Movement planning of mobile vehicles group in the two-dimensional environment with obstacles[1]

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Abstract: - The problem of distributed control of heterogeneous group of vehicles in the environment with obstacles is considered. The control algorithms are based on vehicles kinematics in a two dimensional environment. The proposed algorithms are based on the principle of consideration all neighboring objects as repeller. The proposed method of decentralized group control is based on the simple local control algorithms. A new approach for forming repellers is discussed. This approach is based on the formation of unstable states in the phase space of vehicles. Results of the proposed algorithms are velocities and course angles of the controlled vehicles. Analysis of the received movement trajectories on stability is carried out by Lyapunov functions. Existence and asymptotic stability of the vehicles group steady state is shown. The planning algorithms modification which isn't demanding a preliminary reference is offered. The developed algorithms are realized within the decentralized structure of a control system. Simulation of the group consisting of five vehicles in the environment with motionless obstacles is carried out. On the basis of the carried-out analysis and simulation conclusions results about applicability of the offered method in practice are drawn. The development of the offered movement trajectories planning method assuming use as the kinematics and dynamics is discussed. This method allows considering convergence velocities with obstacles. Also potential of use the offered method in three-dimensional environments and environments with mobile obstacles is discussed.
Key-Words: - group control, vehicle, decentralized control, repeller, unstable state, Lyapunov function.

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1 Introduction

The idea of using repelling and attracting sets in control systems of vehicles was implemented for the first time in A.K. Platonov's research in 1970 [1, 2] where the potentials method was presented as a solution of the path finding problem. Abroad the main references are made to Brooks and Khatib works which were published in 1985 and 1896 [3 – 5]. At the same time paper of Hitachi company about mobile robot`s control in which ideas of "force field" are used was published in 1984 [6]. The method of potential fields is widely adopted now. The overview and the analysis of the methods where potential fields are used can be found in work [7]. In papers [8, 9] the idea of conversion of dot obstacles to repeller is explained, using Lyapunov theorem of instability. Such approach allows realizing movement in the environments with obstacles without mapping. In [10] this approach was extended to three-dimensional space, and in [7] the movement task in the environment with obstacles which can form various configurations was considered.

The idea of obstacles representation as repellers can be also used at the solution of group control tasks [11]. Therefore homogeneous or heterogeneous groups [12, 13] can be considered. Groups often consist of intelligent robots which can be presented as systems supplied with the powerful computer system, or as systems constructed on the basis of intelligent methods, such as fuzzy logic of L.Zade, artificial neural networks and expert systems [14, 15].

Clusters (subgroups) are formed [16, 17] when for the solution of a specific objective not all robots in group are needed or when several tasks are set for group.

In systems of robots group control methods of the centralized, decentralized or hybrid strategy of control can be implemented. At the centralized strategy each control system of vehicles receives algorithm of actions through information channels and realizes it. In this case control systems of active robots, actually, solve local problems of executive mechanisms control; therefore the main part of robots group may have not complex computer systems.

The decentralized strategy of control which leads to the distributed systems of group control is represented as more perspective one. In this regard in this paper the problem of the distributed control of heterogeneous vehicles group in the two-dimensional environment with obstacles is considered with use of repeller ideology.

2 Algorithm with a predetermined trajectory

We consider vehicle which have the following kinematics: (fig. 1)

$$\dot{y}_{1i} = V_i \cos\varphi_i, \quad \dot{y}_{2i} = V_i \sin\varphi_i, \qquad (1)$$

where y_{1i}, y_{2i} are coordinates of vehicle; V_i is velocity of vehicle; φ_i is course angle of vehicle; $i = \overline{1,n}$.

The position of vehicle is characterized by coordinates y_{1i}, y_{2i} in an external Oy_1y_2 system. Velocity V_i and course angle φ_i are controlled variables. Each vehicle measures coordinates of adjacent objects and has information about coordinates y_L, y_R of area L in which the group functions. The number n of vehicles in group isn't known. The task of group relocation in the direction of an axis Oy_2 with uniform distribution of objects along an axis Oy_1 is set.

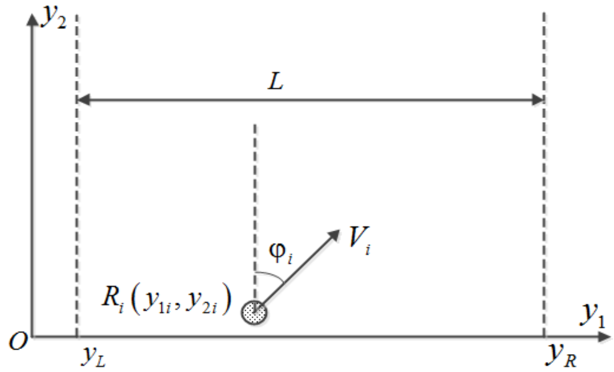

Fig. 1 – Variable conditions of vehicle and coordinate system

Let`s $y_{2i} = 0$, $y_{1i} \neq y_{1j}, \forall i \neq i, i, j = \overline{1,n}$. We specify vehicles so that the index $i = \overline{1,n}$ increased with increase in y_{1i} coordinate. In this case the local algorithm of control for vehicle can be synthesized as follows.

