Model Based Automatic Regulating Time Series for Feedback Control Systems

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Abstract: - In this literature, a model based nonlinear digital control scheme is proposed for analyses and designs of stable feedback control systems. It is derived from the converging characteristic of two specified numerical time series. The ratios of neighborhoods of the series are formulated as a function of the output of the plant and the reference input command, and will be converged to be unities after the output has tracked the reference input command. Lead compensations are found by another numerical time series to speed up the system responses on the on-line adjusting manner and matching the reference model. A servo system, a time-delay system, a high order system and a very high order system are used to show effectiveness of the proposed nonlinear digital controller. Comparisons with other conventional methods are also made.

Key-Words: - Discrete time series, Model based auto tuning, Nonlinear control

1 Introduction

For unit feedback discrete-time control systems, the control sequences are usually functions of the difference between the sampled reference input and output of the plant [1-5]. The discrete-time control sequence can be generated by Finite Impulse Response (FIR) filter or Infinite Impulse Response (IIR) filter. The input of FIR or IIR filter is the difference between the sampled reference input and output of the plant. The output of FIR or IIR will be the input of the plant. In general, they are linear controllers.

A nonlinear discrete-time control sequence described by periodic numerical series $G(jT_s)$ for analyses and designs of industry processes was developed by Tsay [6]. T_s represents the sampling interval. The ratios of $G((k+1)T_s)$ to $G(kT_s)$ of the series are formulated as a function of the reference input command and the output of the plant. The value of $G(kT_s)$ is the control input of the plant $G(jT_s)$. The output of the plant tracks the reference input command exactly after ratios $G((k+1)T_s)$ / $G(kT_s)$ of the series being converged to unities. It implies that $G(kT_s)$ will be converged to a steady state value for a constant reference input applied. Another time series was used to find lead compensators for speeding up the systems performance in the off-line manner. It gives good results for considered system. However, on-line computing and control is usually expected. This is the motivation of this paper.

In this literature, reference models will be used to find lead compensators in the on-line manner. The parameter of the lead compensator is found by another time series which is a nonlinear function of outputs of plant and the reference model. It will be seen that the proposed method gives an easy and effective way for the considered system.

In following sections, basic concepts of the proposed nonlinear discrete-time control sequence is discussed first, and then a servo system, a time-delay system, a high order system and a very high order system are used to illustrate their tracking behavior and performance. Simulating results will show that the proposed nonlinear digital controller gives another possible way for analyses and designs of industry processes.

2 The Basic Approach

2.1 The model reference control configuration

Fig.1 shows the proposed model reference control configuration in which auto regulating controller and auto tuning algorithm are two nonlinear auto regulating time series. The auto regulating controller is function of the reference input R and output of plant. The auto tuning algorithm is function of outputs of plant and reference model. It adjusts the parameter of the lead compensator. Note that the computing period of auto tuning algorithm is slower than that of the auto regulating controller. It is similar to the adaptive control. The ideas of the time series will be discussed in following subsections.

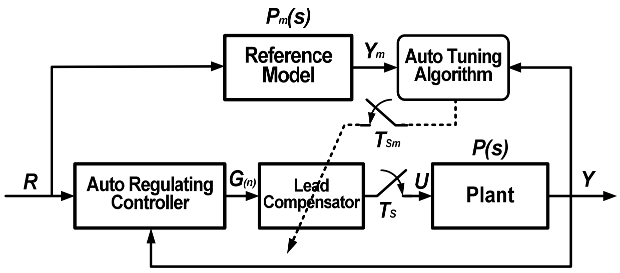

Fig.1 Auto Regulation Control Configuration.

2.2 Automatic regulation time series for feedback control system

A numerical series with time interval T_S [1-6] can be written as in the form of

$$G(jT_S),\ j = 1,2,3,...,n,n+1,..., \tag{1}$$

where $G(jT_S)$ represents a constant value between time interval from $(j-1)T_S$ and $j\,T_S$. For simplicity, the representation of $G(jT_S)$ will be replaced by $G(j)$ in following evaluations. The ratios $G(j+1)\,/\,G(j)$ of the series are defined as in the form of

$$F(j) = G(j+1)/G(j),\ j = 1,2,3,...,n,n+1,.... \tag{2}$$

Eq.(2) gives the value of $G(n+1)$ approaches to be a constant value when the value of $F(n)$ approaches to be unity. Now, the problem for closing the considered system is to find the formula of $F(j)$ which is the function of the reference input command *R* and the output of the plant *Y*. $G(n+1)$ is used as the input of the considered system. Considering a series given below:

