A Set of Speed Sensorless Observers For Estimation of the Rotor Flux and Load Torque For Asynchronous Machine

CHOUYA AHMED, CHENAFIA MOHAMED, MANSOURI ABDELLAH
Department of Genie Electrical
ENPO, BP 1523 El Mnaouer, Oran
ALGERIA
ahmedchouya@gmail.com
mchenafa@yahoo.fr
amansouri@yahoo.fr

Abstract: A state observer is proposed for asynchronous machine; with this observer it possible to observe rotor flux rotating speed and load torque. The gain of this observer involves a design function that has to satisfy some mild conditions which are given. Different expressions of such a function are proposed. Of particular interest, it is shown that high gain observers and sliding mode observers can be derived by considering particular expressions of the design function.

Key–Words: Nonlinear observer, High gain observer, Sliding mode observer, State transformation, LYAPUNOV equation candidate, Sensorless speed.

1 Introduction

The removal of the mechanical speed sensors offers an economic interest and may improve the reliability in the field of low power applications.

This article has as a principal objective to study the technique of determination mechanical speed and rotor flux of the asynchronous machine without velocity sensor.

The robustness, the low cost, the performances and the maintainability make the advantage of the asynchronous machine in many industrial applications or general public. Joint progress of the power electronics and numerical electronics makes it possible today to approach the controlling of axis at variable speed in applications low powers. Jointly with these technological projections, the scientific community developed various approaches of order to control in real time the flux and the speed of the electric machines.

That it is the vectorial control, the scalar control or DTC control, to control the speed of the load it is necessary to measure this one by means of a mechanical sensor. For economic reasons and/or of safety of operation, certain applications force to be freed some. The information speed must then be rebuilt starting from the electric quantities. Multiple studies were undertaken, and without claim of exhaustiveness, we can distinguish several approaches (see [1], [2] and [3]).

The control speed sensorless must however have performances which do not deviate too much from those that we would have had with a mechanical sensor. It is thus significant during the development of an approach of velocity measurement without sensor to lay the stress on the static precise details and dynamics of this one according to the point of operation of the machine.

The article is organized in four sections:

1. Dynamic model of asynchronous machine;
2. Problem formulation;
3. Observers design;
4. Results and simulations.
5. Conclusion

2 Dynamic model of asynchronous machine

In this study, the model of the motor rests on the following hypothesis as [4]:

- The fluxes and the currents are proportional by the intermediary of inductances and the mutual.
- The losses iron are neglected.
- The air-gap is constant (squirrel-cage rotor).
- The homopolar components are null.
It results from these assumptions that the various mutual between rotor and stator can be expressed like functions sinusoidal of the rotor position. 

Its vector state is composed by the stator currents, rotor fluxes and speed, as follows:

\[
\begin{align*}
\dot{i}_s &= -\gamma i_s + KA(\Omega)\psi_r + \frac{1}{\sigma L_s} u_s \\
\dot{\psi}_r &= \frac{1}{L_r} i_r - A(\Omega)\psi_r \\
\Omega &= \frac{p}{L_r} i_r - J_2 \dot{\psi}_r - \frac{1}{\tau} T_L \\
T_L &= \varepsilon T_1
\end{align*}
\] (1)

The states variables accessible to measurement are the stator currents \( i_s = [i_{s\alpha} \ i_{s\beta}] \) but to in no case rotor flux \( \psi = [\psi_{s\alpha} \ \psi_{s\beta}]^T \) and possibly rotating speed. The source of energy \( u = [u_{s\alpha} \ u_{s\beta}] \).

The parameters are define as follows:

\[
\begin{align*}
T_r &= \frac{L_r}{R_r}; \quad \sigma = 1 - \frac{M^2}{L_s L_r} \\
K &= \frac{M}{\sigma L_s L_r}; \quad \gamma = \frac{R_r}{\sigma L_s} + \frac{R_s M^2}{\sigma L_s L_r} \\
I_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad J_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \\
A(\Omega) &= \frac{1}{\tau} I_2 - p \Omega J_2
\end{align*}
\]

In these equations:

\( L_s \) : Stator inductance cyclic, \\
\( L_r \) : Rotor inductance cyclic, \\
\( M \) : Cyclic mutual inductance between stator and rotor \\
\( R_s \) : Stator resistance, \\
\( R_r \) : Rotor resistance, \\
\( \sigma \) : Scattering coefficient \\
\( T_r \) : Time constant of the rotor dynamics, \\
\( J \) : Rotor inertia, \\
\( T_I \) : Resistive torque, \\
\( p \) : Pole pair motor, \\
\( I_2 \) : is the 2-dimensional identity matrix, \\
\( J_2 \) : is a skew - symmetric matrix.

