Frequency identification of Hammerstein-Wiener systems with Backlash input nonlinearity

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Abstract: -The problem of system identification is addressed for Hammerstein-Wiener systems that involve memory operator of Backlash type bordered by straight lines as input nonlinearity. The system identification of this model is investigated by using easily generated excitation signals. Moreover, the prior knowledge of the nonlinearity type, being Backlash or Backlash-Inverse, is not required. The nonlinear dynamics and the unknown structure of the linear subsystem lead to a highly nonlinear identification problem. Presently, the output nonlinearity may be noninvertible and the linear subsystem may be nonparametric. Interestingly, the system nonlinearities are identified first using a piecewise constant signal. In turn, the linear subsystem is identified using a frequency approach.

Key-Words: - Hammerstein-Wiener systems, Backlash operator, Backlash-Inverse operator, Fourier expansions

1 Introduction

Nonlinear systems exist widely in industry and science applications (Nelles, 2001), among which the Hammerstein-Wiener (HW) model is one of the most typical cases (Vörös, 2004). The Hammerstein-Wiener models consist of a linear dynamic block sandwiched by two nonlinear elements (Fig.1). Identification of block-oriented nonlinear systems has been studied for decades (Nelles, 2001; Bruls et al., 1999; Wills and Ljung, 2010).

The Hammerstein-Wiener like models are used in a wide range of applications such as ionospheric dynamics (Palanthandalam-Madapusi et al., 2005) and RF power amplifier modelling (Taringou et al., 2010). The identification of block-oriented nonlinear systems has been dealt with following different approaches including half-substitution iterative technique (Vörös, 2004), subspace and separable nonlinear least-squares methods (Bruls et al., 1999), output error and maximum likelihood algorithms (Wills and Ljung, 2010), frequency identification (Giri et al., 2014).

The paper here is an expansion of the work (Brouri et al., 2014a). Relative to (Brouri et al., 2014a), this paper is more general in its consideration of the input and output nonlinearities. Indeed, the input nonlinearity can be Backlash or Backlash-Inverse, is not required. Further, the output nonlinearity is not necessarily invertible, except at a known point. In particular, the paper provides details of commonly used nonlinearities and has expanded the simulation study to include two other commonly used nonlinearities.

The Backlash nonlinearity (Fig.2) can be classified as a dynamic (i.e., with memory) and hard nonlinearity, commonly occurs in hydraulic servo-valves, electric servomotors, magnetic suspensions, bearings and gears (e.g. Figs.3-4). Backlash often occurs in transmission systems, it caused by the small gaps which exist in transmission mechanisms (Figs.3-4), e.g. the play between the teeth of the drive gear and those of the driven gear. Then, the Backlash operators limit the overall performance of control systems by causing delays, undesired oscillations and inaccuracy (Ramamurthy and Prabhakar, 2012; Walha et al., 2009). Backlash influence identification and modeling is necessary to design a precision controller for this nonlinearity (Kalantari and Foomani, 2009; Lewis and Selmic, 2000).

Nonlinear system identification based on HW models has been an active research topic especially over the last decade, including the recent contributions (Wills and Ninness, 2009; Brouri et al., 2014a; Brouri et al., 2014b). Amongst that are: iterative approaches (Zhu, 2002; Vörös, 2004), overparametrization methods (Schoukens et al., 2012), frequency dommaine methods (Brouri et al., 2014a; Crama and Schoukens, 2004), subspace algorithms (Goethals et al., 2005), and stochastic methods (Wang and Ding, 2008).

As a matter of fact, all proposed identification methods are based on several, more or less restrictive, assumptions concerning the system nonlinearities (invertible, memoryless), the linear subsystem (FIR, known structure), the input signals (Gaussian, PE). Then, most previous works, where that subsystem is supposed to be a transfer function of known order (e.g. Bai, 2002; Ni et al., 2013; Wang et al., 2009; Schoukens et al., 2012).

This paper addresses the more complex case of nonparametric HW systems involving Backlash operators (Fig.2). Apart from stability, no assumption is made on the (nonparametric) linear subsystem $G(s)$ which may thus be infinite order. The third main factor motivating this approach lies in the fact that the output nonlinearity is not globally invertible (unlike most previous works) but satisfied $h^{-1}(0) = 0$. This latter is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of unknown degree and parameters. Unlike many studies that consider the invertibility of output nonlinearity is a usual assumption. The degree p and the parameters of the polynomial can vary from one subinterval to another. In this paper we assume a general model of Backlash which is not required to be symmetric.