$$G(n+1) = \left[\sum_{i=0}^{m} a_i \left(R(n)/Y_S(n)\right)^i\right] G(n) \tag{3}$$

where $R(n)$ represents the reference input command and $Y_S(n)$ represents the non-zero sampled output of the plant at the sampling interval $n\,T_S$. Note that this non-zero constraint will be removed later by level shifting. Eq.(3) is a possible way to close the considered system as a sampled-data feedback control system. Assume the reference input command has been tracked by applying control effort $G(j)$, Eq. (3) becomes

$$G(n+1) = \sum_{i=0}^{m} a_i G(n) \tag{4}$$

For steady-state condition, $G(n+1)$ approaches to be a constant value, it gives

$$\sum_{i=0}^{m} a_i = 1 \tag{5}$$

Rearranging Eq.(3) and taking the derivative of it with respect to $Y_S(n)/R(n)$, one has

$$F(n) = \sum_{i=0}^{m} a_i \left(\frac{Y_S(n)}{R(n)}\right)^{-i} \tag{6}$$

and

$$\frac{\partial F(n)}{\partial (Y_S(n)/R(n))} = -\sum_{i=0}^{m} i a_i \left(\frac{Y_S(n)}{R(n)}\right)^{-1-i} \tag{7}$$

The sufficient but not necessary condition for Eq.(7) less than zero is $a_i > 0$ for $Y_S(n)/R(n) \cong 1$ and Eq. (6) can be rewritten as in the form of

$$F(n) = \sum_{i=0}^{m} a_i \left|\frac{Y_S(n)}{R(n)}\right|^{-i} \tag{8}$$

$a_i > 0$ will be used in the following evaluations. Negative value of Eq.(7) represents the closed-loop system using Eq.(3) activated as a negative feedback system around the equilibrium condition; i.e., $Y_S(n) = R(n)$. This statement will be illustrated and discussed by a graph in the next paragraph. The first order polynomial described in Eq.(3) can be written as in the form of

$$G(n+1) = \left[\beta\left|\frac{R(n)}{Y_S(n)}\right| + 1 - \beta\right] G(n)\,; \tag{9}$$

where β satisfies constrains stated above and becomes an adjustable parameter. Thus, the ratios $F(n)$ becomes

$$F(n) = \beta\left|\frac{R(n)}{Y_S(n)}\right| + 1 - \beta \tag{10}$$

$F(n)$ can be called as "Regulation Function" also. Taking the derivative of Eq.(10) with respect to $Y_S(n) = R(n)$, one has

$$\frac{\partial F(n)}{\partial (Y_S(n)/R(n))} = -\beta\left(\frac{Y_S(n)}{R(n)}\right)^{-2} \tag{11}$$

For negative value of Eq.(11), the value of β must be greater than zero. This implies the range of β is $0 < \beta < 1$. The suitability of the proposed nonlinear adaptive digital controller is based on this negative regulation characteristic. Fig.2 shows ratios $F(n)$ versus $R(n)/Y_S(n)$ represented by Eq.(9) for β = 0.9, 0.7, 0.5,0.3 and 0.1, respectively.

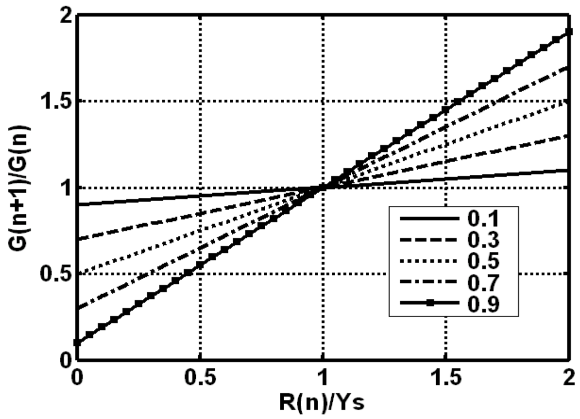

Fig.2. G(n+1)/G(n) Versus R/Y_S for β=0.9, 0.7, 0.5,0.3 and 0.1.