We need to transform system (1) to the triangular form. One will introduce the change of variable according to:

\[
\begin{align*}
\hat{z}_1 &= -\gamma z_1 + K z_2 + \frac{1}{\sigma L_s} u_s \\
\hat{z}_2 &= \frac{1}{T_r} \left( \frac{M}{T_r} z_1 - z_2 \right) - p z_3 J_2 \frac{M}{T_r} z_1 - z_2 \\
\hat{z}_3 &= \left[ \begin{array}{c}
\frac{p}{T_r} z_3 J_2 \psi_r + \varepsilon \Omega \\
\varepsilon T_1
\end{array} \right]^T
\end{align*}
\] (2)

With \( dw = \frac{M}{T_r} z_3 J_2 \psi_r \); and \( \varepsilon = -\frac{1}{\tau} T_L \).

\section{Problem Formulation}

Consider the nonlinear uniformly observable class of systems as the following form:

\[
\begin{align*}
\dot{z} &= f(u, z) + \varepsilon(t) \\
y &= Cz = z_1
\end{align*}
\] (4)

with

\[
\begin{align*}
z &= \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \\
f(z, u) &= \begin{bmatrix} f_1(u, z_1, z_2) \\ f_2(u, z_1, z_2, z_3) \\ f_3(u, z) \end{bmatrix}
\end{align*}
\]

where the state \( z \in \mathbb{R}^3 \), the input \( u \in \mathbb{R}^2 \) the set of bounded absolutely continuous functions with bounded derivatives from \( \mathbb{R}^n \) into \( \mathbb{R} \) a compact subset of \( \mathbb{R}^2 \); \( \varepsilon(t) \in \mathbb{R}^3 \). Our objective consists in designing state observers for system (4). We pose the following hypothesis (see [7]).

\( \mathcal{H}1 \) : There are four positive constants \( \alpha \) and \( \beta \) such as:

\[
\begin{align*}
0 < \alpha^2 &\leq \left( \frac{\partial f_1(u, z)}{\partial z_2} \right)^T \frac{\partial f_1(u, z)}{\partial z_2} \leq \beta^2 \\
0 < \alpha^2 &\leq \left( \frac{\partial f_2(u, z)}{\partial z_3} \right)^T \frac{\partial f_2(u, z)}{\partial z_3} \leq \beta^2
\end{align*}
\] (5)

E-ISSN: 2224-2856 131 Volume 12, 2017

Chouya Ahmed, Chenafa Mohamed, Mansouri Abdellah

\[ E-ISSN: 2224-2856 131 Volume 12, 2017 \]
4 Observers Design

One shall first introduce an appropriate state transformation allowing to easily design the proposed observers. Then, the equations of these observers will be derived in the new coordinates before being given in the original ones.

4.1 State Transformation

Consider the following change of states:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \mapsto x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \Phi(z) = \begin{bmatrix} \Phi(z)_1 \\ \Phi(z)_2 \\ \Phi(z)_3 \end{bmatrix}$$

where the \( \Phi_j(z) \), \( j = 1, 3 \) are defined as follows:

$$x_1 = \Phi(z)_1 = z_1$$

$$x_2 = \Phi(z)_2 = Kz_2$$

$$x_3 = \Phi(z)_3$$

$$= K \begin{bmatrix} -p\varphi_2 \left( \frac{M}{T} z_1 - z_2 \right) \\ \frac{\varphi_2}{\varphi_1} \varphi_2 \partial_{\varphi_1} z_3 \end{bmatrix}^T z_3$$