We present the neighboring objects in the form of repeller for vehicles. Thus, the object neighboring objects at the left has to form force which is pushing out vehicle to the right, and neighboring object on the right – to the left. Functions which are formed repeller for the vehicle are presented in fig. 2.

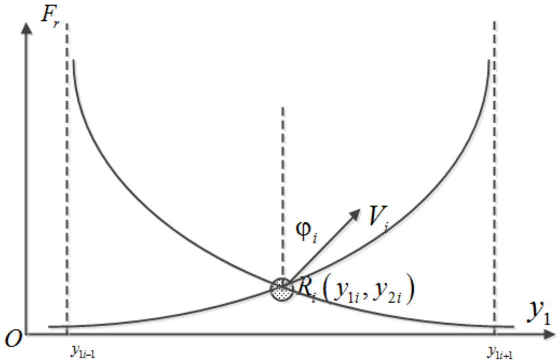

Fig. 2 – Formation of repeller

In this work it is offered to form repellent forces in the form of the dynamic variable which is result of functions integration presented in fig. 2.
Let`s the functions presented in fig. 2 are power functions. Then the specified idea is realized with expansion of system by the following equations (2):

$$\dot{z}_i = \frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}, \qquad (2)$$

where $y_{10} = y_L$; $y_{1n+1} = y_R$.

As appears from the equation (2), variables depend on values, which are inversed to distances between R_{i-1}, R_i and R_{i+1} vehicles. Let`s initial requirements to the vehicles movement trajectory presented in the following vector form:

$$\psi_i = \begin{bmatrix} y_{1i} - y_{i0} \\ \dot{y}_{2i} - V_k \end{bmatrix}$$

Where y_{i0}, V_k are some numbers not equal to zero.
To consider influence of repeller, we create vehicle control system purpose. It looks like this:

$$\psi_i = \begin{bmatrix} y_{1i} - y_{i0}\left(1 + z_i\right) \\ \dot{y}_{2i} - V_k \end{bmatrix} \qquad (3)$$

Thus, at occurrence of the repeller to the left of vehicle the variable z_i increases, therefore, the component $y_{i0}\left(1+z_i\right)$ also increases. At occurrence of the repeller on the right, as appears from (2) variable z_i decreases, therefore, expression $y_{i0}\left(1+z_i\right)$ also decreases.
The derivative on time from the first element of a vector (3) according to the equations (1) (2) is described by the following expression:

$$\dot{\psi}_i[1] = V_i \cos\varphi_i - y_{i0}\left(\frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}\right). \qquad (4)$$

We demand that the vehicle closed control system satisfies to the following differential equations:

$$\dot{\psi}_i[1] + T_{0i}\psi_i[1] = 0,$$
$$\psi_i[2] = 0. \qquad (5),$$

where T_{0i} is constant positive number.
Having substituted in the equation (5) expressions (3), (4), we receive:

$$\begin{bmatrix} u_{ix} \\ u_{iy} \end{bmatrix} = \begin{bmatrix} y_{i0}\left(\frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}\right) - T_{0i}\left(y_{1i} - y_{i0}\left(1+z_i\right)\right) \\ V_k \end{bmatrix} \qquad (6)$$

$$\begin{bmatrix} V_i \\ \varphi_i \end{bmatrix} = \begin{bmatrix} \sqrt{u_{ix}^2 + u_{iy}^2} \\ \arctan\left(\frac{u_{iy}}{u_{ix}}\right) \end{bmatrix} \qquad (7)$$

We carry out the behavior analysis of the closed control system which has the following form:

$$\begin{bmatrix} \dot{y}_{1i} \\ \dot{y}_{2i} \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} y_{i0}\left(\frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}\right) - T_{0i}\left(y_{1i} - y_{i0}\left(1+z_i\right)\right) \\ V_k \\ \frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}} \end{bmatrix} \qquad (8)$$

From (8), the closed control system is divided on two independent subsystems. First subsystem is described by the second equation, and second subsystem is described by the first and third equation.
We carry out the analysis of the closed control system in relation to variables y_{1i} and z_i using the following equations:

$$\begin{bmatrix} \dot{y}_{1i} \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} y_{i0}\left(\frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}\right) - T_{0i}\left(y_{1i} - y_{i0}\left(1+z_i\right)\right) \\ \frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}} \end{bmatrix} \qquad (9)$$