Fig. 2 shows that the value of $F(n)$ is less than one for that of $Y_S(n)$ greater than that of $R(n)$, then the value of $G(n+1)$ will be decreased; and the value of $F(n)$ is greater than one for that of $Y_S(n)$ less than that of $R(n)$, the value of $G(n+1)$ will be increased. This implies that the controlled system connected using Eq.(9) will be regulated to the equilibrium point ($Y_S(n)/R(n)=1$) and gives a negative feedback control system for deviation from the equilibrium point. From Fig.2, it can be seen that one can adjust β to get desired regulating slope; i.e., regulating characteristic.

The constraint of non-zero $Y_S(n)$ can be removed by $R(n)/Y_S(n)$ of Eq.(9) replaced by $(R(n)+Y_O)/(Y_S(n)+Y_O)$. Y_O is a positive value for negative value of *R(n)* only and represents the negative maximal control swing, The modified equation of Eq.(9) becomes

$$G(n+1)=\{\beta\left|\frac{R(n)+Y_o}{Y_S(n)+Y_o}\right|+1-\beta\}G(n)\,. \qquad (12)$$

Eq.(12) implies ratios $G(n+1)/G(n)$ are in the form of

$$F(n)=[\beta\left|\frac{R(n)+Y_o}{Y_S(n)+Y_o}\right|+1-\beta],\ \text{n}=1,2,3,.,j,j+1,.. \qquad (13)$$

Control inputs of the plant are in the form of

$$u(n+1)=G(n+1)-Y_o/P(0) \qquad (14)$$

for the negative swing control using positive values of β, $G(n)$ and $F(n)$. Eq.(13) gives negative regulation characteristics also for $R(n)=Y_S(n)$ is corresponding to $R(n)+Y_O=Y_S(n)+Y_O$.

An adaptive value of Y_o can be selected at $|R(n)|$ for the system is well controlled. Then Eqs.(13) and (14) can be rewritten as

$$G(n+1)=\left\{\beta\left|\frac{R(n)+|R(n)|}{Y_S(n)+|R(n)|}\right|+1-\beta\right\}G(n) \qquad (15)$$

and

$$u(n+1)=G(n+1)-|R(n)|/P(0) \qquad (16)$$

respectively. The maximal value of $G(n)$ can be limited by an adaptive constraint $|R(n)+|R(n)||$ to minimize the control effort. The control input U of the plant is now described by Eq.(16).

Considering a illustrating example [6] represented by the following transfer function

$$P(s)=\frac{30}{s^2+10s+30} \qquad (17)$$

DC gain of $P(s)$ is unity. The sampling period T_S is selected to be equal to 0.1 second for illustrating variations of $G(n)$ and $F(n)$. Time responses of the overall system using the nonlinear digital controller for $\beta=0.2$, $Y_o=|R(n)|$ and $C(z)=1$ are shown in Fig.3. Magnitudes of reference inputs between 0 and 5 seconds are equal to 1; between 5 and 10 seconds are equal to -0.7, between 10 and 14 seconds are equal to 0.5, and between 14 and 17 seconds are equal to -0.3, in which gives reference input $R(n)$ (dash-line), output Y(solid-line), Time series $G(n)$ (dotted-line), and ratios $F(n)$ (dash-dotted-line) of $G(n)$. Fig.3 shows that all values of $G(n)$ and $F(n)$ are positive while the value of output Y tracking the negative value of the reference input $R(n)$. The value of $R(n)$ can be positive or negative. Fig.3 shows also that ratios $F(n)$ are converged to be unities quickly; i.e., the controlled output tracks the reference input quickly. The proposed method gives a good performance and zero steady-state error without integration.

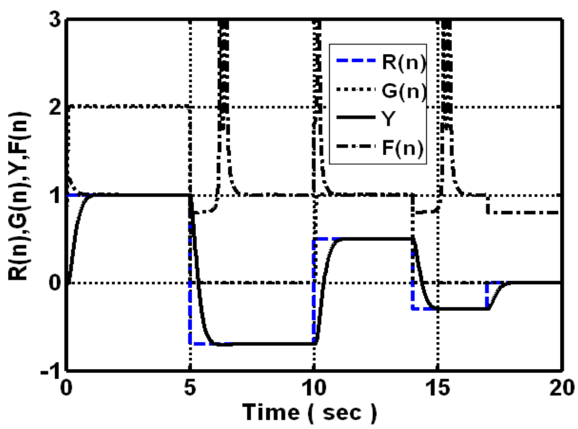

Fig.3. Time Responses of the illustrating example for β=0.20 and Ts=0.01sec.

