Study of Ferromagnetic Dispersed Samples and Particles Magnetic Susceptibility by Faraday Method (Magnetometer with Polar Hemispheres)

SANDULYAK ANNA, SANDULYAK DARIA, ERSHOVA VERA, SANDULYAK ALEXANDER, POLISMAKOVA MARIA
Moscow Technological University
20 Stromynka Street
RUSSIAN FEDERATION
anna.sandulyak@mail.ru

Abstract: - To study the magnetic susceptibility of samples (in particular, dispersed ones), a Faraday magnetometer version with pole-hemispherical tips is preferred. The approach to identification of the gradient stability zone (for positioning small samples) is shown: by the obtained coordinate characteristic of induction (meandering, with the possibility of an objective linear approximation of its section in the vicinity of the inflection) and the coordinate characteristic of the gradient – with stable gradient values in the vicinity of the extremum.

Experimental data on the magnetic susceptibility of dispersed samples containing a dispersed phase of magnetite particles revealed a nonlinear character of the dependence of their susceptibility on the volume fraction of this phase – with an almost linear initial section. The possibility of obtaining information on the susceptibility of the particles themselves using data for this region is discussed, in the range of a volume fraction up to 0.2, and to increase the accuracy of such information to 0.02-0.05 (narrowed range), where the concentration dependences of the susceptibility of dispersed samples obey an independent linear trend.

Keywords: - Faraday magnetometer, magnetic susceptibility, gradient extremum zone, dispersed sample, volume fraction of ferroparticles.

1 Introduction

Among the very popular methods for controlling the magnetic properties of various samples (in particular, magnetic susceptibility), the ponderomotive Faraday force method continues to remain for a long time. Its use by means of appropriate measuring tools (the so called Faraday balance, Faraday magnetometer), unlike other methods, makes it technically possible to solve the problem of obtaining the necessary information over a wide range of temperatures. It is important, however, that it is sufficient to use only small (often justifiably small) samples to fully implement the method [1-3]: not only continuous, but also dispersed [2-6] samples (in particular, powder). For example, this circumstance can often become the key in solving various problems of magnetophoresis and magnetocontrol of ferroimpurities of natural and technogenic mediums [7-13], when it is necessary to deal with objectively small volumes of a dispersed impurity phase of these types of environment.

The possibility of a relatively simple and fairly accurate solution of a wide range of problems, advantages, and, in some cases, preference of Faraday magnetometer in comparison with alternative, for example, widely used SQUID and vibrational magnetometers (including the possibilities of useful complementarity of data) is in [3, 14-20]. In particular, in [17] it is positioned as more sensitive than a vibrational magnetometer, and in [14, 19, 20] a good agreement (small difference) is observed between measurements by it and the SQUID magnetometer.

The variants of the working body of the Faraday magnetometer implementaton, which are responsible for the gradient field creation, being completely diverse constructively, can have three fundamental differences concerning the approach to field generation. One of them (the most common) is the use of an electromagnetic system, i.e. block, consisting of magnetization coils, cores and pole tips (of one or another form) [2, 17, 20-26]. Another approach is the use of short, able to create nonhomogeneous field, coils (solenoids) [3, 5, 15-18, 27, 28] without cores and pole tips. It should be mentioned about the approach (discussed below), connected with the use of strong permanent magnets [19, 29], capable of creating an nonhomogeneous field, in particular, of a magnet turned by the polar surface to the sample under study [19].

When using any of the Faraday magnetometers, it is always important to ensure (in the working area of the sample placement) a stable field heterogeneity, in particular, a stable gradient, i.e. *gradB* ≅ Const or *gradH* ≅ Const [20]. However, this provision has long been largely only declared, without sufficient justification for the theoretical and practical prerequisites for its implementation. At the same time, the available recommendations on the choice of the appropriate shape of the pole pieces are accompanied by comments of a predominantly general plan and are not supported by the necessary evidentiary data.

2 Stable gradient zone (for sample in study of its magnetic susceptibility): the case of Faraday magnetometer with pole-hemispheres

A necessary starting action in justifying the zone of a stable gradient (to place a sample subject to the study of its magnetic susceptibility) is to obtain in the region of the gradient field being created a coordinate (in the *x* direction: along the action of the ponderomotive force and opposite to it) of the induction *B* or the field strength *H*. Thus, in [30-32], with the example of a system with pole-hemispherical tips (Fig. 1) with a diameter of 100*mm*, an effective approach to solving this problem is shown. Having precisely such a coordinate characteristic, for example, of induction *B* (Fig. 2*a*), even by its very form it is quite possible to judge the presence and location of the zone of stable gradient values – in case this characteristic is tortuous, with a clearly visible inflection (in mathematical sense) [30-32]. Hence, the section (comparatively short) of this tortuous characteristic in the vicinity of the inflection point, which can be linearly approximated, indicates a stable (approximately constant) value in this section of the parameter *gradB*=*dB/dx*. This is well demonstrated [30-32] by the presence of an extremum on the obtained (by differentiating the characteristic *B* approximated by a fourth-degree polynomial) coordinate characteristic *gradB* (Fig. 2*b*) with the abscissa of the extremum $x=x_{extr}$, in the neighborhood of which *gradB* is practically stable.