On the other hand, note that several approaches are proposed in literature in order to compensate the undesirable effect of Backlash (Nordin and Gutman, 2002). Although most of available control synthesis techniques require a model of the plant to be controlled, only few contributions can be found in literature addressing the identification of HW systems with Backlash. Then, it is not surprising that only a few methods are available that deal using a Backlash operator bordered by two straight-lines. In (Dong et al., 2009; Cerone et al., 2009), the nonlinearity is a Backlash operator with straight-line borders, but the problem of identification is addressed to Wiener systems. From an identification viewpoint, the difficulty lies not only in the systems nonlinearities of the model dynamics but also in its interconnected structure making its internal signals inaccessible to measurements.

The proposed identification method performs in two stages. The systems nonlinearities (i.e. the output nonlinearity and the Backlash borders) are identified first using a piecewise constant input. The linear subsystem is identified in the second stage using a frequency identification method. The weak decoupling between the two stages entails an increased parameter estimation accuracy. The identification method also enjoys the simplicity of the required input signals (constant and sinusoidal excitations are sufficient) and the consistency of all involved estimators.

The paper is organized as follows: relevant mathematical tools are described in Section 2; Section 3 formulates the problem and derives some preliminary results; the main results are given in Section 4 along with some remarks and proposition concerning the scheme applies to identify the system nonlinearities applied; the linear subsystem identification is coped with in Section 5. The performances of the identification method are illustrated by simulation in Section 6.

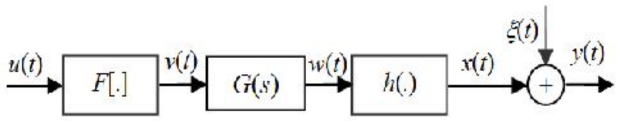

Fig.1. Hammerstein-Wiener Model structure with Backlash input nonlinearity.

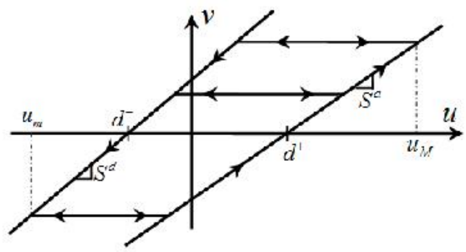

Fig.2. Backlash bordered by straight lines.

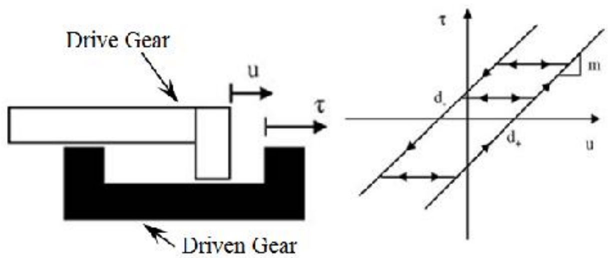

Fig.3. Example of Backlash nonlinearity occurring in mechanical systems.

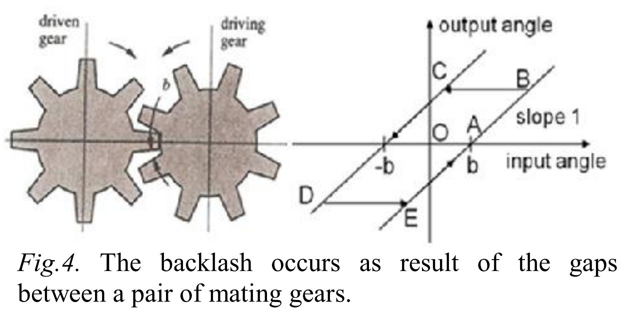

Fig.4. The backlash occurs as result of the gaps between a pair of mating gears.