(7)

where \( \Lambda \) block diagonal matrix and \( \Lambda^{-1} \) is his left inverse:

$$\Lambda = diag \left( I_2, K I_2, -Kp \begin{bmatrix} \varphi_2 \partial_{\varphi_1} z_1 \\ -\varphi_1 \varphi_2 \partial_{\varphi_1} z_3 \end{bmatrix}^T \right)$$

Using this transformation and a time derivative of these states, we can rewrite from model (1), a following model:

$$\begin{align*}
\dot{x}_1 &= x_2 + \varphi_1(u, x_1) \\
\dot{x}_2 &= x_3 + \varphi_2(u, x_1, x_2) \\
\dot{x}_3 &= \varphi_3(u, x) \\
y &= Cx = x_1
\end{align*}$$

(8)

Proceeding as in [5, 6], one can show that the transformation \( \Phi \) puts system (1) under the following form:

$$\begin{align*}
\dot{x}_1 &= Ax + \varphi(u, x) + \frac{\partial \Phi(z)}{\partial z} \varepsilon(t) \\
y &= Cx = x_1
\end{align*}$$

(9)

where \( \varphi(u; x) \) has a triangular structure i.e.

$$A = \begin{bmatrix} 0 & I_2 & 0 \\ 0 & 0 & I_2 \\ 0 & 0 & 0 \end{bmatrix}$$

and \( \varphi(u, x) = \begin{bmatrix} \varphi_1(u, x_1) \\ \varphi_2(u, x_1, x_2) \\ \varphi_3(u, x) \end{bmatrix} \)

4.2 Observers synthesis

As in the works related to the high gain observers synthesis (see [6, 7, 8, 9]), one pose the hypothesis:

\( \mathcal{H}_3 : \) The functions \( \Phi(z) \) and \( \varphi(u, x) \) are globally LIPSCHITZ with respect to \( x \) uniformly in \( u \).

Before giving our candidate observers, one introduces the following notations:

1) Let \( \Delta_\theta \) is a block diagonal matrix define by:

$$\Delta_\theta = \text{diag} \left( I_2, \frac{1}{\theta} I_2, \frac{1}{\theta^2} I_2 \right) : \theta > 0 \text{ is a real number.}$$

3) Let \( S = S_{\theta=1} \) is a definite positive solution of the algebraic Lyapunov equation:

$$S + A^T S + S A - C^T C = 0$$

(10)

Note that (10) is independent of the system and the solution can be expressed analytically. For a straightforward computation, its stationary solution is given by:

$$C_p^n = \frac{\theta}{p(n-p)}$$

where \( C_p^n \) is the positive definite solution of the algebraic Lyapunov equation.

4) \( \forall \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}, \) set \( \bar{\xi} = \Delta_\theta \xi \) and let

$$\Upsilon(\xi) = \begin{bmatrix} \Upsilon_1(\xi_1) \\ \Upsilon_2(\xi_2) \end{bmatrix}$$

be a vector of smooth functions satisfying:

$$\forall \xi \in \mathbb{R}^2 : \xi^T \Upsilon(\xi) \geq \frac{1}{2} \xi^T C^T C \xi$$

(12)
\( \exists \kappa > 0; \ \forall \xi \in \mathbb{R}^2: \quad \| \Upsilon(\xi) \| \leq \kappa \| \xi \| \quad (13) \)

The system
\[
\dot{\hat{x}} = A\hat{x} + \varphi(u, \hat{x}) - \theta \Delta_\theta^{-1} S^{-1} \Upsilon(\hat{x}_1) - \frac{\partial \Phi(z)}{\partial z} \left( \Lambda^{-1} - \left( \frac{\partial \Phi(z)}{\partial z} \right)^{-1} \right) \theta \Delta_\theta^{-1} S^{-1} \Upsilon(\hat{x}_1) \quad (14)
\]

is an observer for (9); Where \( \hat{x} = \hat{x} - x \) error in estimation; \( \Upsilon(\hat{x}) \) satisfies conditions (12) and (13); \( u \) is the input of system (9) and \( \theta > 0 \) is a real number.