Believing in the equations (9) derivatives on time are equal to zero, we find the following equations of the set state:

$$\begin{cases} 0 = \frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}, \\ 0 = y_{0i}\left(\frac{1}{y_{1i} - y_{1i-1}} - \frac{1}{y_{1i+1} - y_{1i}}\right) - T_0\left(y_{1i} - y_{0i}\left(1+z_i\right)\right). \end{cases} \qquad (10)$$

We express from (10) variables y_{1i} and z_i:

$$\begin{cases} y_{1i} = \dfrac{y_{1i-1} + y_{1i+1}}{2}, \\ z_i = \dfrac{y_{1i-1} + y_{1i+1}}{2y_{0i}} - 1. \end{cases} \quad (11)$$

Let`s express recurrence relations (11) through y_L, y_R parameters. We write down the first equation from (11) for $i=1$:

$$y_{11} = \frac{y_L + y_{12}}{2}. \quad (12)$$

Similarly for $i=2$ we have:

$$y_{12} = \frac{y_{11} + y_{13}}{2} = \frac{\frac{y_L + y_{12}}{2} + y_{13}}{2} \Rightarrow y_{12} = \frac{y_L}{4} + \frac{y_{12}}{4} + \frac{y_{13}}{2}, \Rightarrow$$
$$\Rightarrow \frac{3}{4} y_{12} = \frac{y_L}{4} + \frac{y_{13}}{2} \Rightarrow y_{12} = \frac{y_L + 2y_{13}}{3}. \quad (13)$$

For $i=3$ we receive:

$$y_{13} = \frac{y_{12} + y_{14}}{2} = \frac{\frac{y_L + 2y_{13}}{2} + y_{14}}{2} \Rightarrow y_{13} = \frac{y_L}{6} + \frac{2y_{13}}{6} + \frac{y_{14}}{2}, \Rightarrow$$
$$\Rightarrow \frac{2}{3} y_{13} = \frac{y_L}{6} + \frac{y_{14}}{2} \Rightarrow y_{13} = \frac{y_L + 3y_{14}}{4}.$$

(14)

Analyzing sequence (12) – (14) we can write:

$$y_{1i} = \frac{y_L + iy_{1i+1}}{i+1}. \quad (15)$$

Now we write down expression (15) for $i=n$:

$$y_{1n} = \frac{y_L + ny_R}{n+1}. \quad (16)$$

Further for $i = n-1$ from (15) taking into account (16) we receive:

$$y_{1n-1} = \frac{y_L + (n-1)y_{1n}}{n} = \frac{y_L + (n-1)\frac{y_L + ny_R}{n+1}}{n} =$$
$$= \frac{(n+1)y_L + (n-1)y_L + (n-1)ny_R}{n(n+1)} = \frac{2y_L + (n-1)y_R}{n+1} \quad (17)$$

Similarly, for $i = n-2$ from (15) taking into account (17) we receive:

$$y_{1n-2} = \frac{3y_L + (n-2)y_R}{n+1} \quad (18)$$

Carrying out the analysis of sequence (16) – (18), taking into account the second equation (11), let`s receive the following expressions for the equations of the closed control systems unstable state:

$$\begin{cases} y_{1i} = \dfrac{(n-i+1)y_L + iy_R}{n+1}, \\ z_i = \dfrac{(n-i+1)y_L + iy_R}{(n+1)y_{0i}} - 1. \end{cases} \quad (19)$$

Expressions (19) define values of variables y_{1i} and z_i, $i = \overline{1,n}$ in the steady state. From (19) it is obvious that the established values of coordinates y_{1i} depend only on number of vehicles (n) and borders of functioning area y_L, y_R.

Let`s analyze stability of the closed system (1), (2), (6), (7) of rather set mode (19). For this purpose we write down the following square form as Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{n}\left\{\left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)^2 + \left(z_i - \frac{(n-i+1)y_L + iy_R}{(n+1)y_{0i}} + 1\right)^2\right\}$$

(20)

Apparently from expression (20) the sum of square functions of the set state deviations described by coefficients (19) is used as Lyapunov function.

The derivative on time from function (20) taking into account the equation of the closed system (1), (2), (6), (7) is equal to:

$$\dot{V} = \sum_{i=1}^{n}\left\{\left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)\left(y_{0i}\dot{z}_i - T_0\left(y_{1i} - y_{0i}(1+z_i)\right)\right) + \right.$$
$$\left. + \left(z_i - \frac{(n-i+1)y_L + iy_R}{(n+1)y_{0i}} + 1\right)\dot{z}_i\right\}.$$

(21)