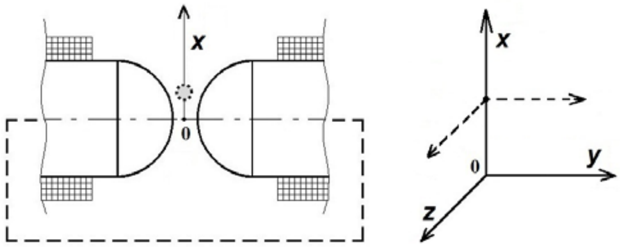
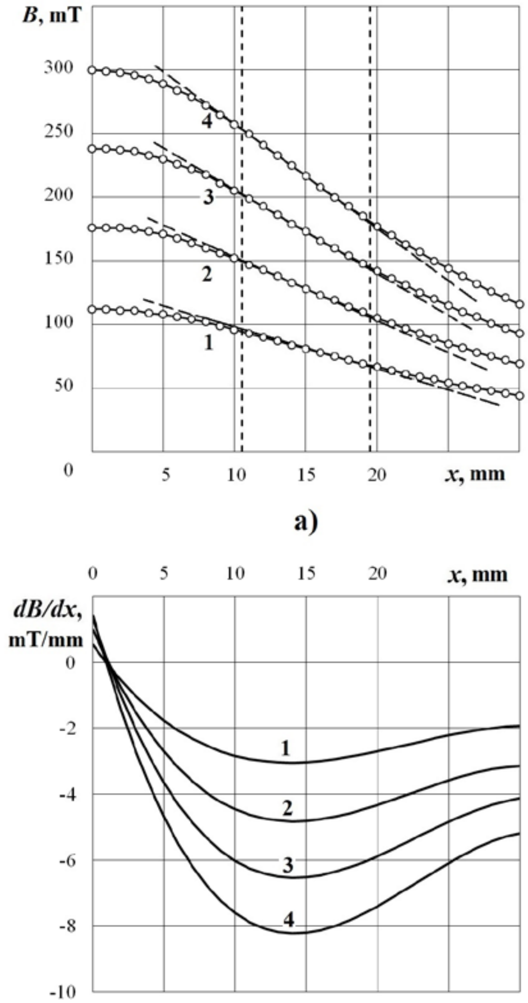


Fig.1. Illustration of pole-hemispherical tips and directions of the measuring sensor displacement: along the *x* axis lying in the plane of symmetry of the interpolar region (main axis), and also along the transverse directions *y* and *z* for values of *x* in the neighborhood of extrema *gradB*.



a)

b)

Fig.2. Coordinate characteristics between pole pieces-hemispheres: *a*) induction of the field *B*, points – experiment, lines – calculation using a polynomial of the fourth degree; *b*) gradient induction *gradB* - the result of differentiation of this polynomial; 1 – *I* = 4*A*, 2 – *I* = 8*A*, 3 – *I* = 16*A*, 4 – *I* = 30*A*; *b* = 10*mm*.

Being independent of the supply current of the windings *I*, the values of $x=x_{extr}$ depend on the mutual removal of the tips *b* (Fig. 3, 4*a*), increasing with increasing *b* according to, as seen in Fig. 4*b*, the logarithmic function [30].

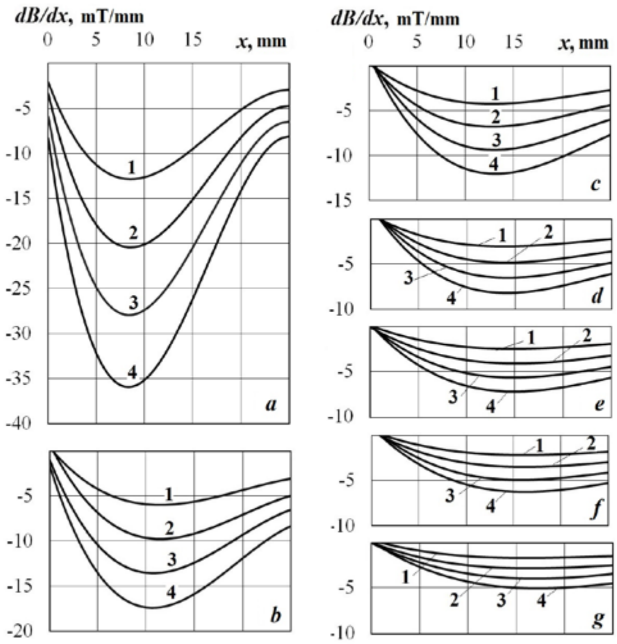


Fig.3. Coordinate characteristics of the gradient between the pole-hemispheres at their mutual removal *b*: *a*) *b* = 3.5*mm*, *b*) *b* = 6*mm*, *c*) *b* = 8*mm*, *d*) *b* = 10*mm*, *e*) *b* = 11.5*mm*, *f*) *b* = 13*mm*, *g*) *b* = 15.3*mm*; 1 – *I* = 4*A*, 2 – *I* = 8*A*, 3 – *I* = 16*A*, 4 – *I* = 30*A*.

The foregoing approach to the identification of the work area [30-32] of the (preferred) Faraday magnetometer variant with pole-hemispherical tips also makes it possible to make a conceptual comment about the feasibility of implementing a version of the Faraday magnetometer with a system based on the use of a permanent magnet (for example, Nd-Fe-B), which is turned by the pole surface to the sample under study (the normal suspension of this sample) [19] (Fig. 5). Thus, in this case [19] (Fig. 6*a*), and in other similar cases (Fig. 6*b*), each of the coordinate characteristics of the induction *B* of the created gradient field decreases monotonically with distance from the pole surface of the magnet. It is all or at least part of it is not linear, it is also not tortuous: it has no signs of an inflection, which would allow us to distinguish a section that can be linearly approximated. As a consequence: there is no zone of stable values of the parameter *gradB* = *dB/dx* - throughout the entire range of these characteristics. In other words, if we additionally obtain (by differentiating such a coordinate characteristic *B*) the coordinate characteristic *gradB*, then the necessary signs – "plateau" or extremum, which should be indicative of the presence of a zone of stable gradient values, so necessary here, will simply be absent.

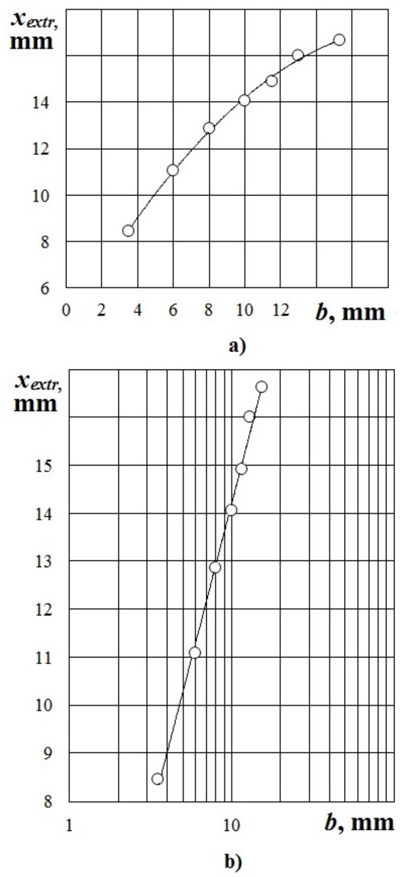


Fig.4. Dependence of the abscissa of the gradient induction extremum (the conditional center of the working zone) on the mutual removal of the pole pieces (*a*) and illustration of the quasi-linearization of these data in semilogarithmic coordinates (*b*).