Finally, one gives the following lemma (see[7]):

**Lemma 1** Assume that system (9) satisfies hypothesis \( \mathcal{H}1 \) to \( \mathcal{H}3 \). Then,
\[
\exists \theta_0 > 0; \quad \forall \theta > \theta_0; \quad \exists \lambda > 0; \quad \exists \mu_0 > 0; \quad \exists M_0 > 0;
\]
\[
\forall u \in U; \quad \forall \hat{x}(0) \in \mathbb{R}; \quad \text{one has : }
\]
\[
\| \hat{x} \| \leq \lambda \theta^{-2} \exp^{\eta(t)} \| \hat{x}(0) \| + M_0 \delta
\]

where \( x \) is the unknown trajectory of (9) associated to the input \( u, \hat{x} \) is any trajectory of system (14) associated to \( (u, y) \) and \( \delta \) is the upper bound of \( \varepsilon_3(t) \). Moreover, one has \( \lim_{\theta \to 0} \{ \mu_0 \} = +\infty \) and \( \lim_{\theta \to \infty} \{ M_0 \} = 0 \).

### 4.2.1 Analyse Stability

One has:
\[
\dot{\hat{x}} = A\hat{x} - \theta \Delta_\theta^{-1} S^{-1} \Upsilon(\hat{x}_1) + \varphi(u, \hat{x}) - \varphi(u, x) - \frac{\partial \Phi(\Phi^{-1}(\hat{x}))}{\partial z} \varepsilon(t) - \Gamma(\hat{x}) \theta \Delta_\theta^{-1} S^{-1} \Upsilon(\hat{x}_1)
\]

where
\[
\Gamma(\hat{x}) = \frac{\partial \Phi(\Phi^{-1}(\hat{x}))}{\partial z} \left( \Lambda^{-1} - \left( \frac{\partial \Phi(z)}{\partial z} \right)^{-1} \right)
\]

Notice that \( \Gamma(\hat{x}) \) is a lower triangular matrix with zeros on its main diagonal. Moreover, using hypothesis \( \mathcal{H}1 \) and \( \mathcal{H}3 \), one can easily deduce that \( \Gamma(\hat{x}) \) is bounded.

Now, one can easily check the following identities: \( \theta \Delta_\theta^{-1} A \Delta_\theta = A \) and \( C \Delta_\theta = C \). Set \( \bar{x} = \Delta_\theta \hat{x} \). One obtains:
\[
\dot{\bar{x}} = \theta A\bar{x} - \theta S^{-1} \Upsilon(\bar{x}_1) + \Delta_\theta (\varphi(\hat{x}) - \varphi(u, x)) - \frac{\partial \Phi(\Phi^{-1}(\hat{x}))}{\partial z} \varepsilon(t) - \theta \Delta_\theta \Gamma(\hat{x}) \Delta_\theta^{-1} S^{-1} \Upsilon(\hat{x}_1) \quad (15)
\]

To prove convergence, let us consider the following equation of \( \text{LyaPunov} \ V(\bar{x}) = \bar{x}^T S \bar{x} \). By calculating the derivative of \( V \) along the \( \bar{x} \) trajectories, we obtain:
\[
V' = 2 \bar{x}^T S \bar{x} = 2 \theta \bar{x}^T S \Delta_\theta \Upsilon(\hat{x}_1) + 2 \bar{x}^T S \Delta_\theta \varphi(\varphi(u, \hat{x}) - \varphi(u, x)) - 2 \bar{x}^T S \Delta_\theta \frac{\partial \Phi(\Phi^{-1}(\hat{x}))}{\partial z} \varepsilon(t)
\]

By taking account of the (10) and (12) the derivative of \( V \) becomes:
\[
V' = -\theta V + 2 \left( \frac{1}{2} \bar{x}^T C^T C \bar{x} - \bar{x}^T \Upsilon(\bar{x}_1) \right)
\]

