation of the cost, and the short time duration of the energy of each node and the constraint of the harsh conditions, such as the GPS denial environment, each node in practice could not be equipped with a GPS receiver, and even it is possible to be equipped with GPS, the accuracy could not be met either. So, in this paper, the usage of TDOA (Time Difference of Arrival) will be put forward instead to locate the unknown acoustic source. The traditional way of TDOA positioning method is that two different propagation speed wireless signals are transmitted between the transmitting node and the receiving node, and the time difference is measured, and then the distance from the signal point to the receiving point is calculated. And the common wireless sensor network ranging methods include: TOA[7] (Time of Arrival), which requires a high requirement of time synchronization in hardware, and AOA[8] (Angle of

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Arrival) requires the sensor node to be capable of the measurement of the angle. Up to now, the positioning results are too singlet to be used [9-10], this paper proposes a novel TDOA method, where location of the sound source will be achieved by using time difference of arrival of the sound signal emitted by the sound source to different sensor networks nodes.

This paper is arranged like the following, at first, location algorithm will be introduced, and then follows the simulation results and its analysis. Finally is the conclusion of this paper.

# 2 Sensor networks distributed localization algorithm

## 2.1 Preliminary knowledge

### 2.1.1 Algebraic graph theory

In this work, $R$ denotes the real numbers, and $R \geq 0$ denotes nonnegative real numbers, $Z \geq 0$ denotes nonnegative integers, $R^N$ the Euclidean space of dimension $N$, and $R^{N \times N}$ the set of $N \times N$ square matrices with real coefficients and $\boldsymbol{I}_N$ is the $N \times N$ identity matrix, $\boldsymbol{1}$ is the column vector with every cell equal to 1.

In the sensor networks, the identifiers of the nodes are $1, 2, ..., N$, respectively. The set of nodes is denoted as $\boldsymbol{V} = \{1, 2, ..., N\}$. For node $i$, the set $N_i$ denotes its neighbor set. $G$ denotes a graph and $\boldsymbol{V}(G)$ is its vertex set, $\boldsymbol{E}(G)$ is its edge set. Its adjacency matrix is $\boldsymbol{B}_f = \left[b_{ij}\right]_{N \times N} \in \boldsymbol{R}^{N \times N}$, its components are not negative, where

$$b_{ij} = \begin{cases} 1 & \boldsymbol{V}_i \boldsymbol{V}_j \in \boldsymbol{E}(G) \\ 0 & \boldsymbol{V}_i \boldsymbol{V}_j \notin \boldsymbol{E}(G) \end{cases}, i\ j = 1, 2, 3, ..., N \qquad (1)$$

In the following sections, $P_i \in \Omega \subset R^2$ is the position of the node $i$, where $\Omega$ is the region of the space we want to check. And $\boldsymbol{P} := (p_1 \cdots p_N)^T$ can be assigned to represent the position vector of the sensor networks. We can say that the sensor network is in formation $f$ if $\boldsymbol{P}$ satisfies all the distance constraints of $\boldsymbol{B}_f$, and we will denote that as $\boldsymbol{P} \sim f$. TOA based localization scheme can be successfully used to locate an acoustic source in $R^N$ if and only if the sensors do not lie on the plane of an hyperbola[11-12].

### 2.1.2 Pulse-coupled clock synchronization algorithm for wireless sensor networks

The pulse-coupled clock synchronization algorithm for wireless sensor networks like [13-14] will be presented. The node in the network is equivalent to an adjustable oscillator, and the control node periodically triggers the oscillator. And a phase variable $x_i$ associated with one of the oscillator $i \in \boldsymbol{V}$ is defined. And each oscillator $i \in \boldsymbol{V}$, which follows its natural frequency $w_i$, will use the received pulses to modify its own internal phase. When the pulse phase reaches the maximum value of $2\pi$, the oscillator emits a pulse and resets its phase to zero. The oscillator updates its phase $x_i$ by using the coupling strength $l \in (0,1]$ and a function of current value $Q(x_i)$ upon receiving a pulse. It can be expressed as follows:

$$x_i^{new} = x_i + l \cdot Q(x_i) \qquad (2)$$

Where, the coupling strength $l$ is the slope of the phase response function. In this algorithm, the clock synchronization is made possible by the following phase response function:

$$Q(x_i) = \begin{cases} -x_i & 0 \leq x \leq \pi \\ -x_i + 2\pi, & \pi < x < 2\pi \end{cases} \qquad (3)$$

The value of the counter at a certain time is the phase of the node. The maximum value of the counter is the phase maximum $x_{th} = CL$ of the node. The trick can be found in [15].

## 2.2 Distributed positioning algorithm

This paper studies the distributed positioning algorithm and the algorithm can also be called as concurrency algorithm. During the positioning process, it only needs to communicate with the neighbor nodes. And it is necessary for us to know the coordinates of the eight anchor nodes to perform distributed positioning on the specified unknown acoustic source. The algorithm can be listed as Figure 1.

The distributed localization algorithm of multi-sensor networks is mainly to measure the time of arrival of the acoustic source to each node, and the emitted sound source is located in $S \in \boldsymbol{R}^2$. As shown in Fig.2, the acoustic source emits a spherical acoustic wave. The measured TOA of the acoustic wave is originated from $S$ at a given time $t_0$. To sensor $i$, located at $p_i$, the sound source location with radial distance $r$ is given as:

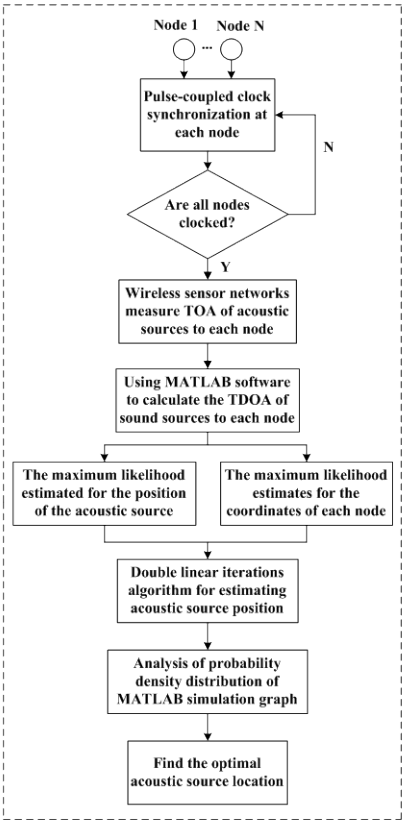


Fig.1The distributed positioning algorithm

$$TOA_i = t_0 + \frac{r}{c} + \delta_i = t_0 + \frac{d(S,P_i)}{c} + \delta_i \qquad (4)$$

Where $c$ is the speed of sound, and $\delta_i$ is the measurement noise of zero-mean white noise with variance $\sigma^2$ . Where $d(x,y) := \|x-y\|_2$ is the Euclidean distance in $R^2$ . Where $x$ and $y$ represent two-dimensional abscissa ordinate. In order to accurately determine the position of the acoustic source $S$, measurement by a single sensor is not possible, and therefore, it is necessary to perform integrated positioning in conjunction with the entire sensor networks. For the initial value $t_0$ can be viewed as an additional unknown independent variable, the position can be calculated by considering the TDOA between the sensor $i$ and the reference sensor $r_f \in V$ . Which can be described as

$$TDOA_i = TOA_i - TOA_{r_f} = \frac{d(S,p_i)-d(S,p_{r_f})}{c} + \delta_{ir_f} \qquad (5)$$