2 Mathematical preliminaries

2.1 Backlash Operator

A Backlash operator is a memory element characterized by two borders, and is denoted $B\left[v,u,\dot{u}\right](t)$; where $u(t)$ is the input signal, $v(t)$ the generated signal and $\dot{u}(t)$ designates the first derivative of $u(t)$. General characteristic of Backlash operator bordered by two straight-lines is shown in Fig.2, and a mathematical model is given by (Tao and Kokotovic 1996):

$$\dot{v}(t)=\begin{cases} S\dot{u}(t) & \text{if } \dot{u}(t)>0 \text{ and } v(t)=S^a(u-d^+) \\ & \text{if } \dot{u}(t)<0 \text{ and } v(t)=S^d(u-d^-) \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

One can see that, whenever the input of Backlash $u(t)$ changes its direction, the output $v(t)$ is delayed from motion of $u(t)$.

2.2 Backlash-Inverse Operator

The Backlash-Inverse operator is also a memory element (dynamic system) characterized by a couple of straight lines (Fig.5), and is denoted $B_{inv}\left[u,z,\dot{z}\right](t)$. When submitted to an input signal $z(t)$, it generates an output signal $u(t)$ defined as follows:

$$u(t)=\begin{cases} \frac{1}{S^a}z(t)+d^+ & \text{if } \dot{z}(t)>0 \\ \frac{1}{S^d}z(t)+d^- & \text{if } \dot{z}(t)<0 \\ u(t-1) & \text{otherwise} \end{cases} \qquad (2)$$

This definition entails no condition on the borders couple. In particular, these latter may be nonsymmetrical.

2.3 Compound Operators

In order to cancel the effect of Backlash (or Backlash-Inverse) in the system, the Backlash pre-compensator needs to generate the inverse of the Backlash operator (Tao and Kokotovic, 1996; Tao and Kokotovic, 1993). The Backlash-Inverse compensator $B_{inv}\left[u,z,\dot{z}\right](t)$ of $B\left[v,u,\dot{u}\right](t)$ (Fig.2) is shown in Fig.5.

Lemma 1. (Tao and Kokotovic, 1996; Tao and Kokotovic, 1993):

The characteristic $B_{inv}\left[u,z,\dot{z}\right]$, defined by (2) is the inverse of the characteristic $B\left[v,u,\dot{u}\right]$ (defined by (1)) in the sense:

- $B\left(B_{inv}\left[u,z,\dot{z}\right](t_0)\right)=z(t_0)$ gives $B\left(B_{inv}\left[u,z,\dot{z}\right](t)\right)=z(t)\ \forall\ t>t_0$,
- $B_{inv}\left(B\left[u,z,\dot{z}\right](t_0)\right)=z(t_0)$ gives $B\left(B_{inv}\left[u,z,\dot{z}\right](t)\right)=z(t)\ \forall\ t>t_0$. □

Lemma 1 above states that if for some time t0 the Backlash-Inverse operator is able to invert the Backlash, then it serves a Backlash-Inverse for all time thereafter; i.e. the compound operators $B_{inv}\left[.\right]\circ B\left[.\right]=I$ and $B\left[.\right]\circ B_{inv}\left[.\right]=I$, where I being the identity operator.

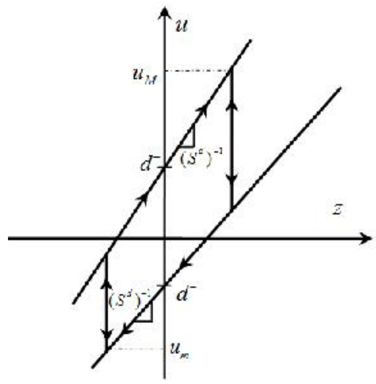

Fig.5. Backlash-Inverse operator.

3 Identification Problem formulation

Hammerstein-Wiener systems consist of a linear dynamic block sandwiched by two nonlinear elements (Fig. 1). In this study, the input nonlinearity $F[.]$ is allowed to be a memory operator of Backlash type bordered by two straight-lines (Fig.2). The output nonlinearity $h(.)$ is memoryless and so is entirely characterized by a single function, denoted $h(.)$.