2.3 Auto-tuning Phase Lead Compensation

A conventional digital filter $C(z)$ can be applied for filtering $G(n)$, if it is necessary. In general, phase-lead is used for speeding up the time response. The first order phase lead can be expressed as

$$C(z) = \left. \frac{T_n s + 1}{T_n s / \rho + 1} \right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \qquad (18)$$

for $\rho > 1$. The parameter T_n can be found by another numerical time series. It is

$$W(n+1) = \left[\eta \left(\frac{Y_{ms} + Y_o}{Y_s + Y_o} \right)^j + 1 - \eta \right] W(n) \qquad (19)$$

$$T_n = W(n+1)$$

where Y_{ms} is the sampled output of the reference model and the sampling interval is T_{sm}. Fig.4 shows time responses of the auto tuning time series is added to tuning the parameter of $C(z)$. The reference model used is

$$P_m(s) = \frac{81}{s^2 + 18s + 81} \qquad (20)$$

Parameters of Eqs.(15), (18) and (19) are β=0.2, $T_s = 0.01\sec$ η=0.7; $T_{sm} = 0.05\sec$, $j = 1$, and $\rho = 50$. The initial value of T_n is equal to 0.005 and converged to 0.11807. Fig.4 shows the controlled output is tracking the output of the reference model quickly.

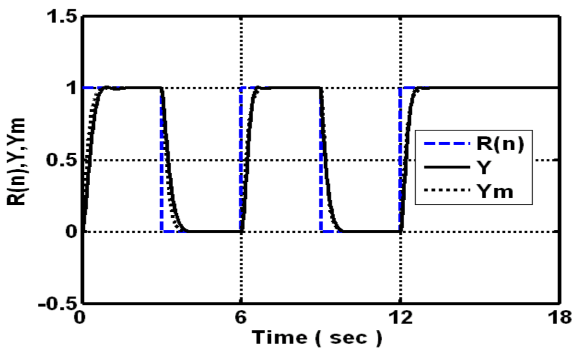

Fig.4. Time responses of the illustrating example for finding $C(z)$.

The proposed control scheme using numerical time series will be applied to three numerical SISO (single-input single-output) examples in next section on on-line adjusting manner. Eq.(19) will be used for finding phase-lead compensators $C(z)$ to meet design specifications.

3 Numerical Examples

Example 1: Consider a stable plant has the transfer function[7, 8]:

$$P_1(s) = \frac{e^{-s}}{(s+1)^2} \qquad (21)$$

It has pure time delay 1 second. The reference model used is

$$P_m(s) = \frac{e^{-s}}{(s+1.3)^2} \qquad (22)$$

Parameters of Eqs.(15),(18) and (19) are $\beta = 0.1$, $T_S = 50ms$, $\rho = 50$, $\eta = 0.3$ and j=1. $T_{sm} = 500ms$. Fig.5 shows on-line adjusting processes for finding $C(z)$. The initial guess of T_n is equal to 2.00 and converged to 0.61218 after third adjusting processes. It gives good performance and tracks the reference model quickly.

Simulation results of the proposed method and four other methods are presented for comparisons. They are Ziegler-Nichols method[9-12] for finding PI and PID compensators, Tan et al[13,14] for finding PID compensator and Majhi[7,8] for finding PI compensator. The controller is in the form of

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{d}{dt} e(t); \qquad (23)$$

Parameters of four found compensators are given below:

(1)ZN(PI) : $K_p = 1.240$ and $K_i = 0.251$.

(2)ZN(PID) : $K_p = 1.6367, K_i = 0.4187$ *and* $K_d = 0.5972$.

(3)Tan's(PID): $K_p = 0.620, K_i = 0.5636$ *and* $K_d = 0.1705$.

(4)Majhi's(PI): $K_p = 0.864$ *and* $K_i = 0.3653$.

Integral of the Square Error(ISE), and Integral of the Absolute Error (IAE) are given in Table 1. Time responses are shown in Fig.6. From Table 1 and Fig.6, one can see that the proposed method gives faster and better performance than those of other methods presented.

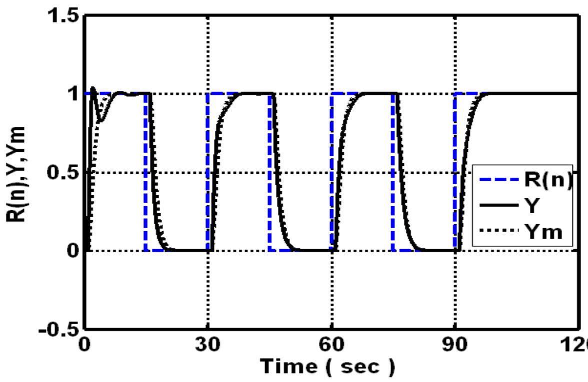

Fig.5. Time responses of the Example 1 for finding $C(z)$.