Where, $\delta_{ir_f}$ is the measurement noise with a zero-mean and variance of $2\sigma^2$. Assume sensor $p_1$ as the reference node, then the maximum likelihood estimation (MLE) for the location of the acoustic source, can be listed as follows[16]

$$\hat{S} = \arg\min_{z\in\Omega} \sum_{i=2}^{N} \left[ TDOA_i - (\frac{d(z,p_i)}{c} - \frac{d(z,p_1)}{c} \right]^2 \qquad (6)$$

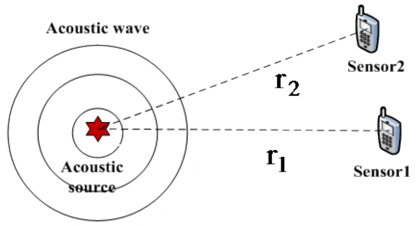


Fig.2. Schematic diagram of acoustic waves

However, in practice, the position of $p_i$ cannot be accurately determined because of the unavoidable errors. Therefore, the position of formula (6) can only be viewed as the estimated value, which is expressed as in the following

$$\hat{S} = \arg\min_{z\in\Omega} \sum_{i=2}^{N} \left[ TDOA_i - (\frac{d(z,\hat{p}_i)}{c} - \frac{d(z,\hat{p}_1)}{c} \right]^2 \qquad (7)$$

Where $\hat{p}_i = p_i + \varepsilon_i$ is the estimated location of node $i$ , and $\varepsilon_i \in [-\bar{p}, \bar{p}]$ is the bounded position error.

In a distributed system, a sensor node $i$ only needs to count neighbor nodes $N_i$ and collect TOA measurements. So, the exchange of information between every node plays a crucial role in distributed systems. Based on the derivation of formula (7), each sensor solves the local locating of the acoustic source, and then combines with the other positioning results from the other nodes, and finally averages to obtain the global solution. For each node $i$

$$\bar{S}_i = \arg\min_{z\in\Omega} \sum_{j\in N_i} \left[ TDOA_j - (\frac{d(z,\hat{p}_j)}{c} - \frac{d(z,\hat{p}_i)}{c} \right]^2 \qquad (8)$$

It is clear that only its neighbor nodes are considered to estimate the position of the acoustic source. In order to ensure the accuracy of the distributed average algorithm convergence, the following assumptions are made:

**Assumption 1:** The communication delay is small enough and there is a positive finite time interval $T$ . At each time interval $T$ , every node broadcasts its pulses.

**Assumption 2:** The sensor in each node $i \in V$ knows its neighbors set, $N_i$ , and also knows the number of its neighbors, $|N_i|$ .

In order to average all the positioning results and obtain the $\bar{S}_{avg}$ , we will introduce double linear iterative algorithm in this paper[17]. And the algorithm combines the information of the neighbor nodes as follows:

Each node has two positioning variables $s_i$ and $z_i$, initializes $s_i = \overline{S}_i$ and $z_i = 1$, and a timer variable $x_i$. Variable $x_i$ is initialized $x_i = 0$ and increased at a fixed rate. When the timer phase reaches the threshold value as $x_i = x_{th}$, the node $i$ updates its positioning variable and broadcasts. When receives the neighboring information, for each node $j \in N_i$, it will use the received information to update its own positioning variable. When $x_i = x_{th}$ is reached, the sensor network's positioning variables are updated according to the following rules:

$$
\begin{aligned}
s_i^{new} &= \frac{s_i}{1+|N_i|} \\
s_{\mathrm{j}}^{new} &= s_j + b_{ij}\frac{s_i}{1+|N_i|} \\
z_i^{new} &= \frac{z_i}{1+|N_i|} \\
z_{\mathrm{j}}^{new} &= z_j + b_{ij}\frac{z_i}{1+|N_i|}
\end{aligned}
\tag{9}
$$