Analytically, the Hammerstein-Wiener system is described by the following equations:

$$v(t)=F\left[v,u,\dot{u}\right](t) \qquad (3a)$$

$$w(t)=g(t)*v(t) \qquad (3b)$$

$$x(t)=h\left(w(t)\right);\ \ y(t)=x(t)+\xi(t) \qquad (3c)$$

Where $F\left[.\right]\in\left\{B\left[.\right],B_{inv}\left[.\right]\right\}$, $g(t)=L^{-1}\left(G(s)\right)$ and * refers to the convolution operation. The identification problem at hand consists in accurately identifying the transfer function $G(s)$ as well as the system nonlinearities (i.e. the nonlinear operator parameters (S^a, d^+, S^d, d^-) and the output nonlinearity $h(.)$). The identification must only rely on the use of the input and output signals, i.e. the system input and output signals $(u(t), y(t))$ are accessible to measurement while the internal signals $v(t)$, $w(t)$ and $x(t)$ are not (Fig.1). The equation error $\xi(t)$ accounts for external disturbances (or measurement noise) and other modelling effects; it is supposed to be stationary and uncorrelated with the control input $u(t)$.

The input nonlinearity $F[.] \in \{B[.], B_{inv}[.]\}$ undergoes the Backlash $B[.]$ or Backlash-Inverse $B_{inv}[.]$ forms. At this stage, the transfer function $G(s)$ assumes no known structure, but it must be asymptotically stable to make possible open-loop system identification and $G(0) \neq 0$.

The complete system model is analytically described by equations (3a-c). At this point, it is worth emphasizing the plurality of the model $(F[v,u,\dot{u}], G(s), h(w))$ (i.e. the considered identification problem does not have a unique solution), defining the Hammerstein-Wiener systems. Indeed, if $(F[v,u,\dot{u}], G(s), h(w))$ represents a solution then, any model of the form $(F[v,u,\dot{u}]/k_1, G(s)/k_2, h(k_1 k_2 w))$ is also a solution of the above identification problem, whatever $k_1 > 0$ and $k_2 > 0$. To reach this goal, it will prove judicious to focus on the model $(\bar{F}[.], \bar{G}(s), \bar{h}(.))$ defined as follows:

$$\bar{F}[v,u,\dot{u}] = F[v,u,\dot{u}] / S^a \tag{4a}$$

$$\bar{G}(s) = G(s) / G(0) \tag{4b}$$

$$\bar{h}(w) = h\left(G(0) S^a w\right) \tag{4c}$$

Property (4b) means that $G(s)$ is made unit static gain. To avoid multiplying notations, the unique model satisfying (4a-c) will still be denoted $(F[.], G(.), h(.))$.

Recall that the Backlash behavior is such that, when the input $u(t)$ starts increasing and the backlash working point $(u(t), v(t))$ moves, e.g. on the descendant border, the point $(u(t), v(t))$ moves first along a horizontal path. Once reaches the ascendant lateral border, the backlash working point $(u(t), v(t))$ moves along it until the input $u(t)$ stops increasing. Similar interpretations hold for the backlash working point descendant stages. Obviously, if the input signal $u(t)$ spans monotonically, in both senses, a sufficiently wide working interval then, the working point will span a closed backlash cycle, passing from one border to the other along two connecting horizontal paths (Fig.2). If the input working interval $[u_m \;\; u_M]$ is not sufficiently large, the resulting steady-state internal signal $v(t)$ will be constant i.e. the backlash working point $(u(t), v(t))$ will move along a horizontal segment (Fig.6a). Then, the system output $y(t)$ becomes constant (up to noise) after a transient period. This observation can be based upon in practice to discard non-suitable choices of $[u_m \;\; u_M]$.. In the case of Backlash-Inverse operator, the working point will span a closed backlash cycle, passing from one border to the other (Fig.6b), regardless of the working interval. Presently, one considers a memory operator of Backlash type; a symmetrical procedure could similarly be described in the case of Backlash-Inverse type.

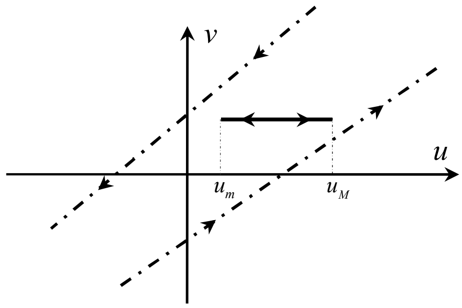

Fig.6a. The Backlash cycle reduces to a horizontal segment.