We transform expression (21) to:

$$\dot{V} = \sum_{i=1}^{n}\left\{\left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)y_{0i}\dot{z}_i + \right.$$
$$+ \left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)\left(-T_0\left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1} + \frac{(n-i+1)y_L + iy_R}{n+1} - y_{0i}(1+z_i)\right)\right) +$$
$$\left. + \left(z_i - \frac{(n-i+1)y_L + iy_R}{(n+1)y_{0i}} + 1\right)\dot{z}_i\right\} = \sum_{i=1}^{n}\left\{\left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)y_{0i}\dot{z}_i - \right.$$
$$- T_0\left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)^2 + \left(y_{1i} - \frac{(n-i+1)y_L + iy_R}{n+1}\right)T_0\left(y_{0i}(z_i+1) - \frac{(n-i+1)y_L + iy_R}{n+1}\right) +$$
$$\left. + \left(z_i - \frac{(n-i+1)y_L + iy_R}{(n+1)y_{0i}} + 1\right)\dot{z}_i\right\}.$$

(22)

We allocate full squares in expression (22) using the second and the third subexpressions:

$$\dot{V}=\sum_{i=1}^{n}\left\{-T_0\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}-\frac{y_{0i}}{2}\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)\right)^2-\right.$$
$$-\frac{y_{0i}^2}{4}\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)^2+\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)\dot{z}_i+$$
$$\left.+\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}\right)y_{0i}\dot{z}_i\right\}.$$
(23)

Again we will allocate full squares, using the second and the third subexpressions from expressions (23):

$$\dot{V}=\sum_{i=1}^{n}\left\{-T_0\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}-\frac{y_{0i}}{2}\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)\right)^2-\right.$$
$$\left.-\left(\frac{y_{0i}}{2}\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)-\frac{\dot{z}_i}{y_{0i}}\right)^2-\frac{\dot{z}_i^2}{y_{0i}^2}+\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}\right)y_{0i}\dot{z}_i\right\}.$$
(24)

Applying once again operation of full squares allocation, from expression (24), taking into account the equation (2), we receive:

$$\dot{V}=\sum_{i=1}^{n}\left\{-T_0\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}-\frac{y_{0i}}{2}\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)\right)^2-\right.$$
$$-\left(\frac{y_{0i}}{2}\left(z_i+1-\frac{(n-i+1)y_L+iy_R}{(n+1)y_{0i}}\right)-\frac{\frac{1}{y_{1i}-y_{1i-1}}-\frac{1}{y_{1i+1}-y_{1i}}}{y_{0i}}\right)^2-$$
$$\left.-\left(\frac{\frac{1}{y_{1i}-y_{1i-1}}-\frac{1}{y_{1i+1}-y_{1i}}}{y_{0i}}-\frac{y_{0i}}{2}\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}\right)\right)^2-\frac{y_{0i}^2}{4}\left(y_{1i}-\frac{(n-i+1)y_L+iy_R}{n+1}\right)^2\right\}.$$
(25)

Thus, from (25) follows that equilibrium position (19) is asymptotically steady in the closed system (1), (2), (6), (7). Thus it is necessary that:

$$y_{1i}\neq y_{1i-1},\ \forall i=\overline{1,n}.$$

3 Algorithm with maintenance of the formation

The algorithm of control (6), (7) demands a preliminary given motion trajectory. Besides, vehicles without algorithm of control (6), (7) move with constant velocities therefore do not adhere to one line. In the conditions of obstacles various vehicles trajectories length may strongly differ, therefore it is required to modify algorithms of control. For this purpose we enter into consideration the following vector of control errors:

$$\psi_i=\begin{bmatrix} y_{1i}-\dfrac{y_{1i-1}+y_{1i+1}-z_i}{2},\ i=\overline{1,n} \\ \dot{y}_{21}-V_k,\ y_{2i}-y_{2i-1},\ i=\overline{2,n}\end{bmatrix} \quad (26)$$

Let's demand that errors (26) satisfy to the following system of the differential equations:

$$\dot{\psi}_i[1]+T_{1i}\psi_i[1]=0,\ i=\overline{1,n},$$
$$\psi_1[2]=0,\ \dot{\psi}_i[2]+T_{2i}\psi_i[2]=0,\ i=\overline{2,n}, \quad (27),$$

where T_{1i}, T_{2i} are constant parameters.

Let`s differentiate a vector (26) and substitute it in (27). Having resolved system of the algebraic equations, we receive:

$$u_{ix}=\frac{\dot{y}_{1i-1}+\dot{y}_{1i+1}-\frac{1}{y_{1i}-y_{1i-1}}+\frac{1}{y_{1i+1}-y_{1i}}}{2}-T_{1i}\left(y_{1i}-\frac{y_{1i-1}+y_{1i+1}-z_i}{2}\right),\ i=\overline{1,n},$$
$$u_{1y}=V_k,$$
$$u_{iy}=\dot{y}_{2i-1}-T_{2i}\left(y_{2i}-y_{2i-1}\right),\ i=\overline{2,n}.$$
(28)