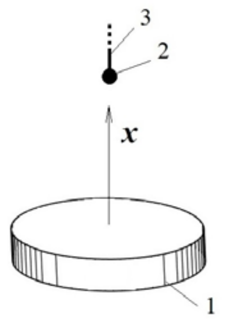


Fig.5. Variant of positioning of a permanent magnet (source of a gradient field) in a Faraday magnetometer - according to [19] (1 - magnet, 2 - sample, 3 - sample stock connected to a force meter); this option is unacceptable for obtaining characteristics with stable values of the parameter *gradB* (or *gradH*).

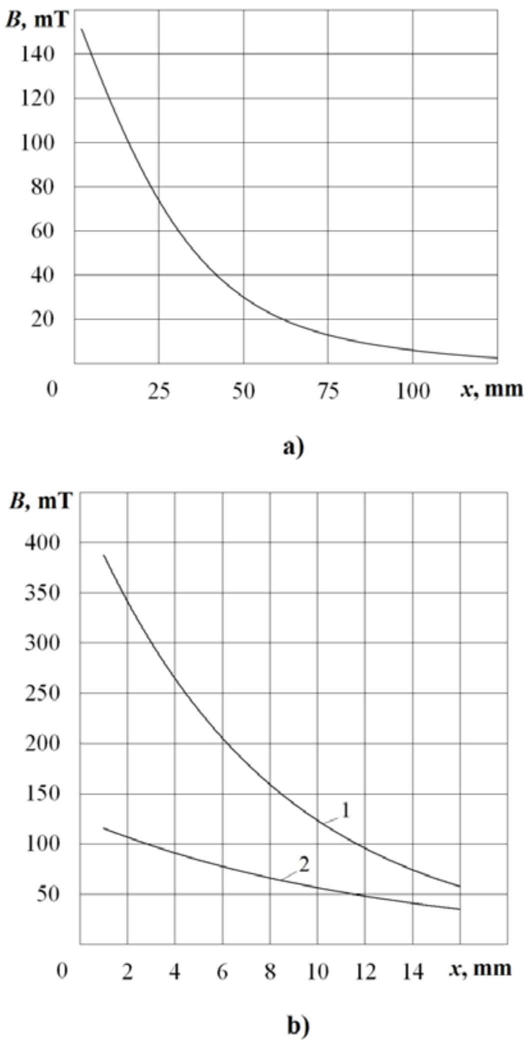


Fig.6. Change in the induction of the gradient magnetic field produced by a permanent magnet, as it moves away from its pole surface: *a*) according to the data of [19] for a Nd-Fe-B magnet, *b*) the obtained data for a stronger (1) and relatively weak (2) magnets.

3 Estimation of 3D gradient zone

The data in Fig. 2,3 shows the location and longitudinal dimension of the gradient stability zone, namely along the central axis *x* lying in the symmetry plane of the interpolar region. Such a model (one-dimensional) requires expansion: it is equally important to have the corresponding data on the detected zones in the transverse direction, i.e. perpendicular to the *x*-axis (in order to have information on the corresponding restriction not only of the longitudinal but also of the transverse dimension of the sample being studied).

Fig. 7 shows such characteristics *B* obtained by step-by-step moving the measuring sensor from the characteristic points on the central axis *x* (from the points *x* within the stability zones of the parameter $\partial B/\partial x$) in the transverse directions *y* and *z* (Fig. 1).

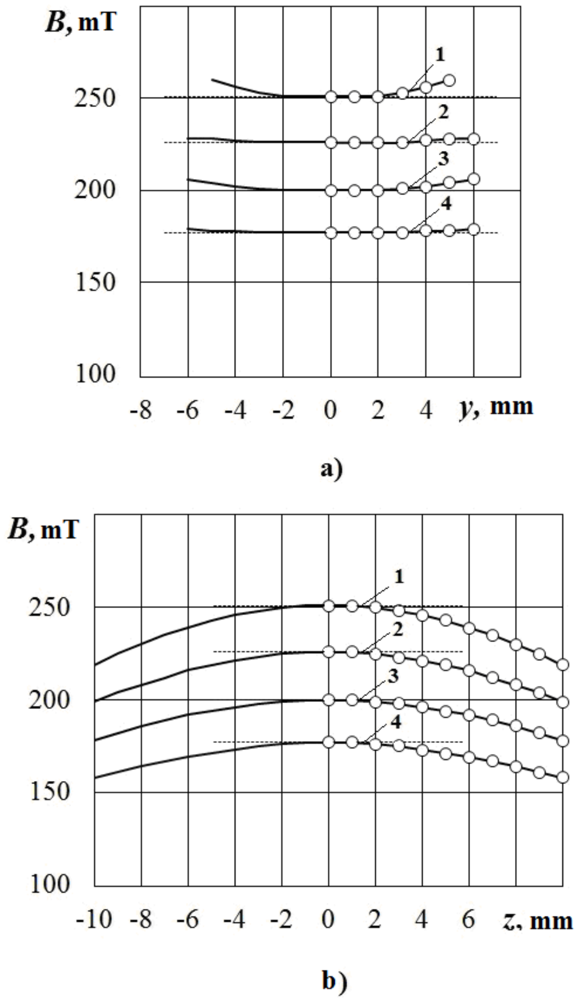


Fig.7. Coordinate characteristics of the induction of the field along the directions *y* (*a*) and *z* (*b*) for certain values of *x* in the neighborhood of the stable values of the parameter $\partial B/\partial x$; I = 30A; 1 - x = 11mm, 2 - x = 14mm, 3 - x = 17mm, 4 - x = 20mm.

