

Optimal PID Parameters based Ant Colony Optimization algorithm of power system with non-linearity and boiler dynamics

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Abstract: - Proportional-Integral-Derivative (PID) and Proportional-Integral (PI) controllers are applied into the power system to examine the controller performance. Therefore, this work describes the application of an Artificial Intelligence (AI) optimization technique to design Proportional-Integral-Derivative (PID) controller for Load Frequency Control (LFC) of single area re-heat thermal power system. The PI-/ PID- controllers gain values are optimized using conventional method and AI optimization technique; respectively. Further, the proposed technique effectiveness is analyzed by considering non-linearity and Boiler dynamics into the same power system. Comparing the power system response with/ without non-linearity, it is proved that the response will have more oscillation in the case of using non-linearity and boiler dynamics (BD). Moreover, robustness of the analysis is carried out via varying the governor's time constants, turbine, re-heater and power system in about +50% to -50% from its nominal value by the 25% step.

Key-Words: - Artificial Intelligence; Ant Colony Optimization; Load Frequency Control; Non-linearity; Proportional-Integral-Derivative (PID) controller

1 Introduction

Generally, electric power systems with interconnected areas, Load Frequency Control (LFC) have a significant role. The LFC is endeavoured to preserve the system frequency of each area and the inter-area tie line power with acceptable limits to deal with the variation of the load demands and system disturbances. Consequently, analyzing Load Frequency Control (LFC) crisis in power generating unit is a fascinating topic that has received more attention in literature. Much reliable and economical operation of the power system requires power balance between the total load demand of the power generation and the system associated losses. The goal of LFC in power system is to establish system frequency during sudden load disturbance. Load demand value is not constant or predictable as it varies randomly due to the enormous development in technology and industries [1-29].

The power system is equipped with optimized PI controller and controller gain values using recent Self Adaptive Modified Bat Algorithm (SAMBA) technique. Quasi-Oppositional Harmony Search algorithm (QOHS) [10] is applied to tune the PID

controller gain values in the automatic generation control (AGC) of multi-area non re-heat thermal power system. The AGC of multi-area Solar Thermal-Thermal power system with PID controllers is discussed in [13]. As the Grey Wolf Optimizer algorithm (GOW) is used for optimizing the controller gain values with different conditions and parameter variations. Load frequency control issue in the multi-area power system is discussed in [15].

In recent years, several controllers have been developed for regulation of the power system operation and parameters (frequency and tie-line power flow) within the specified or scheduled value [1-9]. To achieve better dynamic response in multi-/ single area power systems, controllers have been proposed such as: Proportional – Integral (PI) controller [2], Integral Double Derivative (IDD) [6], Proportional-Integral-Derivative (PID) controller [7] and Fractional Order PID (FOPID) [17] and 2DOF-PID [22]. The conventional controllers have slow response, lack of efficiency and poor handling when non- linearities are added into the system.

The scheme of conventional controller based LFC system is discussed in [1]. Many researchers have

applied different control methods such as Fuzzy logic control [2], optimal control and variable Structure control for improving LFC of large scale and small scale power system.

Recently, the difficulties and drawbacks in the classical and conventional tuning methods are effectively eliminated by introducing population based optimization techniques. A promising alternative solution of the LFC problem is to use meta-heuristic algorithms such as: Artificial Bee colony (ABC) [5], Stochastic Particle Swarm Optimization (SPSO) [7], Imperialist Competitive Algorithm (ICA) [8], Firefly Algorithm (FA) [9, 25], Quasi-Oppositional Harmony Search algorithm (QOHS) [10], Fuzzy C-Means clustering technique (FCM) [11], Emotional learning-based intelligent controller [12], Grey Wolf Optimizer algorithm (GOW) [13], Cuckoo Search (CS)[14,26], Self Adaptive Modified Bat Algorithm (SAMBA) [15], hybrid Firefly Algorithm and Pattern Search (hFA-PS) [16], Multi-Objective Optimization (MOO)[17], Teacher Learning Based Optimization (TLBO) [18], Modified Harmony Search Algorithm (MHSA) [19], Minority Charge carrier Inspire algorithm (MCI) [20], hybrid PSO-PS optimization (hPSO-PS) [21].