Where, $b_{ij}$ is the corresponding term of the neighboring matrix $\boldsymbol{B}_G$. At any point in time $\hat{S} = \frac{s_i}{z_i}$ gives node $i$ its own estimate. If each node updates its variable according to the above rules, node $i$ estimates that $\hat{S}_i$ will converge to the average of the initial estimate $\overline{S}_i$. Node $i$ updates in matrix form as:

$$
s^{new} = \boldsymbol{B}_i s z^{new} = \boldsymbol{B}_i z \tag{10}
$$

Where, the $\boldsymbol{B}_i$ is the $N \times N$ identity matrix. The $i$ th column of adjacency matrix $\boldsymbol{B}_G$ replace the $i$ th column of $\boldsymbol{B}_i$ and scaled by $\frac{1}{1+|N_i|}$. Therefore, the $\boldsymbol{B}_i$ is diagonal matrix and random matrix in column. We assume that the broadcast is completed in one cycle. And then, let $\boldsymbol{P}_N$ be the product of the $N$ matrix $\boldsymbol{B}_i$. So the $\boldsymbol{P}_N$ is also a diagonal matrix and random in column. Finally, update it after $N$ broadcasts.

$$
s^{+N} = \boldsymbol{P}s \quad z^{+N} = \boldsymbol{P}z \tag{11}
$$

Where, $\boldsymbol{P} = \boldsymbol{P}_N$, and $+N$ indicates $N$ times update. And $\boldsymbol{P}$ is corresponding to the product of $N$ matrices $\boldsymbol{B}_i$ in any order, the graph is a combination of $\boldsymbol{B}_i$ matrices of $N$ [18]. Because the composite graph contains the union of all the edges of $\boldsymbol{B}_i$, every underlying graph will be connected. It is well-known fact that in consensus theory, an infinite product of column stochastic matrices with positive diagonal and connected underlying graph will converge exponentially to a matrix of the form $\lambda \boldsymbol{1}^T$. So, when time comes to infinity, we can have

$$
s \to \lambda \boldsymbol{1}^T \overline{S} = (\sum_{i\in \boldsymbol{V}} \overline{S}_i)\lambda Z \to \lambda \boldsymbol{1}^T \boldsymbol{1} = N\lambda \tag{12}
$$

So for each sensor

$$
\frac{s_i}{z_i} = \frac{\lambda_i \sum_{i\in \boldsymbol{V}} \overline{S}_i}{\lambda N_i} = \frac{1}{N}\sum_{i\in \boldsymbol{V}} \overline{S}_i \tag{13}
$$

Since the convergence process is gradually in nature, a tolerance error $\varepsilon$, which is an small enough positive number, should be determined in advance to tell when will be the stop station.

**Localization protocol:** When we start to detect a source sound signal, each sensor broadcasts a TOA and collects TOA measurements from its neighbor nodes, obtains $\overline{S}_i$ by solving its positioning problem (8), initializes its positioning variables to $s_i = \overline{S}_i$ and $z_i = 1$, and monitors a timer variable $x_i$.

**1.** When the timer reaches the threshold $x_i = x_{th}$, sensor $i$ updates its positioning variable using the following rules:

$$
s_i^{new} = \frac{s_i}{1+|N_i|} \tag{14}
$$

$$
z_i^{new} = \frac{z_i}{1+|N_i|} \tag{15}
$$

Then, it broadcasts its updated values $s_i^{new}$ and $z_i^{new}$, and restarts timer $x_i$.