The very form of these dependences (shown in Fig. 7, with allowance for symmetric branches), namely the extreme one, indicates the presence of a "plateau" of the parameter *B* in the directions y and *z* for the selected values of *x*. This means that the zone of stable values of $\partial B/\partial x$ is localized not only strictly on the central axis *x*; this zone extends to a certain space in the neighborhood of this axis (along the *y* and *z* directions), thereby ensuring the presence of a 3D zone of stable gradient values. In this case, the transverse dimensions of the zone-volume are quite comparable with the *x*-axis dimensions established above, which indicates the possibility of conducting studies with samples (on the study of their magnetic susceptibility), for example, a spherical shape up to 5*mm* in diameter here.

The obtained results, in particular, shown in Fig.7, also allow us to substantiate frequently used simplifications (in relation to the problem under consideration) about such a used concept as the gradient of the field *gradB* (or *gradH*). Thus, within the zones of practical stability of the parameter $\partial B/\partial x$, the frequently occurring partial derivatives $\partial B/\partial y$ and $\partial B/\partial z$ (near the central axis *x*, small distances *y* and *z*, which are acceptable for studying the magnetic properties of a small volume sample) can be neglected: $\partial B/\partial y \rightarrow 0$ and $\partial B/\partial z \rightarrow 0$. Then it is quite right to suppose (as it is often done): $\partial B/\partial x = dB/dx = gradB$ (or $\partial H/\partial x = dH/dx = gradH$).

4 Role of polar hemispheres diameter

As follows from Fig. 2-4 the coordinates of the extremum of the gradient x_{extr} are obtained using the example of half-sphere tips with a diameter D = 100 *mm*.

It is important to study using hemispherical tips of an another diameter, which would make it possible to compare the corresponding coordinates of the gradient stability zones. This is necessary mainly to determine the eligibility of operating relative (as more universal) parameters: b/D and x_{extr}/D.

Fig. 8 shows the coordinate dependences of the induction of the field *B* in the region between the pole-hemispherical tips with a diameter D = 135*mm* when they are disposed of, in particular, b = 4.7*mm*. The supply current of the winding *I* varied from 4*A* to 30*A*. It can be seen that each of the induction curves *B* has an inflection (in the vicinity of which, as before, its section can be linearized), which indicates the presence of an extremum of the coordinate characteristic of the gradient dB/dx (Fig. 9), and for various *b* - from 4.7 *mm* to 17.6 *mm*, with the corresponding values of the coordinates of the extremum x_{extr}.

As for the effect on x_{extr} of the mutual removal *b* of the hemispherical tips, at much larger x_{extr}-values on Fig. 10 (than x_{extr}-values earlier for D = 100*mm* - see Fig. 4*a*), it is noteworthy that the data obtained in relative coordinates (x_{extr}/D from b/D) are generalized (Fig. 11). This indicates the universality of the approach to identifying zones of stability (gradient and force factor) between pole pieces of exactly this (spherical) shape, regardless of their diameter.

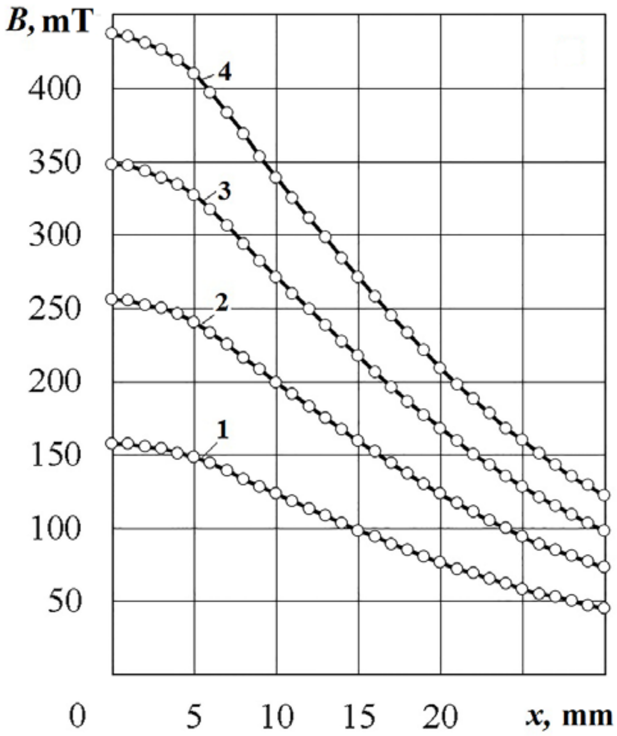


Fig.8. Coordinate characteristics of field induction between pole-hemispherical tips of diameter D = 135 *mm*, points - experiment, lines - calculation using a polynomial of the fourth degree; 1 - I = 4*A*, 2 - I = 8*A*, 3 - I = 16*A*, 4 - I = 30*A*; b = 4.7*mm*.

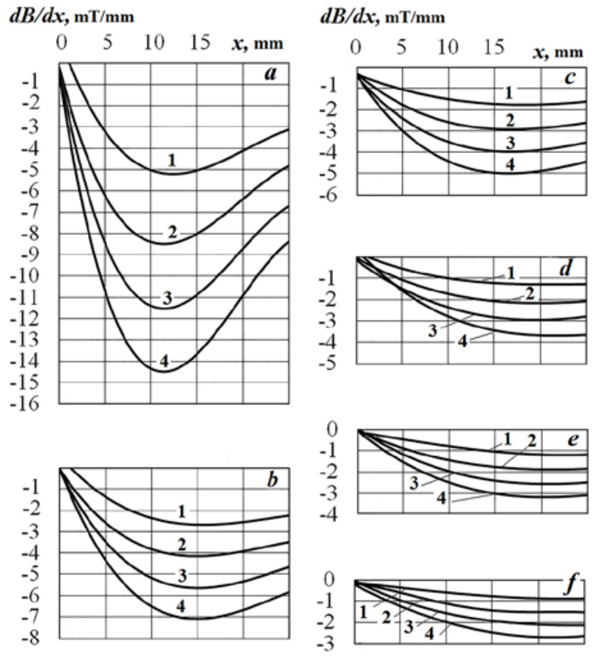


Fig. 9. Coordinate characteristics of the gradient between pole-hemispherical tips of diameter D = 135 *mm* at their mutual removal *b*: *a*) b = 4.7*mm*, *b*) b = 8.1*mm*, *c*) b = 10.8*mm*, *d*) b = 13.5*mm*, *e*) b = 15.5*mm*, *f*) b = 17.6*mm*; 1 - I = 4*A*, 2 - I = 8*A*, 3 - I = 16*A*, 4 - I = 30*A*.