This paper presents a Ant Colony Optimization (ACO) technique for optimal tuning of the PID controllers parameters. The inspiration for this study is to prove and reveal the robustness of ACO based PID controller in the single area reheat thermal power system under different loading conditions in the presence and absent of nonlinearities and boiler dynamics. The rest of the paper is organized as follows: an open loop and closed loop single area thermal power systems are designed in section II, Section III discusses the design of PI/PID controller and Conventional/Ant colony Optimization algorithm tuning procedure, simulation results are discussed in section IV, finally the conclusion is described in section V.

2 Investigated Power System and Modelling

Thermal power plant converts high temperature and high pressure heat steam energy into constructive mechanical energy with the aid of turbine that feeds the electrical generator. Then, this electrical generator is used to convert the mechanical energy to electrical energy in the power system [27-29]. As shown figure 1, the typical components of the thermal power plants are: governor, re-heater, turbine, generator and load/ speed regulator.

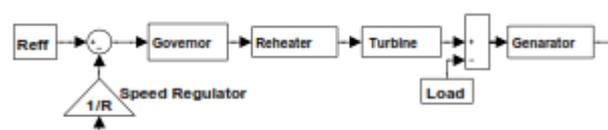


Fig.1 General block diagram of open loop reheat thermal power system

The water is transformed into steam with high pressure and temperature using steam generator [27-29]. The essential components of the steam turbine is re-heater, condenser, boiler feed pump, steam generator (Boiler), drum and control valve. The overall efficiency of the turbine is increased by dividing the steam pressure stages into two or more stages using a re-heater unit. Based on the steam stages, turbines are classified into three different types such as: non reheat turbine, single stage reheat turbine and double stage reheat turbine [27-29]. The steam turbine transfer function is given by:

$$G_T(S) = \frac{1}{(1 + ST_i)} \left(\frac{1 + \alpha ST_r}{1 + ST_r} \right) \quad (1)$$

Where, T_r is the re-heat time constant in sec. and T_i is the steam chest time constant in sec., S is Laplace domain function. The governor dead band nonlinearity produces a significant effect on the LFC performance that is defined as “The total magnitude of speed changes within which there is no resulting changes in value position”.

In this work, the LFC performance effect is considered to be 0.05%. The permissible rate of generation constraint for thermal power system is $0.0017 \text{ pu MW sec}^{-1}$. The drum type model of boiler dynamics produces steam under pressure. In addition, oil/gas filled boiler dynamics respond quickly during sudden load demand than coal fired boiler dynamics. Rating of the investigated thermal power system is 2000MW.

Figures 2 and 3 illustrate the block diagram of the transfer function model of both single area open loop and closed loop re-heat thermal power systems; respectively. Here, R refers to the self regulation parameter for the governor in p.u. Hz; T_g represents the speed governor time constant in sec; T_r is the reheat time constant in sec; K_r is the reheat gain; T_p , K_p is the load frequency constant ($T_p=2H/f*D$, $K_p=1/D$). Stability of the open loop system varies depending on the variation of the output response according to the input.

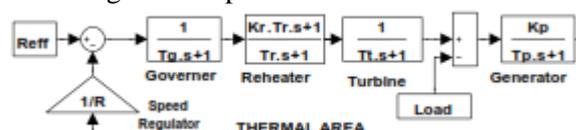


Fig.2 Transfer function model of open loop reheat thermal power system

In the power generating system, damping oscillations with steady state error in their response occur due to the load demand. The parameters of the open loop response are tabulated in table 1.

Table 1 Parameters of the open loop response

Parameters	Load demand = 0%	Load demand = 1% SLP
Frequency Peak (Hz)	0	-0.05744
Settling time (s)	0	24
Steady state error (Hz)	0	-0.02353

In order to mitigate and compensate error in response and load demand, proper assortment of controller is more crucial in the case of LFC problem. In this investigation, industrial Proportional- Integral (PI) and Proportional-Integral-Derivative (PID) controllers are designed and implemented.