Now, assume that \( \theta \geq 1 \), then, because of the triangular structure and the LIPSCHITZ assumption on \( \varphi \), one can show that:
\[
\| \Delta_\theta (\varphi(u, \hat{x}) - \varphi(u, x)) \| \leq \zeta \| \bar{x} \|
\]

where \( \zeta \) is a constant of LIPSCHITZ. Similarly, according to hypothesis \( \mathcal{H}1 \) and to the LIPSCHITZ assumption on \( \Phi \) (hypothesis \( \mathcal{H}3 \)), \( \Gamma(\hat{x}) \) is bounded. Moreover, and since \( \Gamma(\hat{x}) \) is lower triangular with zeros on the main diagonal, one has:
\[
\| \theta \Delta_\theta \Gamma(\hat{x}) \Delta_\theta^{-1} \| \leq \varrho \text{ for } \theta \geq 1
\]
where $\rho > 0$ is a constant that does not depend on $\theta$. Finally, according to the structure of $\bar{z}(t)$ and since $\frac{\partial \Phi(\Phi^{-1}(x))}{\partial z}$ is triangular, one can show that:

$$\left\| \Delta_\theta \frac{\partial \Phi(\Phi^{-1}(x))}{\partial z} \bar{z}(t) \right\| \leq \frac{\beta^2}{\theta^2} \delta$$

(19)

where $\delta = \sup \{\bar{z}(t)\}$ given in hypothesis $H2$; $\beta$ is given in hypothesis $H1$. Using inequalities (17), (18), (13) and (19) inequality (16) becomes:

$$V \leq -\theta V + 2\lambda_{\max}(S) \| \bar{x} \| (\zeta \| \bar{x} \| + \rho \eta(S) \| \bar{x} \|)$$

$$+ 2\lambda_{\max}(S) \frac{\delta^2}{\theta^2} \| \bar{x} \| \leq - (\theta - c_1) V + \frac{\rho^2}{\theta^2} \delta \sqrt{V}$$

where $c_1 = 2\rho^2(S)(\zeta + \rho \eta(S))$ and $c_2 = 2\beta^2 \eta(S) \lambda_{\max}(S)$ with $\lambda_{\min}(S)$ and $\lambda_{\max}(S)$ being respectively the smallest and the largest eigenvalues of $S$ and $\eta(S) = \sqrt{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}}$.

Now taking $\theta_0 = \max \{1, c_1\}$ and using the fact that for $\theta \geq 1$, $\| \bar{x} \| \leq \| \bar{x} \| \leq \theta^2 \| \bar{x} \|$, one can show that for $\theta > \theta_0$, one has:

$$\| \bar{x} \| \leq \theta_0(S) \exp \left[ - \frac{\theta - c_1}{2} t \right] \| \bar{x}(0) \| + 2\beta^2 \frac{\eta^2(S)}{(\theta - c_1)} \delta$$

It is easy to see that $\lambda, \mu_0$ and $M_0$ needed by the result 1 are: $\lambda = \eta(S), \mu_0 = \frac{\lambda_{\min}}{\lambda_{\max}}$ and $M_0 = 2\beta^2 \frac{\eta^2(S)}{(\theta - c_1)}$. This completes the proof.

### 4.3 Observers equations in the original coordinates

Proceeding as in [6], one can show that observer (14) can be written in the original coordinates $x$ as follows:

$$\dot{z} = f(u, \bar{z}) - \theta \Lambda^{-1} \Delta_\theta^{-1} \Sigma^{-1} \Psi(\bar{z}_1 - z_1)$$

(20)

Some expressions of $\Psi(\bar{z}_1 - z_1) = \Psi(\bar{z}_1)$ that satisfying conditions (12) and (13) shall be given in this section and the so-obtained observers are discussed. These expressions will be given in the new coordinates $x$ in order to easily check conditions (12) and (13) as well as in the original coordinates $z$ in order to easily recognize the structure of the resulting observers.

### 4.4 High gain observer

Consider the following expression of $\Psi(\bar{z})$:

$$\Psi_{HG}(\bar{z}) = C^T C \bar{z} = C^T \bar{x}_1$$

(21)

One can easily check that expression (21) satisfies conditions (12) and (13). Replacing $\Psi(\bar{z})$ by expression (21) in (20) gives rise to a high gain observer (see e.g. [9, 5, 6]):

$$\dot{z} = f(u, \bar{z}) - \theta \Lambda^{-1} \Delta_\theta^{-1} \Sigma^{-1} C^T (\bar{z}_1 - z_1)$$

(22)