**2.** After receiving a set of node $j$ variables $\frac{s_j}{1+|N_j|}$ and $\frac{z_j}{1+|N_j|}$ from a neighboring node $j$, node $i$ updates its location variable by using the following rules:

$$
s_i^{new} = s_i + \frac{s_j}{1+|N_j|} \tag{16}
$$

$$
z_i^{new} = z_i + \frac{z_j}{1+|N_j|} \tag{17}
$$

When the error satisfies $\left|\frac{s_i^{new}}{z_i^{new}} - \frac{s_i}{z_i}\right| < \varepsilon$, select

$\hat{S}_i = \frac{s_i}{z_i}$ as the location of the acoustic source, and that is

$$\hat{S}_i \to \bar{S}_{avg} = \frac{1}{N}\sum_{i \in V} \bar{S}_i, \quad \forall i \in V \tag{18}$$

## 3 Simulation results and analysis

This article focuses on the positioning of the acoustic produced by the explosion. The positioning of the acoustic source produced by an explosion of a small weapon depends entirely on the acoustic produced by the explosion of the muzzle. The start time of the explosion is obviously unknown. Therefore, the TDOA positioning method is much more reasonable to be selected. The listed following figure shows the use of four different forms of sensor networks configuration to locate the same unknown source, and for each configuration, it consists of 8 sensors. It can be seen in Figure 3. For (a), it is the classical wedge formation; and for (b), it is the quad formation; and as for (c), it is corresponding to the symmetric wedge formation; and at last, for (d), it is the circle formation. For formation (d), some results have been reported[19-20]. The length, which is assumed to be 106 meters long, is intended to avoid irrelevant errors caused by positional uncertainty and TDOA. The simulation results of the corresponding sensor networks are shown in the Figure 4.