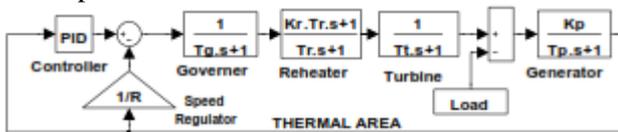


Fig.3 Transfer function model of the closed loop reheat thermal power system

3 Design of PID controller using ACO with different objective functions

Generally, the controller responsibility in LFC crisis preserves the overall system stability and recovers the system performance, while load demand increases/decreases. The above mentioned problem is solved by generating appropriate control signal using the controller. In this study, industrial PI and PID controller are implemented. The structure of PI and PID controller are shown in figures 4 and 5.

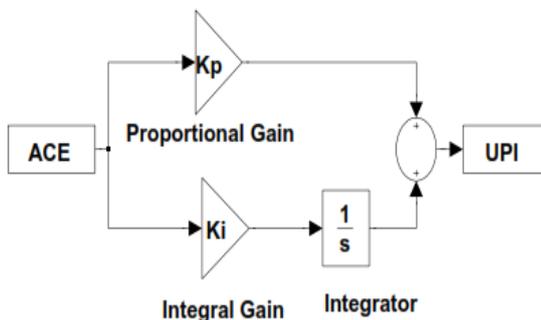


Fig.4 Structure of PI controller

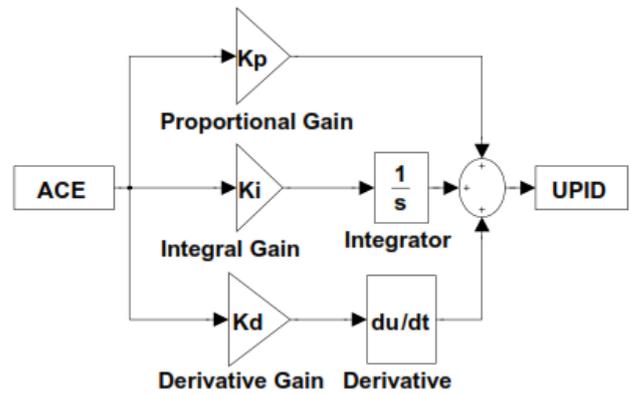


Fig.5 Structure PID controller

The control signal (U) generated by the PI and PID controller is given by U_{PI} and U_{PID} ; respectively as:

$$U_{PI} = -K_i \cdot ACE - \frac{K_p}{T_i} \int ACE \quad (2)$$

$$U_{PID} = -K_i \cdot ACE - \frac{K_p}{T_i} \int ACE - K_d T_d \frac{d}{dt} ACE \quad (3)$$

Where, U_{PI} and U_{PID} are the control signal generated by the PI and PID controllers; respectively, K_i is the integral gain, K_d is the derivative gain, K_p represents the proportional gain, T_i is the integral time constant, T_d refers to the derivative time constant and ACE is the area control error (ACE). The conventional method tuned PI controller gain value and the performance index curve is shown in figure 6. The tabulated vales of figure 6 are shown in Table II. The curve is plot between different values of the gain values with different performance index.

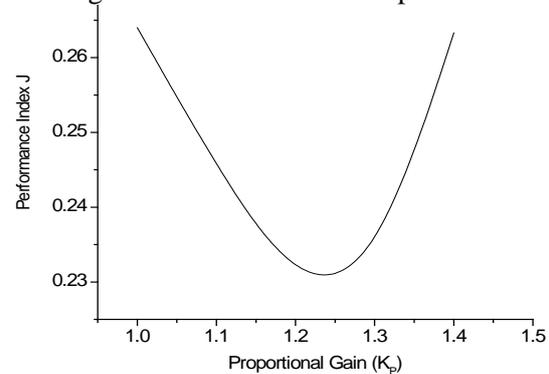


Fig.6 The PI controller Performance Index Curve

In order to sustain the system stability and to recover system performance, a suitable selection of the objective function is more critical to find controller parameters. In this work, an Integral Time Absolute Error (ITAE) objective function is suggested for both conventional and AI tuning technique. This objective function expression is given by:

$$J = \int_0^T t \cdot |\Delta f| dt \quad (4)$$

Thus, the optimization problem is solved using the Ant Colony Optimization (ACO) algorithm to obtain the optimal parameters of the controller gain values.