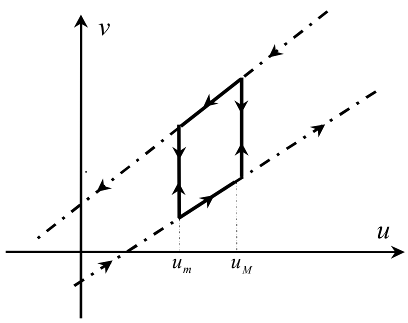

Fig.6b. Example of obtained backlash cycle for a small working interval.

4 Nonlinear elements Identification

In this section, an identification scheme is proposed to get estimates of nonlinear elements. Furthermore, it can be shown that, it is possible to separately identify the system nonlinearities. In other words, the identification of the linear subsystem is not necessary as long as the nonlinear elements are concerned.

Recall that, the output nonlinearity is supposed to be well approximated, within any subinterval, with a polynomial of unknown degree and parameters. The degree p and the polynomial parameters can vary from one subinterval to another. For simplicity of notation, the input nonlinearity $F[v,u,\dot{u}]$ is denoted $F[u]$. Practically, if the HW system is excited by any constant input U_j, then the transient dynamic effect of $G(s)$ vanishes, leading to constant asymptotic values of all system signals. Specifically, as the linear subsystem is asymptotically stable with unit static gain (see (4b)), one gets in the steady state:

$$u(t)=U_j \quad \Rightarrow \quad v(t)=w(t)\approx W_j$$
$$\text{and} \quad x(t)\approx X_j=h(W_j)=h\circ F[U_j] \tag{5}$$

It is readily seen from (5) that, the HW system boils down to the compound function $h\circ F[.]$. Accordingly, to identify the input and output nonlinearities, the suggested protocol involves two main stages, referred to ascendant and descendant. Each stage involves a series of constant inputs. In the ascendant experimental stage, the HW model is successively excited by a set of constant inputs with amplitudes:

$$U_1<U_2<\ldots<U_N \tag{6}$$

Using (4a), since F[.] is a backlash operator and the amplitude sequence $\{U_j;\, j=1\ldots N\}$ is increasing, there is an integer $j_a\in[1\,,N]$ such that:

$$V_j=V_1=F[U_1] \quad \text{for} \quad j=1\ldots j_a \tag{7a}$$

$$V_j=U_j-d^+ \quad \text{for} \quad j=j_a+1\ldots N \tag{7b}$$

Then, as the linear block is asymptotically stable with unit static gain and combining (5) and (7a-b), one immediately gets:

$$W_j=W_1=V_1=F[U_1] \quad \Rightarrow$$
$$X_j=h\circ F[U_1]=X_1 \quad \text{for} \quad j=1\ldots j_a \tag{8a}$$

$$W_j\approx V_j=U_j-d^+ \quad \Rightarrow$$
$$X_j=h\left(U_j-d^+\right) \quad \text{for} \quad j=j_a+1\ldots N \tag{8b}$$

The internal $x(t)$ signal is not directly measurable. Then, using (3c) and the fact that $\xi(t)$ is zero-mean, the steady state undisturbed output X_j can be recovered by averaging $y(t)$ on a sufficiently large interval. Then, just as suggested in (Ljung, 1999), the following averaging is considered:

$$\hat{X}_j(M)=\frac{1}{M}\sum_{i=1}^{M}y(i), \quad \text{with} \quad M>>1 \tag{9}$$

Accordingly, the noise $\xi(t)$ is presently supposed to be a zero-mean ergodic stochastic process. This, together with (9), yields:

$$\hat{X}_j(M)=X_j+\frac{1}{M}\sum_{i=1}^{M}\xi(i)\underset{M\to\infty}{\to}X_j \quad (w.p.\ 1) \tag{10}$$

Remark 1.

1) The set of points $\{(U_j,F[U_j]);\, j=j_a+1\ldots N\}$ belong to the Backlash ascendant border. The set of points $\{(U_j,F[U_j]);\, j=1\ldots j_a\}$ moves along a horizontal segment. These results can be observed practically. Then, the system output $y(t)$ remains constant (up to noise) for $U_j\in\{U_1;\ldots;U_{j_a}\}$.

2) If $h(.)$ is polynomial function of degree p, then the compound function $h(u-d^+)$ is also a pth degree polynomial. Accordingly, to estimate the parameters of the nonlinearity $h(u-d^+)$, the following requirement must hold:

$$N-j_a\ge p+1 \tag{11}$$

Then, the nonlinearity $h(u-d^+)$ will be estimated making use the set of points $\left\{\left(U_j,\hat{X}_j(M)\right);\, j=j_a+1\ldots N\right\}$, where $N\ge p+1+j_a$.