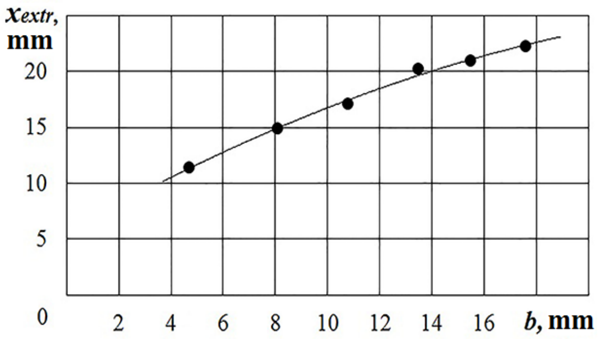


Fig. 10. Dependence of the abscissa of the induction gradient extremum (conditional center of its stability zone) on the mutual removal of pole-hemispherical tips; D = 135*mm*.

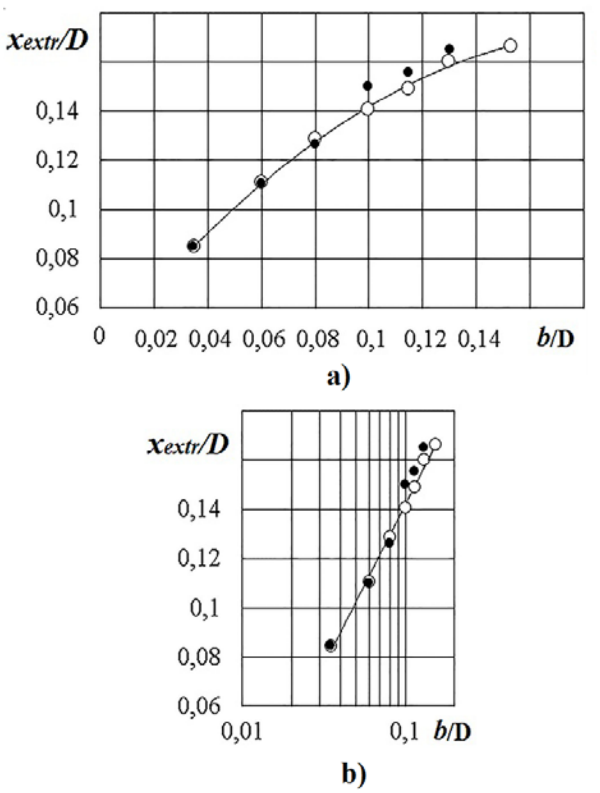


Fig. 11. Influence of relative values of mutual removal of pole-hemispherical tips on the relative coordinate of the induction gradient, ○ - D = 100 mm (according to Fig. 4), • - D = 135 mm (according to Fig. 10); *a*) at simple (nonlogarithmic) coordinates, *b*) in semilogarithmic coordinates the data are quasilinearizable, which indicates their logarithmic form.

5 Data on dispersed samples magnetic susceptibility as function of dispersed phase volume fraction

Dispersed phase of various technological and natural media often features some ferroparticles that possess ferro- and ferromagnetic properties. It is imperative to obtain the data on magnetic susceptibility χ such particles, in particular, in solving many applied scientific tasks of magnetophoresis and/or magnetic control, especially in analysis criteria magnetic capture of such particles [10, 33, 34].

An approach to obtaining such χ data for the given purpose could be based on data of the susceptibility $\langle\chi\rangle$ of the disperse sample (in particular, suspension, colloid, powder) containing such particles. In this case data of the susceptibility $\langle\chi\rangle$ could be obtained by using ponderomotive Faraday method (it is enough having a small volume of sample), data of the susceptibility χ – by using a necessary relation:

$$\chi = \langle\chi\rangle / \gamma , \qquad (1)$$

where γ – volume fraction (concentration) of ferroparticles in sample. We should mention specially, that the relation (1) is true only under condition of sufficient mutual distancing of the ferroparticles in the test specimen (when we virtually exclude any mutual magnetic impact), i.e. with relatively small values of the volume fraction γ.

The issue of allowable value of $\gamma = [\gamma]$ (allowable for this calculation of χ) should be admitted as controversial. Thus, alongside with the somewhat standard concept of rigid limitation of γ to $\gamma = [\gamma]$ = 0.02-0.05 $\cong$ 0.035 (it was mentioned at researches undertaken by Kondorskiy E.I., Dikanskiy Y.I. and systematized at [35]), there are noteworthy values of the so-called demagnetising factor of the disperse (grain) medium sample [36], that are almost zero with $\gamma \leq$ 0.2-0.25. Since this factor can be justly considered as an 'indicator' of magnetic interaction of ferroparticles in the disperse sample, we cannot exclude a possibility of a radically different conceptual assumption of a less rigid limitation of γ: to $\gamma = [\gamma]$ = 0.2-0.25.

The most objective data to solve this debatable issue would be the data of direct experiments on γ impact on $\langle\chi\rangle$, at that values $\gamma = [\gamma]$ are assumed to be the criterion value of γ, which restrict the directly proportional relation of $\langle\chi\rangle$ with γ.