3.1 Ant Colony Optimization technique

Ant colony optimization technique was introduced by M. Dorigo in early 1900s as a novel nature inspired metaheuristic for the solution of combinatorial optimization problem [23-24]. This algorithm is based on the real ant behavior in searching the source of food. It is evident that the shortest path has large pheromone concentrations, so that more ants tend to choose it to travel. There are three major phases in the Ant Colony Algorithm:

- ❖ Initialization
- ❖ Constructing ant solution
- ❖ Updating pheromone

The global updating rule is implemented in the ant system where all ants start their tours and pheromone is deposited and updated on all edges based on:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k \in \text{colony that used edge } (i,j)} \frac{Q}{L_k} \tag{6}$$

Where; P_{ij} is the probability between the town i and j

τ_{ij} is the pheromone associated with the edge joining cities i and j

d_{ij} refers to the distance between cities i and j

Q is constant

L_k is the length of the tour performed by K_{th} ant

ρ is the evaporation rate

In this study, the number of ants=50, pheromone (τ) =0.6, evaporation rate (ρ) =0.95 and the number of iterations=100.

Table II demonstrates the optimal gain values of PID controller using Ant Colony Optimization technique with/without non-linearity and boiler dynamics effect.

Table 2 Optimal Gain values of PID controller by using Ant Colony Optimization Technique

Objective function	Turbine	K_p	K_i	K_d	J
Without non-linearity and Boiler dynamics	Conventional PI controller	1.2	0.17	-	0.2294
	ACO-PID controller	0.98	0.9	0.4	0.1674
With non-		1	1	0.4	0.2370

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4 Simulation results and Discusses

MATLAB/SIMULINK environment is used to model the transfer function of the simulated single area power system discussed in section II. The simulation results demonstrate the effectiveness and robustness of the proposed control method. Simulation process of the investigated power system is divided into the following three different cases for performance analysis:

- A. Power system with conventional PI and ACO PID controller without non-linearity and Boiler dynamics effect.
- B. Power system with ACO PID controller with/ without non-linearity and Boiler dynamics effect.
- C. Power system with ACO PID controller and variations (-50% to +50%) in time constants of the investigated power system parameters.

In case A, simulations are performed for 1% SLP in the system applying the conventional PI and ACO-PID controllers. The frequency deviation and area control error comparisons are shown in figures 7 and 8.

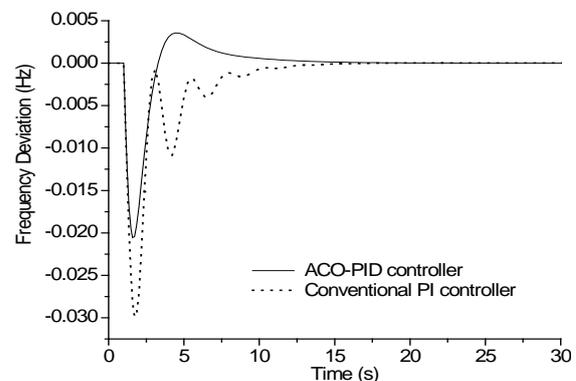


Fig.7 Comparisons of frequency deviations considering Conventional PI and ACO-PID controller without considering non-linearity and boiler dynamics

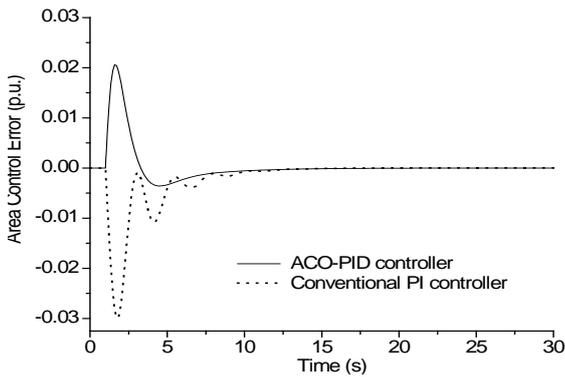


Fig.8 Comparisons of control input deviations Conventional PI and ACO-PID controller without considering non-linearity and boiler dynamics

It is clear from figures 7 and 8 that the damping oscillations and the settling time of the conventional PI controller are more than ACO-PID based controller response. This proves that the ACO-PID controller gives superior and enhanced controlled performance during sudden load demand.