Table 1. IAE and ISE errors of Example 1 using different control methods.

Methods	Proposed	ZN(PI)	ZN(PID)	Tan's	Majhi's
ISE	1.7392	2.2675	1.7694	2.2471	2.4654
IAE	2.3534	4.0107	2.8757	3.0725	4.0659

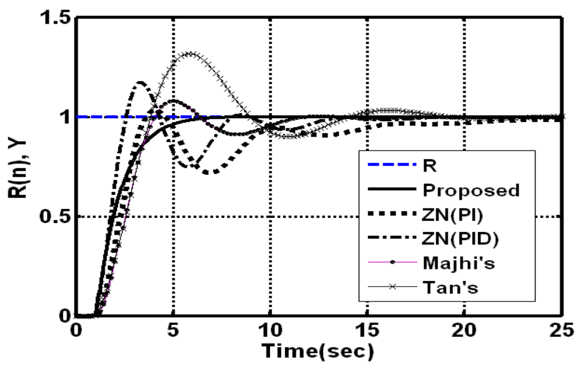

Fig.6. Time responses of Example 1 using different control methods.

Example 2: Consider a sixth order plant[7, 8]

$$P_2(s) = \frac{1}{(s+1)^6} \tag{24}$$

The reference model used is

$$P_m(s) = \frac{e^{-2s}}{(s+0.6)^2} \tag{25}$$

Parameters of Eqs.(15),(18) and (19) are $\beta = 0.3$, $T_s = 100ms$, $\rho = 50$, $\eta = 0.4$ and j=1. $T_{sm} = 1s$. Fig.7 shows on-line adjusting processes for finding $C(z)$. The initial guess of T_n is equal to 2.00 and converged to 0.82948 after third adjusting processes. It gives good performance and tracks the reference model quickly.

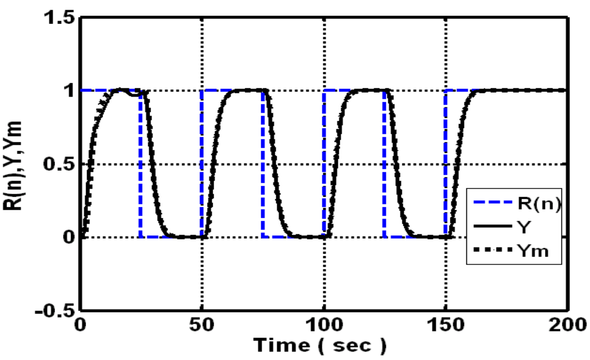

Fig.7. Time responses of the Example 2 for finding C(z).

Simulation results of the proposed and four other methods are presented for comparisons. They are Ziegler-Nichols rule[9-12] for finding PI and PID compensators, Ho et al[15] for finding PID compensator and Majhi[7,8] for finding PI compensator. Parameters of five found compensators are given below:

(1)ZN(PI) : $K_p = 1.079$ *and* $K_i = 0.110$.

(2)ZN(PID) :
$K_p = 1.4248, K_i = 0.1838$ *and* $K_D = 1.360$.

(3)Majhi's(PI): $K_p = 0.7736$ *and* $K_i = 0.1547$.

(4)Ho's(PID) :
$K(s) = 1.3(1 + 0.189/s + 1.3s/(0.13s+1))$.

Integral of the Square Error(ISE), and Integral of the Absolute Error (IAE) are given in Table 2. Time responses are shown in Fig.8. From Table 2 and Fig.8, one can see that the proposed method gives faster and better performance than those of other methods.

Table 2. IAE and ISE errors of Example 2 using different control methods.

Methods	Proposed	ZN(PI)	ZN(PID)	Ho's	Majhi's
ISE	3.9591	5.338	4.023	5.215	3.746
IAE	5.2355	9.557	6.514	7.226	5.425

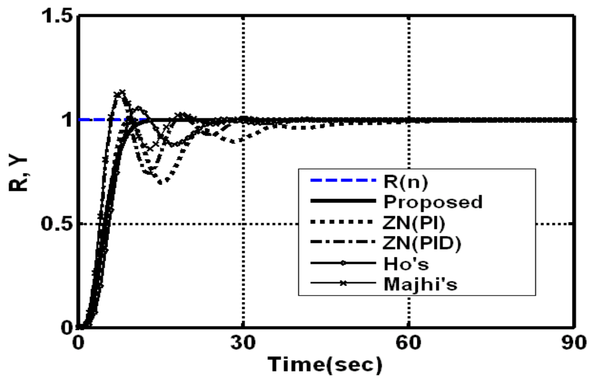

Fig.8. Time responses of Example 2 using different control methods.