Largely, this problem was essentially considered as far back as in papers [35, 37] by systematization the concentration dependences of $\langle\chi\rangle$ for powders and colloids with a disperse phase of the magnetite particles, which are obtained by a Kondorskiy E.I., Bibik E.E., Chekanov V.V., Grebnev S.K. et al. As exemplified by these dependences (for various ranges of γ with the general range of 0.1 – 0.85 and for selected different values of H at range from 25 to 520kA/m), these researches [35, 37] demonstrated that those concentration dependences $\langle\chi\rangle$, which cover quite wide ranges of γ (no limited by values up to γ = [γ] = 0.25), are far from being directly proportional. They are approximated by a power function:

$$\langle\chi\rangle \sim \gamma^n \tag{2}$$

with a degree of n = 1.1-1.3. Therefore, they are inapplicable for solution of the given task to define values χ with the use of the explicitly stated relation (1).

At the researches [35, 37] which was made at the range of γ – from γ = 0,065 to γ = 0.2-0.25 (disperse phase is magnetite) dependences $\langle\chi\rangle(\gamma)$ is near to linear at logarithmic coordinates (Fig.12*a*); average value of degree n really equals $n \cong 1.1$ (by the results of processing the data in MS Excel). So, these dependences at this range of γ [35, 37] are near to linear ($n \cong 1$). This could easily be verified by illustration of these dependences at simple (nonlogarithmic) coordinates (Fig.12*b,c*).

These data point to the possibility of limitation γ-values (in the case using $\langle\chi\rangle$ data for obtaining χ data by (1)) by value $\gamma \leq [\gamma]$= 0.2-0.25 at least. Corresponding experimental data (for extended range of γ) are essential for detection visible change from linear dependence $\langle\chi\rangle(\gamma)$ to nonlinear.

6 Detection of «kink» in a characteristic curve $\langle\chi\rangle(\gamma)$ of disperse sample by experimental method

Conclusion about possible criterion value of γ (γ=[γ]=0.2-0.25) could be proved by special researches (Fig.13). It is necessary to mix define mass of dispersed phase (magnetite) with milled sand to obtain one or another γ value, taking into account mass of mix, its volume in sample and density of particle's material. Test loose sample is situated at the tank with near spheric shape (its diameter is 5mm) between spheric pole pieces at Faraday balance to obtain data of magnetic susceptibility $\langle\chi\rangle$ of sample [30].

Results of measuring a ponderomotive force F and following determination of magnetic susceptibility $\langle\chi\rangle$ of sample are shown in Fig.13 at the range of magnetite volume fraction up to $\gamma \cong$ 0.3 and field intensity H= 84-141kA/m. It is clear that the linear character of dependences between F (and $\langle\chi\rangle$) and γ at the selected range of H is kept while $\gamma \cong 0.2$. Estimated by (1) magnetic susceptibility values of test magnetite particles are χ = 0.9-0.8 (by using data $\langle\chi\rangle$ and γ < 0.2).

Thus, acceptable value of γ could equals γ = [γ] = 0.2 (for estimating χ by (1)). It agrees with the result that is obtained by analysis data [35, 37] on Fig.12*b,c*.

Having these results (Fig.12*b,c*) detailed analysis of this vast range of dependences $\langle\chi\rangle(\gamma)$ is interesting for both ranges: obtained extended range of $\gamma \leq 0.2$ and recommended mentioned earlier rather narrowed range of $\gamma \leq$ 0.02-0.05.

7 The analysis of dependences between magnetic susceptibility and concentration of disperse sample for extended acceptable range of γ

One more confirmation to possibility of using data $\langle\chi\rangle$ (besides Fig.12*b,c*) at the range of $\gamma \leq$ 0.2–0.25 to obtain χ data by (1) is the artificial linear generalization family of dependences $\langle\chi\rangle(\gamma)$ (Fig. 14). This generalization is made in specially created coordinates and it is a result of the data approximation as a direct proportional relationship (in particular, in MS Excel) by using data of Fig. 12*b,c*:

$$\langle\chi\rangle = k_H \times \gamma, \tag{3}$$

then defining particular values (for some dependences with inherent for them field intensity H value) of the proportionality coefficient k_H (Table, $\gamma \leq 0.25$) that enters formula (3); k_H values are necessary for obtaining such parameter (by ordinate in Fig.14) of generalization as relation $\langle\chi\rangle/k_H$.

As expected, this generalization dependence between $\langle\chi\rangle/k_H$ and γ (Fig.14) obeys the right angle bisector for the chosen generalization coordinates in the range studied γ = 0.065-0.25 (Fig. 12*b,c*). Thus, we can conclude about the eligibility of artificial linearization of dependences of $\langle\chi\rangle$ on γ in the range $\gamma \leq$ 0.2-0.25 and, therefore, now the substantiated conclusion on the data $\langle\chi\rangle$ belonging to the range of $\gamma \leq$ 0.2-0.25 can really be used for acquiring the calculated data χ by (1).