Then the equations of the closed control system look like:

$$\dot{y}_{1i}=\frac{\dot{y}_{1i-1}+\dot{y}_{1i+1}-\frac{1}{y_{1i}-y_{1i-1}}+\frac{1}{y_{1i+1}-y_{1i}}}{2}-T_{1i}\left(y_{1i}-\frac{y_{1i-1}+y_{1i+1}-z_i}{2}\right),\ i=\overline{1,n},$$
$$\dot{y}_{21}=V_k,$$
$$\dot{y}_{2i}=\dot{y}_{2i-1}-T_{2i}\left(y_{2i}-y_{2i-1}\right),\ i=\overline{2,n},$$
$$\dot{z}_i=\frac{1}{y_{1i}-y_{1i-1}}-\frac{1}{y_{1i+1}-y_{1i}}$$
(29)

The closed system (29), as well as earlier, is divided on two independent subsystems. The first subsystem consists of the second and the third equations of system (29), and the second consists of the first and the fourth equations of system (29).

Let's consider the first subsystem consisting of the second and third equations of system (29) and write down it in the following form:

$$\dot{y}_{21}=V_k,$$
$$\dot{y}_{22}=\dot{y}_{21}-T_2\left(y_{22}-y_{21}\right)=V_k-T_2\left(y_{22}-y_{21}\right),$$
$$\dot{y}_{23}=\dot{y}_{22}-T_2\left(y_{23}-y_{22}\right)=V_k-T_2\left(y_{22}-y_{21}\right)-T_2\left(y_{23}-y_{22}\right)=V_k-T_2\left(y_{23}-y_{21}\right),$$
$$\ldots$$
$$\dot{y}_{2i}=V_k-T_2\left(y_{2i}-y_{21}\right),$$
$$\ldots$$
(30)

Let's integrate the first equation (30):

$$y_{21}=y_{21}^0+V_kt \quad (31)$$

Then, taking into account (31), the last equation from (30) looks like:

$$\dot{y}_{2i}+T_2y_{2i}=V_k+T_2\left(y_{21}^0+V_kt\right) \quad (32)$$

Solving the equation (32), we receive:

$$y_{2i}(t)=\left(y_{2i}^0-y_{21}^0\right)e^{-T_2t}+y_{21}^0+V_kt \quad (33)$$

From expression (33) follows that:

$$\lim_{t\to\infty}y_{2i}(t)=\lim_{t\to\infty}\left(\left(y_{2i}^0-y_{21}^0\right)e^{-T_2t}+y_{21}^0+V_kt\right)=y_{21}^0+V_kt$$

Thus, eventually position of all vehicles along an axis of Oy2, converge for the position of the most left object, i.e. the group maintains a formation.

Let's consider the second subsystem consisting of the first and fourth equations of system (29). The set state of this subsystem is described by the equations:

$$0=\frac{-\frac{1}{y_{1i}-y_{1i-1}}+\frac{1}{y_{1i+1}-y_{1i}}}{2}-T_{1i}\left(y_{1i}-\frac{y_{1i-1}+y_{1i+1}-z_i}{2}\right),$$

$$0=\frac{1}{y_{1i}-y_{1i-1}}-\frac{1}{y_{1i+1}-y_{1i}}.$$

(34)

Solving system (34), we receive expressions:

$$y_{1i}=\frac{y_{1i-1}+y_{1i+1}}{2},\quad z_i=0, \tag{35}$$

Or

$$y_{1i}=\frac{\left(n-i+1\right)y_L+iy_R}{n+1},\quad z_i=0. \tag{36}$$

For research of the closed control systems stability we consider the following function:

$$V=\frac{1}{2}\left(y_i-\frac{y_{i-1}+y_{i+1}}{2}+\frac{1}{2}z_i\right)^2 \tag{37}$$

The derivative of function (37) on time owing to the equations of the closed system (29) is equal to following:

$$\dot{V}=\left(y_i-\frac{y_{i-1}+y_{i+1}}{2}+\frac{1}{2}z_i\right)\left(\dot{y}_i-\frac{\dot{y}_{i-1}+\dot{y}_{i+1}}{2}+\frac{1}{2}\dot{z}_i\right)=$$

$$=\left(y_i-\frac{y_{i-1}+y_{i+1}}{2}+\frac{1}{2}z_i\right)\left(\frac{\dot{y}_{1i-1}+\dot{y}_{1i+1}}{2}-\frac{1}{2}\dot{z}_i-T_{1i}\left(y_{1i}-\frac{y_{1i-1}+y_{1i+1}}{2}+\frac{1}{2}z_i\right)-\frac{\dot{y}_{i-1}+\dot{y}_{i+1}}{2}+\frac{1}{2}\dot{z}_i\right)=$$

$$=-T_{1i}\left(y_i-\frac{y_{i-1}+y_{i+1}}{2}+\frac{1}{2}z_i\right)^2$$

(38)

Taking into account $y_0=y_L,\ y_{n+1}=y_R$ the first equation from (35) may be modified to

$$y_n=\frac{y_L+ny_R}{n+1},\ y_i=\frac{y_L}{i+1}+\frac{iy_{1i+1}}{i+1},\ i=\overline{n-1,1}.$$