In case B, the ACO tuned PID controller is equipped in the examined power system. The performance is analyzed by adding non-linearities and boiler dynamics into the power system with 1% SLP to evaluate the effectiveness. The system response, frequency deviation and area control error are shown in figures 9 and 10.

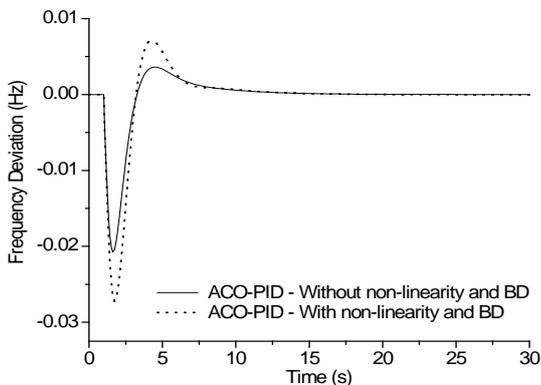


Fig.9 Comparisons of frequency deviations with and without out considering non-linearity and boiler dynamics

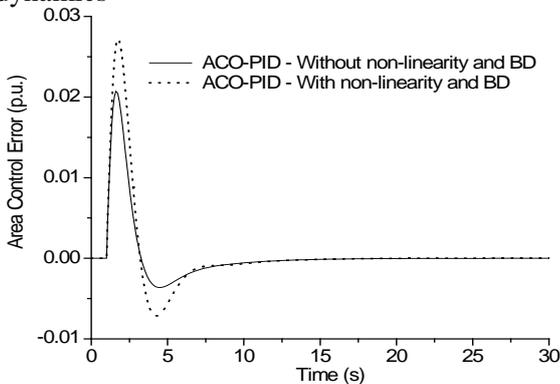


Fig.10 Comparisons of area control error with and without out considering non-linearity and boiler dynamics

Figures 9 and 10 demonstrate that the power system response yields more damping oscillations and consumes more time when the non-linearity and Boiler Dynamics is taken into the account. Thus, it is concluded that, tuning the PID controller using ACO provides better controlled performance with and without non-linearity and boiler dynamics.

Additionally, in case C, the robustness of the ACO-PID controller is examined via varying the power system time constants from its nominal value i.e. from -50% to +50% with 25% step. The system response is demonstrated in figures 11 and 12, as the governor time constant is varied from -50% to +50% using 1% SLP. The PID controller gain value of different criteria is given in the Table II.

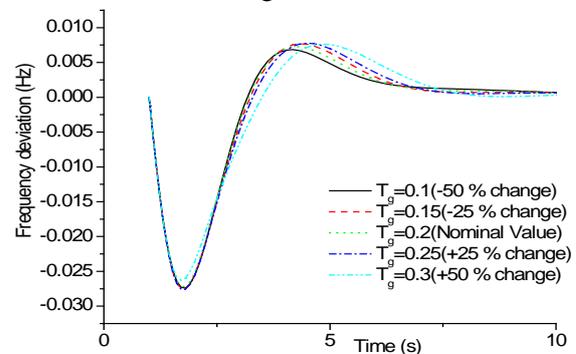


Fig.11 Comparisons of frequency deviations for changes in T_g .

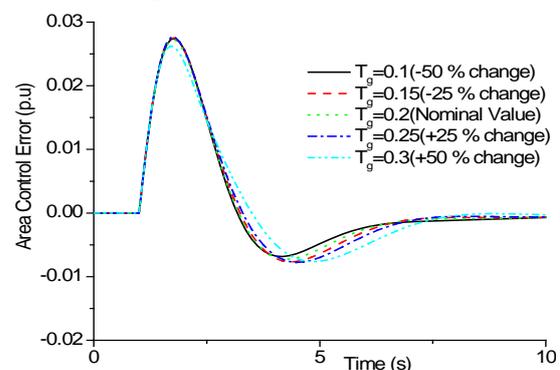


Fig.12 Comparisons of control input deviations for changes in T_g

Figures 13 and 14 illustrates the comparisons of frequency deviations and area control inputs for changes in re-heater time constant in the same tested power system.