Example 3: Consider the very high order plant[7, 8]:

$$P_3(s) = \frac{1}{(s+1)^{20}} \tag{26}$$

The reference model used is

$$P_m(s) = \frac{e^{-12.5S}}{(s+0.3)^2} \tag{27}$$

Parameters of Eqs.(15),(18) and (19) are $\beta = 0.1$, $T_s = 100ms$, $\rho = 50$, $\eta = 0.1$ and j=1. $T_{sm} = 1s$. Fig.9 shows on-line adjusting processes for finding $C(z)$. The initial guess of T_n is equal to 1.75 and converged to 1.0893 after second adjusting processes. It gives good performance and tracks the reference model quickly.

Final results and four other methods are presented for comparison and show the merit of the proposed method. They are Ziegler-Nichols method[9-12] for finding PI and PID compensators, Zhuang and Atherton. [16] for finding PI compensator and Majhi[7,8] for finding PI compensator. Parameters of four found compensators are given below:

(1)ZN(PI) : $K_p = 0.585$ *and* $K_i = 0.0305$.

(2)ZN(PID):
$K_p = 0.77256, K_i = 0.05088$ *and* $K_d = 4.9135$.

(3)Majhi's(PI) : $K_p = 0.5097$ *and* $K_i = 0.0443$.

(4)Zhuang's(PI): $K_p = 0.473$ *and* $K_i = 0.058$.

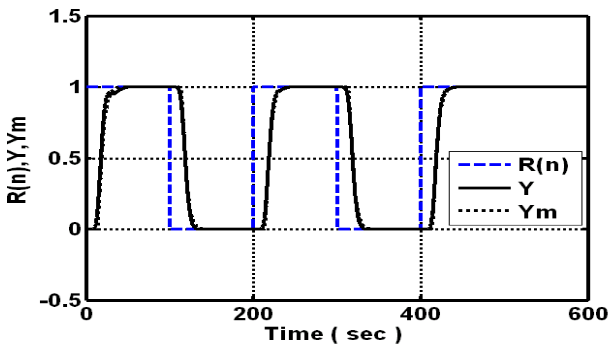

Fig.9. Time responses of the Example 3 for finding $C(z)$.

Time responses are shown in Fig.10. Table 3 gives integration of absolute error(IAE) and integration of square error(ISE) of them. From Table 3 and Fig.10, one can see that the proposed method gives better performance than those of other methods.

Table 3. IAE and ISE errors of Example 3 using different control methods.

Methods	Proposed	ZN(PI)	ZN(PID)	Majhi's	Zhuang's
ISE	16.3715	21.2271	16.2160	20.1908	21.8142
IAE	18.7662	32.7084	22.9707	26.8295	32.9125

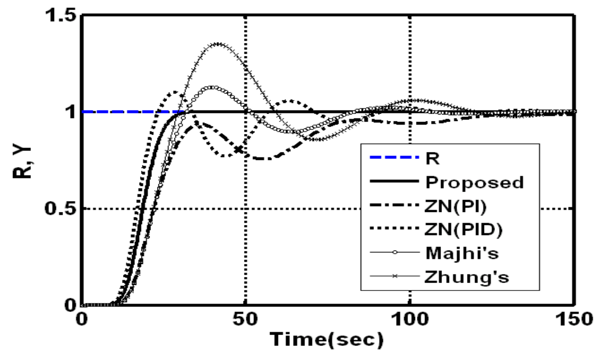

Fig.10. Time responses of Example 3 using different control methods.

4 Conclusions

In this literature, a new model based nonlinear digital controller has been proposed for analyses and designs of industry processes. They are sampled-data feedback control systems. It was applied to four simple and complicated numerical examples to get good performance and zero steady-state errors. No integrations of tracking errors are needed to get zero steady-state errors. Lead compensations are also found by another numerical time series to speed up the system responses on the on-line adjusting manner. From simulation and comparison results with other famous control methods, it can be seen that the proposed method provides another possible control scheme for sampled-data feedback control systems.

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