(39)

Let's place the origin of coordinates in y_L point. Then using expression (39) we can write:

$$y_n=\frac{n}{n+1}y_R,\ y_{n-1}=\frac{n-1}{n+1}y_R,\ \ldots\ y_1=\frac{1}{n+1}y_R.$$

(40)

From (40) we can find distance between the neighboring vehicles:

$$y_{1i}-y_{1i-1}=\frac{1}{n+1}y_R=\frac{L}{n+1}. \tag{41}$$

Thus, the considered control system of vehicles group will function successfully when the following condition is performed:

$$r_p^j<\frac{L}{n+n_p+1}. \tag{42}$$

Where variable r_p^j is obstacle radius.

4 Algorithm with parametrical introduction of unstable states

Let's consider the following expressions defining errors of vehicles:

$$e_1=\begin{bmatrix} y_{11}-\frac{y_{11-1}+y_{11+1}}{2}+\frac{LT_{11}}{2k}z_1 \\ \dot{y}_{21}-V_k \end{bmatrix} \tag{43}$$

$$e_i=\begin{bmatrix} y_{1i}-\frac{y_{1i-1}+y_{1i+1}}{2}+\frac{LT_{1i}}{2k}z_i \\ y_{2i}-y_{2i-1} \end{bmatrix} \tag{44}$$

Let's demand that errors (43), (44) satisfied to the following differential equations:

$$\dot{e}_1\left[1\right]+\left(T_{21}-z_1^2\right)e_1\left[1\right]=0 \tag{45}$$

$$\dot{e}_i+\begin{bmatrix} T_{2i}-z_i^2 & 0 \\ 0 & T_{2i} \end{bmatrix}e_i=0,$$

$$i=\overline{2,n} \tag{46}$$

Let's calculate derivative of errors (43), (44) and, having substituted them together with (43) in (45), (46), we receive the algebraic equations, having solved which we find expressions for change of the vehicles movement velocities and angles:

$$\begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix}=\begin{bmatrix} \frac{\dot{y}_{1L}+\dot{y}_{12}}{2}-\frac{LT_{11}}{2k}\dot{z}_1-\left(T_{21}-z_1^2\right)\left(y_{11}-\frac{y_{1L}+y_{12}}{2}+\frac{LT_{11}}{2k}z_1\right) \\ V_k \end{bmatrix} \tag{47}$$

$$\begin{bmatrix} u_{ix} \\ u_{iy} \end{bmatrix}=\begin{bmatrix} \frac{\dot{y}_{1i-1}+\dot{y}_{1i+1}}{2}-\frac{LT_{1i}}{2k}\dot{z}_i-\left(T_{2i}-z_i^2\right)\left(y_{1i}-\frac{y_{1i-1}+y_{1i+1}}{2}+\frac{LT_{1i}}{2k}z_i\right) \\ \dot{y}_{2i-1}-T_{2i}\left(y_{2i}-y_{2i-1}\right) \end{bmatrix} \tag{48}$$

Then the equations of the closed system look like:

$$\begin{bmatrix} \dot{y}_{11} \\ \dot{y}_{21} \\ \dot{z}_1 \end{bmatrix}=\begin{bmatrix} \frac{\dot{y}_{1L}+\dot{y}_{12}}{2}-\frac{LT_{11}}{2k}\dot{z}_1-\left(T_{21}-z_1^2\right)\left(y_{11}-\frac{y_{1L}+y_{12}}{2}+\frac{LT_{11}}{2k}z_1\right) \\ V_k \\ \frac{k}{L}\left(2y_{11}-y_{1L}-y_{12}\right) \end{bmatrix} \tag{49}$$

$$\begin{bmatrix} \dot{y}_{1i} \\ \dot{y}_{2i} \\ \dot{z}_i \end{bmatrix}=\begin{bmatrix} \frac{\dot{y}_{1i-1}+\dot{y}_{1i+1}}{2}-\frac{LT_{1i}}{2k}\dot{z}_i-\left(T_{2i}-z_i^2\right)\left(y_{1i}-\frac{y_{1i-1}+y_{1i+1}}{2}+\frac{LT_{1i}}{2k}z_i\right) \\ \dot{y}_{2i-1}-T_{2i}\left(y_{2i}-y_{2i-1}\right) \\ \frac{k}{L}\left(2y_{1i}-y_{1i-1}-y_{1i+1}\right) \end{bmatrix}$$

$$,\ i=\overline{2,n}.. \tag{50}$$

Let's carry out the stability analysis of the first and third equations (49), (50), believing that repellent forces are formed by the following equation:

$$\dot{z}_i=k\left(2y_{1i}-y_{1i-1}-y_{1i+1}\right)/L \tag{51}.$$

Believing in (49), (50) signals y_{1i-1}, y_{1i+1} are equal to zero we receive the following system for the analysis of stability:

$$\begin{bmatrix} \dot{y}_{1i} \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} -\frac{LT_{1i}}{2k} y_{1i} - \left(T_{2i} - z_i^2\right)\left(y_{1i} + \frac{LT_{1i}}{2k}\frac{z_i}{2}\right) \\ \frac{2k}{L} y_{1i} \end{bmatrix}. \quad (52)$$