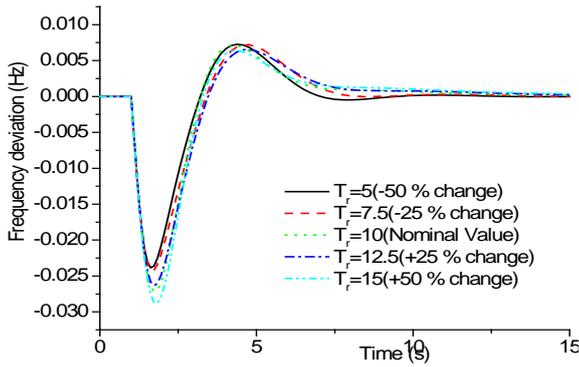


Fig.13 Comparisons of control input deviations for changes in T_r

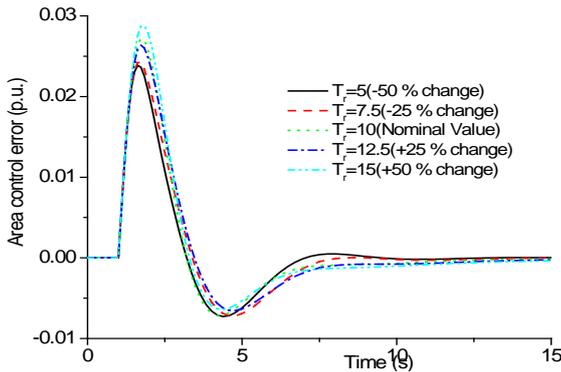


Fig.14 Comparisons of frequency deviations for changes in T_r .

Figures 15 and 16 show the response of frequency deviation and area control error with variations in the turbine time constant. While, figures 17 and 18 show the variations in the frequency deviations and control input with changes in the power system time constant.