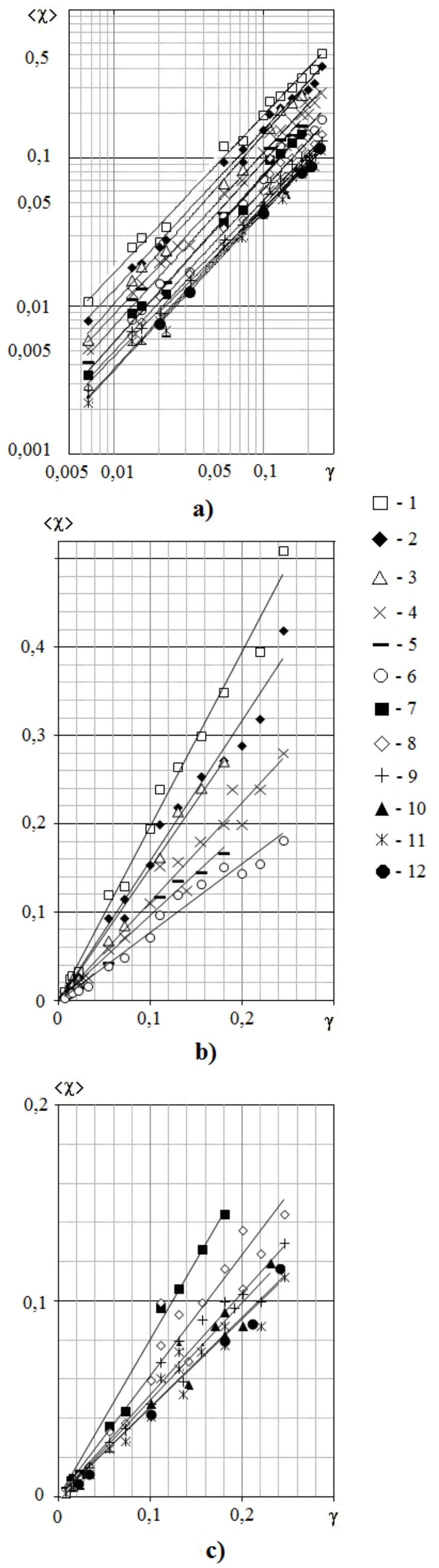


Fig.12. Magnetic susceptibility of the disperse medium $\langle\chi\rangle$ [35, 37] depending on the γ volume fraction of the disperse phase (magnetite) at various values of field intensity H; 1 – H = 90kA/m, 2 – 150, 3 – 190, 4 – 220, 5 – 270, 6 – 340, 7 – 365, 8 – 420, 9 – 550, 10 – 650, 11 – 730, 12 – 780; *a*) at logarithmic coordinates, *b*), *c*) – the same at simple (nonlogarithmic) coordinates.

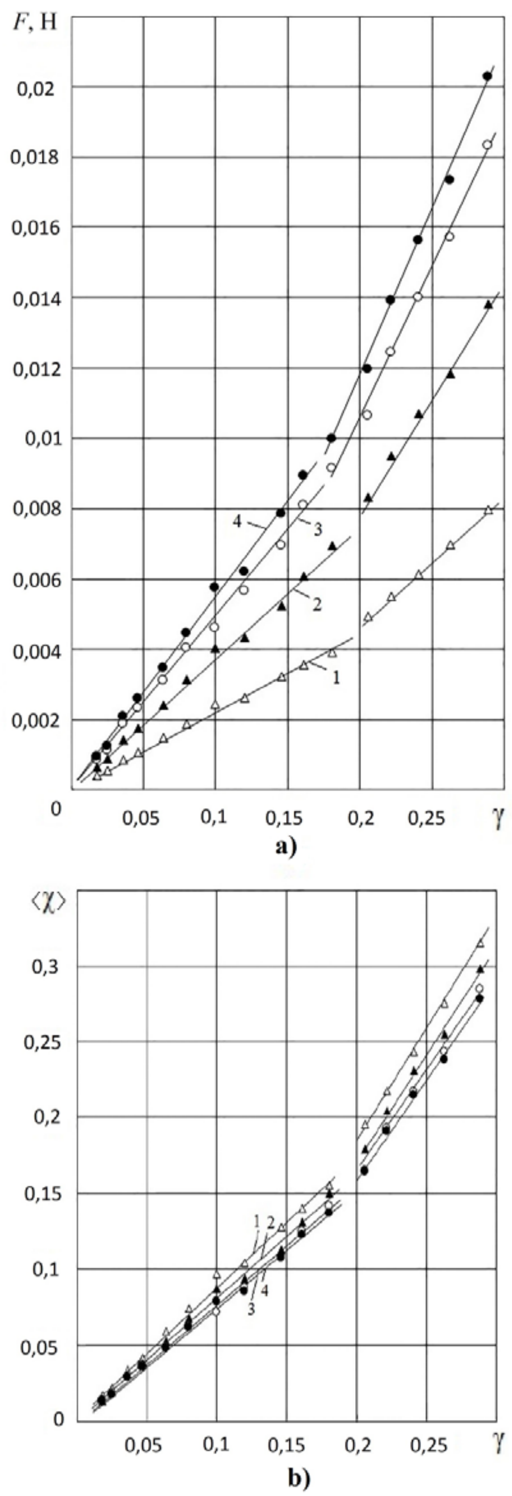


Fig. 13. Influence volume fraction γ of magnetite disperse phase on parameters: *a*) ponderomotive force F which acts on spheric sample (at nonuniform magnetic field of Faraday balance); *b*) magnetic susceptibility $\langle\chi\rangle$ of the disperse medium; average field intensity H values at sample positioning: 1 – H = 84kA/m, 2 – 113, 3 – 133, 4 – 141.

Table. Proportionality coefficient k_H of the dependences in Fig. 12*b,c* for their linear approximation case according to (3).

№№	*H, kA/m*	Values k_H	
		with $\gamma \le 0.25$	with $\gamma \le 0.035$
1	90	1.968	1.603
2	150	1.579	1.286
3	190	1.506	1.136
4	220	1.119	0.963
5	270	0.964	0.749
6	340	0.771	0.618
7	365	0.806	0.599
8	420	0.619	0.541
9	550	0.522	0.469
10	650	0.497	0.376
11	730	0.459	0.369
12	780	0.453	0.388

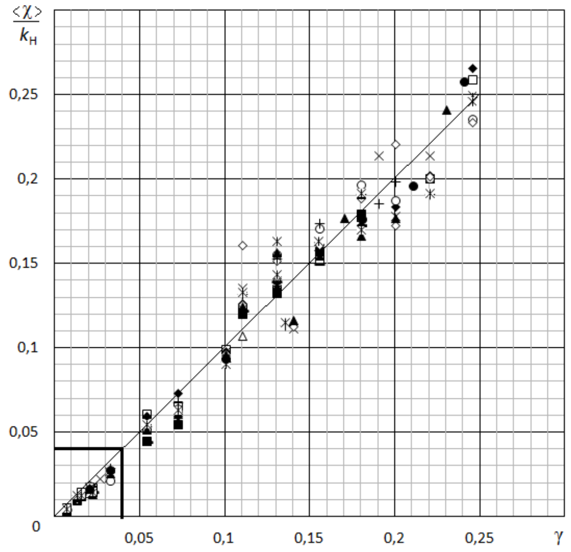
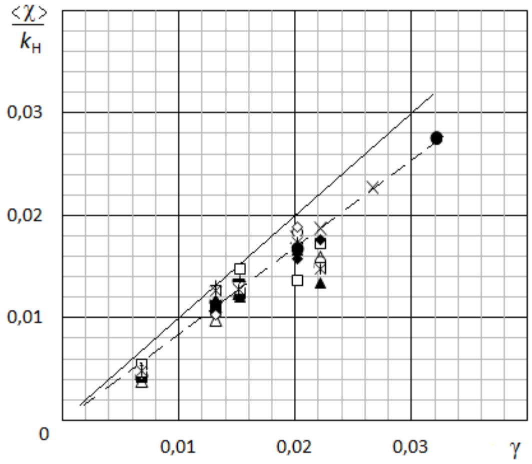