Or

$$\begin{aligned}
\dot{z}_1 &= -\gamma \bar{z}_1 + K \bar{z}_2 + \frac{1}{\mu_1} u_s - 3\theta (\bar{z}_1 - z_1) \\
\dot{z}_2 &= \frac{1}{T_r} \left( \frac{M_r}{T_r} \bar{z}_1 - \bar{z}_2 \right) - p \bar{z}_3 J_2 \left( \frac{M_r}{T_r} \bar{z}_1 - \bar{z}_2 \right) \\
&\quad - p \omega J_2 \psi_r + \frac{2}{\mu_1} J_2 \psi_r - \frac{2\beta^2}{\theta} (\bar{z}_1 - z_1) \\
\dot{z}_3 &= \left[ \frac{2}{T_r} J_2 \psi_r + \frac{2}{\mu_1} J_2 \psi_r \right]^{-1} \bar{z}_2
\end{aligned}$$

(23)

Referring to (7), the rotor flux is governed by the following equations:

$$\hat{\psi}_r = \left( \frac{1}{T_r} J_2 - p \Omega J_2 \right)^{-1} \bar{z}_2$$

(24)

### 4.5 Sliding mode observers

At first glance, the following vector seems to be a potential candidate for the expression of $\Psi(\bar{z})$:

$$\Psi_{sign}(\bar{z}) = C^T C \text{sign}(\bar{z}) = C^T \text{sign}(\bar{z}_1)$$

(25)

$$= C^T \text{sign}(\bar{z}_1) = C^T \bar{C} \text{sign}(\bar{z}_1)$$

where sign is the usual sign function with

$$\text{sign}(\bar{z}_1) = \begin{bmatrix} \text{sign}(\bar{z}_{11}) \\
\text{sign}(\bar{z}_{12})
\end{bmatrix}$$

then:

$$\dot{z} = f(u, \bar{z}) - \theta \Lambda^{-1} \Delta_\theta^{-1} \Sigma^{-1} C^T \text{sign}(\bar{z}_1 - z_1)$$

(26)

Or

$$\begin{aligned}
\dot{z}_1 &= -\gamma \bar{z}_1 + K \bar{z}_2 + \frac{1}{\mu_1} u_s - 3\theta \text{sign}(\bar{z}_1 - z_1) \\
\dot{z}_2 &= \frac{1}{T_r} \left( \frac{M_r}{T_r} \bar{z}_1 - \bar{z}_2 \right) - p \bar{z}_3 J_2 \left( \frac{M_r}{T_r} \bar{z}_1 - \bar{z}_2 \right) \\
&\quad - p \omega J_2 \psi_r + \frac{2}{\mu_1} J_2 \psi_r - \frac{2\beta^2}{\theta} \text{sign}(\bar{z}_1 - z_1) \\
\dot{z}_3 &= \left[ \frac{2}{T_r} J_2 \psi_r + \frac{2}{\mu_1} J_2 \psi_r \right]^{-1} \text{sign}(\bar{z}_1 - z_1)
\end{aligned}$$

(27)
Indeed, condition (12) is trivially satisfied by (25). Similarly, for broken input bounded output systems. However, expression (25) cannot be used due to the discontinuity of sign function (see [10]). Indeed, such discontinuity makes the stability problem not well posed since the Lyapunov method used throughout the proof is not valid. In order to overcome these difficulties one shall use continuous functions which have similar properties as those of the sign function. This approach is widely used when implementing sliding mode observers. Indeed, consider the following function:

4.5.1 Tanh function:

\[ \gamma_{\tanh}(\tilde{x}) = C^T C \tanh(\tilde{x}) = C^T \tanh(\tilde{x}_1) \]
\[ = C^T \tanh(\tilde{z}_1) = C^T \bar{C} \tanh(\tilde{z}) \] (28)

where tanh denotes the hyperbolic tangent function; then:

\[ \dot{z} = f(u, z) - \theta \Delta^\gamma \Delta^\phi S^\gamma C^T \tanh(\tilde{z}_1 - z_1) \] (29)

Or

\[ \begin{cases} \dot{\tilde{z}}_1 = -\gamma \tilde{z}_1 + K \tilde{z}_2 + \frac{1}{\sigma_u} u_s - 30 \arctan(\tilde{z}_1 - z_1) \\ \dot{\tilde{z}}_2 = \frac{1}{\tau_r} \left( \frac{M}{\tau_r} \tilde{z}_1 - \tilde{z}_2 \right) - p z_{31} J_2 \left( \frac{M}{\tau_r} \tilde{z}_1 - \tilde{z}_2 \right) \\ \dot{\tilde{z}}_3 = \left[ \frac{p}{\tau_r} J_2 \psi_r \frac{M}{\tau_r} \tilde{z}_1 - \tilde{z}_2 \right]^T + \frac{\phi^2}{K_p} \arctan(\tilde{z}_1 - z_1) \end{cases} \]