Let's choose the following expression (59) as Lyapunov function.

$$V_i = \frac{1}{2} y_{1i}^2 + \frac{1}{2}\left(y_i + z_i\right)^2. \quad (53)$$

The derivative on time from expression (53) owing to system (52) is:

$$\dot{V}_i = -2\left(T_{1i} + T_{2i} - z_i^2\right) y_{1i}^2 - \\ -\left[\left(T_{2i} - z_i^2\right)\frac{T_{1i}L}{2k} + \left(T_{1i} + T_{2i} - z_i^2\right) - \frac{4k}{L}\right] y_{1i} z_i - \left(T_{2i} - z_i^2\right)\frac{T_{1i}L}{2k} z_i^2. \quad (54)$$

Let's enter the following notations:

$$R_i = T_{2i} - z_i^2,\; a_i = \frac{LT_{1i}}{2k},\; b_i = \frac{2k}{L} \quad (55)$$

Also we rewrite expression (54) taking into account (55):

$$\dot{V}_i = -2\left(T_{1i} + R_i\right) y_{1i}^2 - \left[(1 + a_i)R_i + T_{1i} - 2b_i\right] y_{1i} z_i - R_i a_i z_i^2 \quad . (56)$$

Let's assume that the vector $[y \quad z]^T$ will be subjected to some transformation [18], i.e.

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \quad (57),$$

where η_1 η_2 are components of a new vector. Thus the transformation matrix in (57) such is that function $-\dot{V}_i$ in new variables is equal to the sum of new variables squares. Then taking into account transformation (57) and the accepted assumption expression (56) we register so:

$$\dot{V}_i = -2\left(T_{1i} + R_i\right)\left(c_{11}\eta_1 + c_{12}\eta_2\right)^2 - \\ -\left[\left(1 + a_i\right)R_i + T_{1i} - 2b_i\right]\left(c_{11}\eta_1 + c_{12}\eta_2\right)\left(c_{21}\eta_1 + c_{22}\eta_2\right) - R_i a_i \left(c_{21}\eta_1 + c_{22}\eta_2\right)^2 = -\eta_1^2 - \eta_2^2.$$
(58)

Let's accept $c_{12} = 0$, and we will receive from (58):

$$\dot{V}_i = -2\left(T_{1i} + R_i\right)c_{11}^2\eta_1^2 - \left[\left(1 + a_i\right)R_i + T_{1i} - 2b_i\right]\left(c_{11}\eta_1\right)\left(c_{21}\eta_1 + c_{22}\eta_2\right) - \\ -R_i a_i\left(c_{21}\eta_1 + c_{22}\eta_2\right)^2 = \left[-2\left(T_{1i} + R_i\right)c_{11}^2 - \left(\left(1 + a_i\right)R_i + T_{1i} - 2b_i\right)c_{11}c_{21} - R_i a_i c_{21}^2\right]\eta_1^2 - \\ -R_i a_i c_{22}^2\eta_2^2 - \left[\left(\left(1 + a_i\right)R_i + T_{1i} - 2b_i\right)c_{11}c_{22} + 2R_i a_i c_{21}c_{22}\right]\eta_1\eta_2 = -\eta_1^2 - \eta_2^2.$$
(59).

Equating coefficients at identical degrees η_1, η_2, in the left and right parts (59), we receive system of the equations, solving which we find:

$$c_{22}^2 = \frac{1}{R_i a_i}, \quad (60)$$

$$c_{11} = \frac{-2R_i a_i}{\left(1 + a_i\right)R_i + T_{1i} - 2b_i} c_{21}, \quad (61)$$

$$c_{21}^2 = \frac{1}{2\left(T_{1i} + R_i\right)\left(\frac{-2R_i a_i}{\left(1 + a_i\right)R_i + T_{1i} - 2b_i}\right)^2 - R_i a_i} \quad (62)$$

Thus, transformation (57), (60) – (62) leads expression (56) to the canonical negatively defined form therefore conditions of non-peculiarity of transformation (57) are conditions of asymptotic stability of the closed system (49), (50).