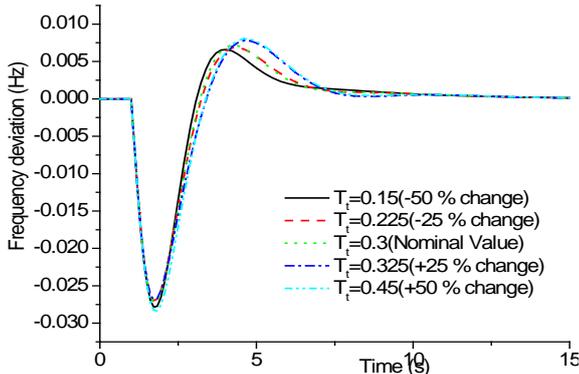


Fig.15 Comparisons of frequency deviations for changes in T_t

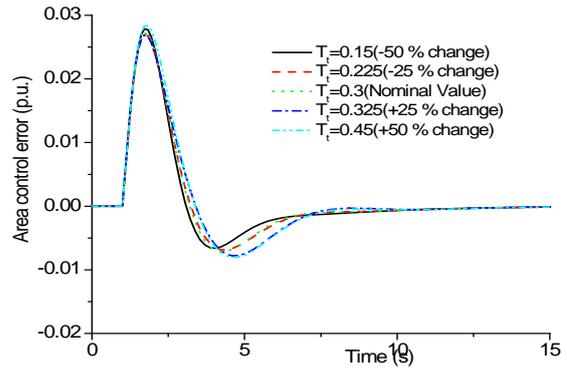


Fig.16 Comparisons of control input deviations for changes in T_t

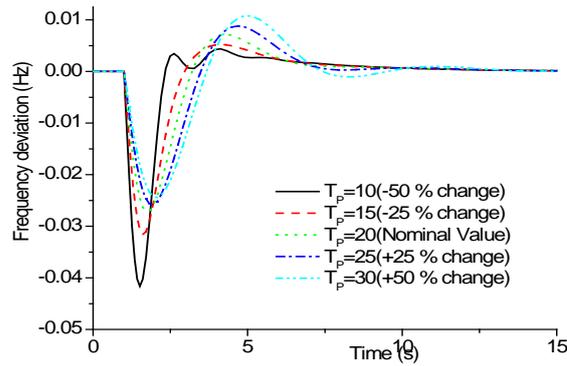


Fig.17 Comparisons of frequency deviations for changes in T_p

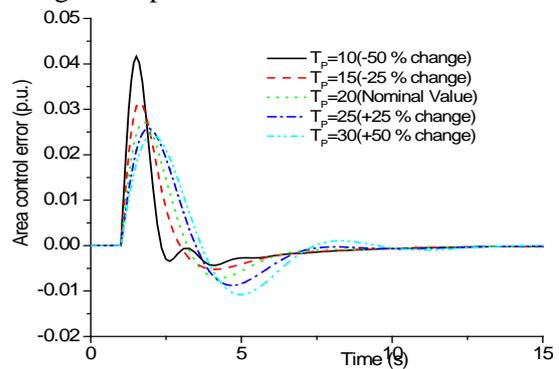


Fig.18 Comparisons of control input deviations for changes in T_p

The parameter variation test is effectively done into the tested power system and the responses are shown in figures 11 and 18. Their numerical values of PID controller gain values are tabulated in Table 3. The settling time and the maximum overshoots of the frequency deviation are shown in Table 4. From the above discussions of all three cases, it is established that ACO tuned PID controllers always give better controlled response in single area thermal power system with different cases. Table 3 Optimal Gain values of PID obtained by parameter variation test in the range of $\pm 50\%$

Parameters	% Change	Parameter Value	PID Controller Gains		
			K_p	K_i	K_d
T_g	Nominal	0.2	1	1	0.4
	-50%	0.1	1	0.97	0.3
	-25%	0.15	0.99	1	0.48
	+25%	0.25	0.99	0.99	0.57
	+50%	0.3	0.99	1	0.76
	Nominal	10	1	1	0.4
T_r	-50%	5	1	0.99	0.67
	-25%	7.5	1	1	0.66
	+25%	12.5	1	1	0.55
	+50%	15	1	0.99	0.42
	Nominal	0.3	1	1	0.4
T_t	-50%	0.15	0.98	1	0.23
	-25%	0.225	0.98	0.99	0.39
	+25%	0.325	0.98	0.98	0.64
	+50%	0.45	0.99	0.92	0.66
	Nominal	20	1	1	0.4
T_p	-50%	10	4	0.99	0.37
	-25%	15	1	1	0.46
	+25%	25	1	0.94	0.44
	+50%	30	0.98	0.96	0.42

Table 4 Settling time and maximum overshoots of the frequency deviation obtained by parameter variation test in the range of $\pm 50\%$

Parameters	% Change	Settling Time	Maximum overshoots (Hz)
		Nominal	15.23
T_g	-50%	18.20	0.0066
	-25%	17.95	0.00764
	+25%	16.23	0.00769
	+50%	17.71	0.00757
T_r	-50%	11.85	0.00727
	-25%	14.86	0.00718

	+25%	17.74	0.00624
	+50%	19.67	0.0064
T_t	-50%	16.83	0.00643
	-25%	17.16	0.00698
	+25%	14.77	0.00798
	+50%	15.69	0.00804
T_p	-50%	15.83	0.00344
	-25%	17.07	0.00516
	+25%	15.84	0.00713
	+50%	17.79	0.00888

5 Conclusion

Through this study, the Load Frequency Control (LFC) of single area re-heat thermal power system is performed using conventional and Artificial Intelligence tuning technique based controllers. The conventional tuning method is used to tune PI controller parameters and its performance is compared to the ACO tuned PID controller. The experimental simulation results reveal that ACO-PD controller provides superior controller response with/ without non-linearity in the case of 1% Step Load Perturbation (1% SLP). Moreover, the performance variation test is performed by varying the time constants (Governor, Turbine, re-heater and power system time constant) of the examined power system from its nominal value, i. e. -0% to +50% by the step of 25%. Finally, simulation result exposes that, the performance of the ACO-PID controller does not affect by the system parameters variation.

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