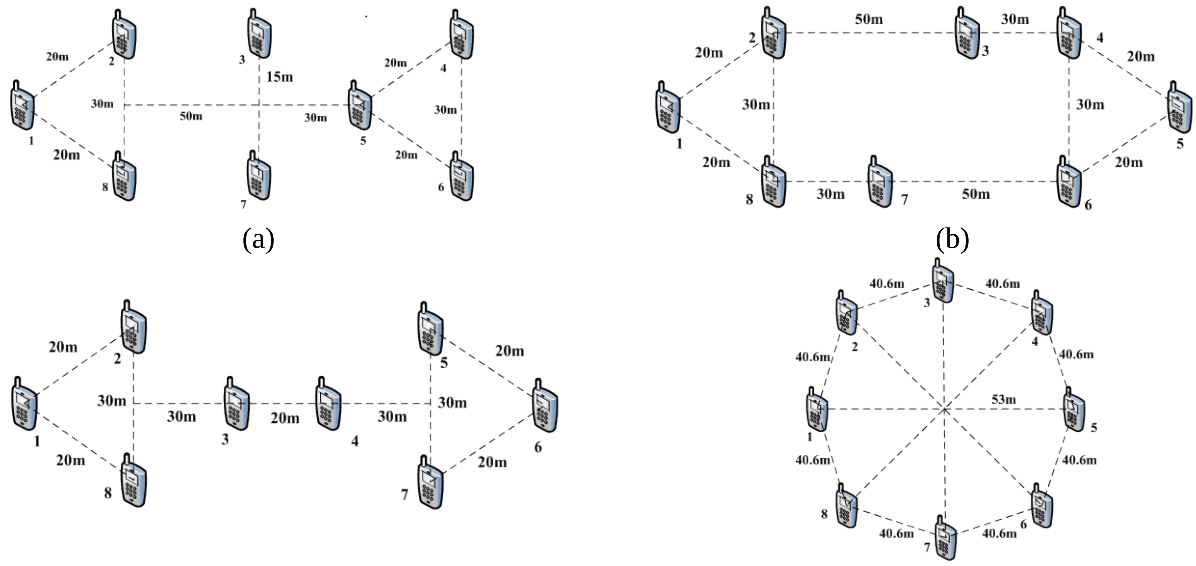


Fig.3. Configuration of the sensor networks

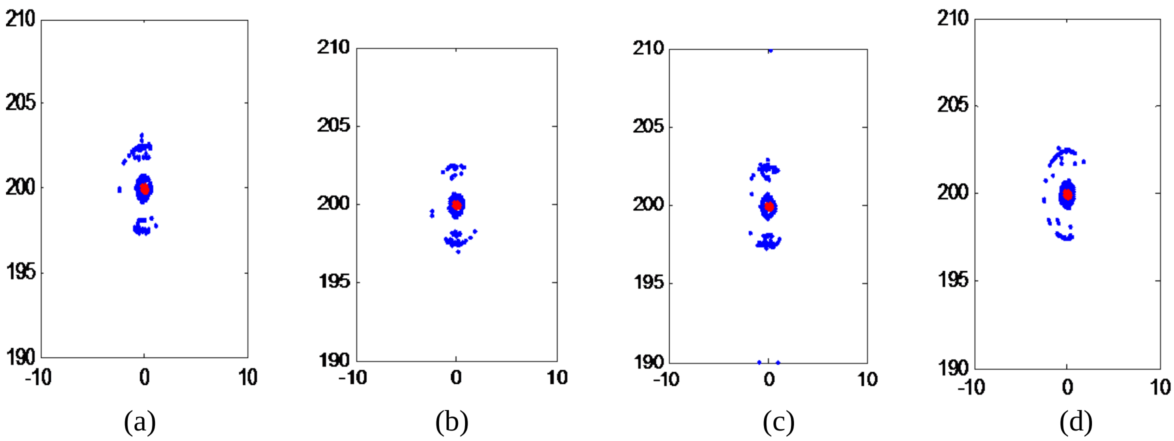


Fig.4. Positioning results

Table 1 Data analysis of the configuration of the four different sensor networks forms

| Set up | Mean $x$ | Std $x$ | Mean $y$ | Std $y$ | MAE | Var |
|---|---|---|---|---|---|---|
| a | 0.230 | 0.331 | 0.080 | 0.762 | 0.503 | 0.496 |
| b | -0.113 | 0.313 | -0.130 | 0.759 | 0.474 | 0.478 |
| c | -0.145 | 0.319 | -0.052 | 1.230 | 0.718 | 1.121 |
| d | -0.133 | 0.366 | 0.007 | 0.612 | 0.427 | 0.343 |

In order to test the influence of the position of the uncertain acoustic source on the positioning result, it is assumed that the simulation is performed without synchronization error. From Figure 4, it can be seen that when the acoustic source is located at $S=(0,200)$, the red point is the position of the real acoustic source. The four different sensor networks in Figure 3 simulate positioning of the same acoustic source respectively. It can be seen that although the location is uncertain, the four distribution methods can accurately locate the acoustic source. Table 1 shown that, the simulation is repeated at 1,000 times. Mean represents the mean of the two-dimensional space horizontal and vertical coordinates, and Std represents the standard deviation of the two-dimensional space horizontal and vertical coordinates. It can be seen that std $x$ in b is 0.313 and std $y$ in d is 0.612. We can derive that b has the highest accuracy for the horizontal coordinate positioning and d has the highest positioning accuracy for the ordinate. And the Mean-Absolute-Error (MAE) is $\frac{1}{1000}\sum d(S,\hat{S})$. In fact, we can obtain from c the maximum MAE is 0.718, the error is 0.677%, and the variance is 1.121. Moreover, the MAE is 0.427, the error is 0.403%, and the variance is 0.343 in d. We can derive that the positioning accuracy of d is the highest and the data of 1000 positioning results is the most stable. However, if the error is within 2% of the range, the four different forms of sensor networks configuration can all accurately locate the same unknown source.

# 4 Conclusions

In order to find the localization of the acoustic source, one kind of distributed location scheme was proposed in helps of the sensor networks. Started at the definition of the reference node and the topology of the sensor networks, continued with the pulse-coupled clock synchronization algorithm, and followed with the measurement of the time difference of arrival of the source sound signal to all the other nodes, the touching task was turned into pieces of small problems, and the later were dealt with finally. The suggested scheme sounds promising for its extension to the real arena, such as pollution source identification, poisonous source tracing and such et al.

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