From expression (60), taking into account designations (55), follows that:

$$T_{2i} > z_i^2. \quad (63)$$

Similarly from (61) and (62) we receive:

$$(1 + a_i)R_i + T_{1i} - 2b_i > 0, \quad (64)$$

$$8R_i a_i (T_{1i} + R_i) > \left[(1 + a_i)R_i + T_{1i} - 2b_i\right]^2 \quad (65)$$

Without reducing a generality, it is possible to assume that the condition $L = k$ is satisfied. In this case the inequality (65) looks like:

$$\left(4T_{1i} - \left(1 + \frac{T_{1i}}{2}\right)^2\right)R_i^2 + \left(4T_{1i}^2 - 2\left(1 + \frac{T_{1i}}{2}\right)\left(T_{1i} - 4\right)\right)R_i - \left(T_{1i} - 4\right)^2 > 0 \quad (66)$$

Thus, search of stability conditions is reduced to the solution of a square inequality (66) with restrictions (63), (64).

The graphic solution of the specified inequalities is given in fig. 3.

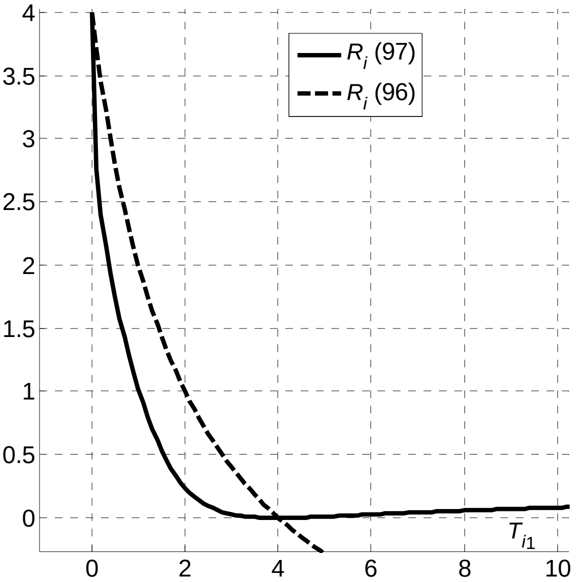

Fig. 3 – Graphic solution of inequalities

From fig. 3 we find:

$$0 < T_{1i} < 4, \quad z_i^2 < T_{2i} < z_i^2 + 4 \quad (67)$$

5 Simulation results

Let`s vehicle model is described by the equations (1), and the control is described by expressions (28).

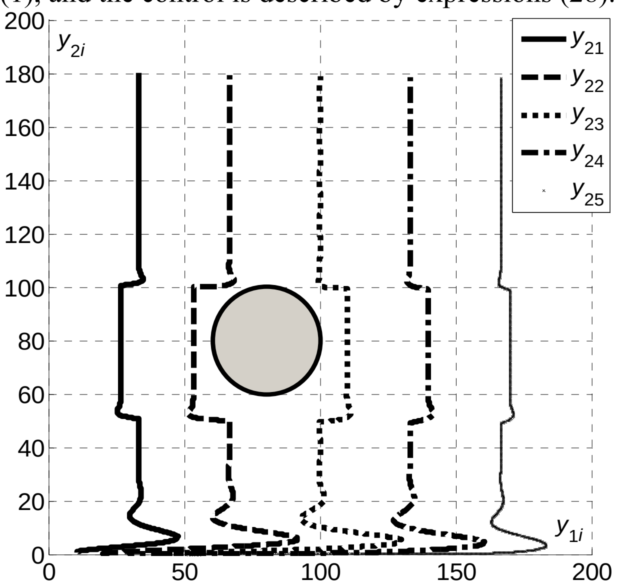

Fig. 4 – Simulation results

Control system parameters are following: width of a working zone $L = 200$ m, $y_L = 0$ m, $y_R = 200$ m; $n = 5$ is number of vehicles; settings on the velocity of $V_{0i} = 1$ m/s; time constants $T_{1i} = 1$ s^{-1}; entry conditions $y_{2i} = 0$, $y_{11} = 10, y_{12} = 20, y_{13} = 30, y_{14} = 40, y_{15} = 50$ m; coordinates of the center are (80, 80); radius of an obstacle is 20 m.

For safety maneuvers of vehicles begin for 10 meters before achievement of an obstacle. At first maneuver starts by the vehicle nearest to the found obstacle. Simulation results are given in fig.4. As it follows from fig.4 the control system carries out equable placement of vehicles along Oy1 axis, and provides avoidance of obstacles.

4 Conclusion

In paper algorithm of the distributed group control of the heterogeneous vehicles functioning in the environment with obstacles is offered and analyzed. The algorithm is under construction on the control principle which allows considering all neighboring objects as repeller.

We propose a method of repellers introduction differing in that repulsive forces are formed by the dynamic element integrating distances to the neighboring obstacles. The carried-out analysis and simulation results show efficiency of the offered methods in environments with obstacles. Thus the offered approach can be applied and to non-stationary environments since obstacles are represented formally as vehicles.

The offered algorithms can be used in the planning movement systems of various objects [19 – 24]. The method of planning provides stability of the movement at of the object kinematics level.

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