3) The non-linearity $h(u-d^+)$ crosses the x-axis at the point $(d^+,0)$. At this point, it is worth emphasizing that, $h^{-1}(0)=0$ allows to determine an

accurate estimate $\hat{d}^+(M)$ of d^+ (Fig.7) using the estimate $\hat{h}_M(u-d^+)$ of $h(u-d^+)$.

4) In the case of Backlash-Inverse operator, one immediately gets $j_a = j_d = 0$. □

Proposition 1. (Kozen and Landau, 1989)
Let the compound polynomial function $h \circ f$. Complete decompositions are not unique. Consider the examples:

- $hof = h(x+d)o(f-d)$.
- $x^p o x^r = x^r o x^p$. □

By Proposition 1, the complete decompositions of hof, where $h(.)$ is polynomial function of degree p and $f(u) = u - d^+$, are not unique. It is readily seen from (4a-c) and $h^{-1}(0) = 0$ that, the complete decompositions of $ho(u-d^+)$ are uniquely determined. Then, an accurate estimate of the output nonlinearity $h(.)$ can be provided (Fig.7). Indeed, using the function changes $h^a(u) = \hat{h}_M(u-d^+)$, the estimate of the output nonlinearity achieved:

$$\hat{h}_M(u) = h^a(u + \hat{d}^+(M)) \tag{12}$$

On the other hand, the parameters of the descendant lateral border S^d and d^- can be provided using the descendant experimental stage. Specifically, the HW model is successively excited by a set of constant inputs with decreasing amplitudes:

$$U_{N+1} = U_{N-1} < U_{N+2} = U_{N-2} < \dots < U_{2N-1} = U_1 < U_{2N} \tag{13}$$

Similarly, following the same undisturbed output estimator (9), a set of points along the path of $ho(S^d(u-d^-)) = h\left(S^d u - S^d d^-\right)$ can be estimated, i.e. $\left\{\left(U_j, \hat{X}_j(M)\right); j = N + j_d + 1 \dots 2N\right\}$ where $1 \le j_d \le N$ is an integer, such that:

$$V_j = V_N = F[U_N] \quad \text{for} \quad j = N \dots N + j_d \tag{14a}$$

$$V_j = S^d(U_j - d^-) \quad \text{for} \quad j = N + j_d + 1 \dots 2N \tag{14b}$$

Accordingly, an estimate $\hat{h}_M\left(S^d u - S^d d^-\right)$ of $h\left(S^d u - S^d d^-\right) = ho(S^d(u-d^-))$ can be easily determined. Finally, the fact that the output nonlinearity is known entails the problem of identifying S^d and d^- a trivial issue (see Kozen and Landau, 1989). Specifically, if $h(u)$ is polynomial of degree p, then $h(S^d(u-d^-))$ is also polynomial of degree p and crosses the x-axis at the $(d^-, 0)$. In which the coefficient of the highest order term is equal to the leading coefficient of $h(u)$ multiplied by $(S^d)^p$.

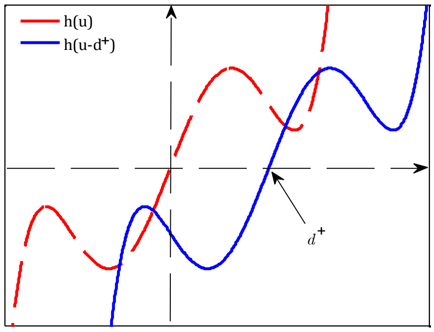

Fig.7. Comparison between $h(u)$ and $h(u-d^+)$.

5 Linear subsystem identification

The result of Section 4 is a quite interesting achievement as it shows that, it is possible to separately identify the nonlinear elements. In the present section, a frequency identification method is proposed to estimate the nonparametric linear subsystem $G(j\omega)$, whatever the frequencies $\omega > 0$. Note that the Adaptive compensation is the most appropriate technique to handle the uncertainty in the Backlash parameters; however, since Backlash is not a differentiable nonlinearity, recent nonlinear control design methodologies cannot be applied.