4.5.2 Arctan function:

\[ \gamma_{\arctan}(\tilde{x}) = C^T C \arctan(\tilde{x}) = C^T \arctan(\tilde{x}_1) \]
\[ = C^T \arctan(\tilde{z}_1) = C^T \bar{C} \arctan(\tilde{z}) \] (33)

5 Comparison of sensorless observers

To examine practical usefulness, the proposed observer has been simulated for a three-phase 1.5kw asynchronous machine (see [11]), whose parameters are depicted in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pair motor</td>
<td>p</td>
</tr>
<tr>
<td>Frequency</td>
<td>f</td>
</tr>
<tr>
<td>Stator inductance cyclic</td>
<td>L_s</td>
</tr>
<tr>
<td>Rotor inductance cyclic</td>
<td>L_r</td>
</tr>
<tr>
<td>Cyclic mutual inductance</td>
<td>M</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>R_s</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>R_r</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>J</td>
</tr>
</tbody>
</table>

Table 1: Asynchronous machine parameters used in simulations

In order to evaluate the observer behaviour in the realistic situation, the measurements of \( \dot{z} \) issued from the model simulation have been corrupted by noise measurements with a zero mean value. The torque lead takes the shape of stair.

**High gain observer**: The adjustment parameter of the observer (23) is to chosen \( \theta = 150 \). The dynamic behaviour of the error of rotor flu is depicted in Figure 1 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flu error. The means of error flu equal \(-3.10^{-3}\) with very small variance \(4.9155 \times 10^{-5}\) this is almost surety. The pace of speed error is given by the figure 2 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b) where means of error rotating speed equal \(10.37 \times 10^{-2}\) and variance equal \(2.2929\); the curve of load torque is illustrated on figure 3 graph (a). In graph (b) appear
gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal $-5.88 \times 10^{-3}$ and variance equal $62.72 \times 10^{-2}$

**Sliding mode observer with tanh**: Estimation results of the proposed algorithm (30) with $\theta = 250$ is reported in Figure 4, 5 and 6. The behaviour of the error of rotor flu is depicted in Figure 4 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flu error. The means of error flu equal $-2.31 \times 10^{-2}$ with very small variance $2.2 \times 10^{-3}$ this is almost surety. The pace of speed error is given by the figure 5 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b) where means of error rotating speed equal $-2.3373$ and variance equal $33.8509$; the curve of load torque is illustrated on figure 6 graph (a). In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal $-1.7535$ and variance equal $14.4642$
Sliding mode observer $\arctan$: Under the same conditions with the function $\tanh$. One simulates for the function $\arctan$. The figure 7, 8 and 9 illustrates the pace of error flux error speed and error load torque in respectively. The behaviour of the error of rotor flux is depicted in Figure 7 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flux error. The means of error flux equal $-2.73 \times 10^{-2}$ with very small variance $2.8 \times 10^{-3}$ this is almost surety. The pace of speed error is given by the figure 8 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal $-2.6758$ and variance equal $38.7251$; the curve of load torque is illustrated on figure 9 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal $-1.9414$ and variance equal $15.9706$

6 Conclusion

In this paper, high gain and alternative form for a sliding mode observers are presented. they is observer makes possible to observe, rotor flux rotor speed and load torque. An observer with high gain and three others with sliding mode which the functions sign, $\tanh$ and $\arctan$. Observer whose sign gives chattering. High gain observer is good for the observation of rotor flux rotating speed and load torque.

References:

[1] A. Chouya. Cascade Observer Interconnected with an Estimator: Application to the Asyn-
chronous Motor Without Mechanical Sensor. (9th International Conference on Modeling and Simulation of Electric Machines, Converters and Systems, ELECTRIMACS2008, Université Laval), ref:33-flfz-1112, Québec-CANADA.