Fig.14. Generalization of data in Fig.12*b,c* for the range $\gamma \le 0.25$: approximation with the use of coefficient k_H values for $\gamma \le 0.25$ (Table) correlates to the bisector of the right angle of the chosen coordinates system.

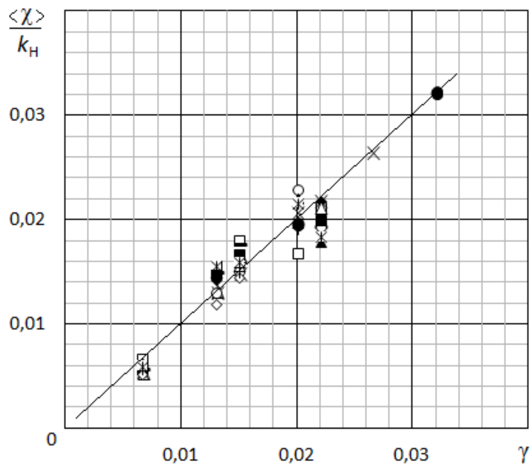
8 The analysis of concentration dependences of disperse sample for a narrowed range of γ

A vast data array (Fig. 12*b,c*) allow to analyse dependences not only for extended acceptable range of γ ($\gamma \le 0.2$-0.25), but for a narrowed range of γ ($\gamma \le 0.035$) mentioned earlier. The issue of allowable value range of γ, where dependence $\langle \chi \rangle (\gamma)$ is near to linear, is still controversial.

If we employ the corresponding data of $\langle \chi \rangle$ which belong to the range of $\gamma \le 0.035$ in Fig. 12*b,c* or in Fig.14 (limited by heave lines area), then their linear generalization as it was done the one performed before with the use of values k_H (Table, $\gamma \le 0.25$) – does not yield the desired result. Thus, Fig.15*a* clearly illustrates that the approximation of data $\langle \chi \rangle$ (dashed line) does not correlate to the right angle bisector of the chosen system of coordinates (solid line). It means that linear concentration dependences $\langle \chi \rangle$ obtained at $\gamma \le 0.035$ differ from the same linear dependences of $\langle \chi \rangle$ received at $\gamma \le 0.25$. Values of the coefficient k_H verify this fact.



a)



b)

Fig. 15. Generalisation of data in Fig. 12*b,c* for the range $\gamma \le 0.035$ (limited by heavy lines area in Fig.14): approximation with the use of coefficient k_H values (Table) for $\gamma \le 0.25$ (*a*) – does not correlate to the right angle bisector of the chosen coordinates system; the same for $\gamma \le 0.035$ (*b*) correlates to the right angle bisector of the chosen coordinates system.

Previous values of the proportionality coefficient k_H (Table, $\gamma \leq 0.25$) are not quite applied to the range of $\gamma \leq 0.035$. They are defined separately by the afore-described procedure based on the data of $\langle\chi\rangle$ which belong to this narrowed range of γ. In this case k_H (Table, $\gamma \leq 0.035$) differ from the previous values of coefficient k_H (Table, $\gamma \leq 0.25$) on average by 18%.

The respective generalization of the data $\langle\chi\rangle$ belonging to the narrowed range of γ with the account for the newly acquired values of coefficient k_H are given in Fig.15*b*. The approximation of the results of this generalisation corresponds to the bisector of the right angle of the chosen coordinate system.

Thus both variants may be acceptable (for obtaining magnetic susceptibility data χ of ferroparticles by expression (1)): the first – range of $\gamma \leq 0.2$, the second – range of $\gamma \leq 0.02$-0.05 (as more accurate result).

7 Conclusion

The example of spherical pole tips (the Faraday magnetometer), between which the coordinate characteristic of the field induction is tortuous (thereby guaranteeing the presence of an extremum zone of the coordinate characteristic of the gradient and its practically stable value in the neighborhood of the extremum). Thus, a working zone has been identified, within which a small sample should be placed to study its magnetic susceptibility. It is shown that for each of the distances between the tips, the coordinates of the extremum of the gradient remain very close to each other irrespective of the current load, which indicates the possibility of constant positioning of the sample when the research regimes change. As the distance between the tips increases, these coordinates increase logarithmically.

It is shown that, having the data of the susceptibility $\langle\chi\rangle$ of a dispersed sample containing these particles, it is possible to obtain data on the magnetic susceptibility χ of the ferroparticles of its dispersed phase (which is necessary, in particular, for solving many applied magnetophoresis and / or magneto-control problems of these particles). An obligatory condition is to provide relatively small values of their volume fraction (concentration) - up to a certain (admissible, in fact - criterial) value of this share, which is currently considered to be controversial. Direct experiments with powder samples in the range of the volume fraction of the dispersed magnetite phase $\gamma \leq 0.3$ showed that the linear dependence portion $\langle\chi\rangle$ on γ corresponding to the condition described above (the subsequent determination of χ) is observed when $\gamma \leq 0.2$-0.25. At the same time, at lower values of $\gamma \leq 0.02$-0.05, an "individual" (with respect to the proportionality coefficient somewhat different from the previous one), a linear link of $\langle\chi\rangle$ and γ, can be detected, indicating the possibility of obtaining more accurate values of χ in principle.

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