Then, one key idea is to neutralize the effect of $F[.]$ (Tao and Kokotovic, 1996; Tao and Kokotovic, 1993) by placing its inverse as pre-compensator (Fig.8). Doing so, the augmented system, including the pre-compensator, boils down to a Wiener model. The result system is excited by the simple sinusoidal input:

$$z(t) = U \sin(\omega t) \tag{15}$$

In view of Lemma 1 and from (15), the steady state of the internal signal can be expressed as follows:

$$v(t) = z(t) = U \sin(\omega t) \tag{16}$$

Then, it follows from (3b) that (in steady-state):

$$w(t) = U\left|G(j\omega)\right| \sin\left(\omega t - \varphi(\omega)\right) \tag{17}$$

where $\varphi(\omega) = -\angle G(j\omega)$. At this point, the output nonlinearity *h*(.) is known and can be approximated with a polynomial of degree *p*. The undisturbed system output *x*(*t*) takes the following form:

$$x(t) = h\left(w(t)\right) = \sum_{i=0}^{p} c_i \left(U\left|G(j\omega)\right|\right)^i \sin\left(\omega t - \varphi(\omega)\right)^i \quad (18)$$

where $c_0 = 0$ (see Section 4). On the other hand, the power formulas $(\sin\alpha)^{2i}$ and $(\sin\alpha)^{2i+1}$ can be expressed as:

$$\left(\sin\alpha\right)^{2i} = \frac{1}{2^{2i}} C_i^{2i} + \frac{(-1)^i}{2^{2i-1}} \sum_{r=0}^{i-1} (-1)^r C_r^{2i} \cos\left(2(i-r)\alpha\right) \quad (19a)$$

$$\left(\sin\alpha\right)^{2i+1} = \frac{(-1)^i}{4^i} \sum_{r=0}^{i} (-1)^r C_r^{2i+1} \sin\left((2i+1-2r)\alpha\right) \quad (19b)$$

For convenience, let us recapitulate the explicit signal relationships obtained from (18), using (19a-b):

$$x(t) = \sum_{k=0}^{p} A_k\left(\left|G(j\omega)\right|\right) \sin\left(k\omega t + \beta_k\left(\varphi(\omega), \left|G(j\omega)\right|\right)\right) \quad (20)$$

It is readily checked that, the amplitude A_k depends on $\left|G(j\omega)\right|$, c_i $(i = 1,\dots,p)$ and *U*. Also, the phase β_k depends on all these parameters in addition to the phase $\varphi(\omega)$. The rule (20) gets benefit from two major facts: (i) the only unknown parameters in (20) are the modulus $\left|G(j\omega)\right|$ and the phase $\varphi(\omega)$; (ii) *x*(*t*) is the sum of sinusoidal signals and it is worth considering a single frequency component. Consequently, an accurate estimate of the complex amplitudes $G(j\omega)$ can be obtained by measuring a single frequency component of *x*(*t*).

On the other hand, one can notice that the steady-state undisturbed output *x*(*t*) is periodic of same period $T = 2\pi / \omega$ as the input, it can be developed in Fourier series:

$$x(t) = s_0 + \sum_{k=0}^{\infty} s_k \sin\left(k\omega t + \psi_k\right) \quad (21)$$

Accordingly, the modulus $\left|G(j\omega)\right|$ and the phase $\varphi(\omega)$ in (20) can be accurately estimated if s_k and ψ_k are known. One difficulty with the considered identification problem is that, the undisturbed system output *x*(*t*) is not accessible to measurement and the system output *y*(*t*) is infected by the disturbance $\xi(t)$ whose stochastic law is not known. Then, it follows from (3c), one immediately gets from (21):

$$y(t) = s_0 + \sum_{k=0}^{\infty} s_k \sin\left(k\omega t + \psi_k\right) + \xi(t) \quad (22)$$

Now, to avoid the above complexity, bearing in mind the fact that $u(t)$ is a $T-$periodic excitation signal. Then, this generates, in steady-state $T-$periodic signal $x(t)$. Then, just as suggested in (Ljung, 1999), the following $T-$periodic averaging of the undisturbed system output $x(t)$ is considered:

$$\hat{x}_N(t) = \frac{1}{N} \sum_{l=0}^{N-1} y\left(t + lT\right), \quad \text{for } 0 \le t \le T \quad (23a)$$

$$\hat{x}_N(t + lT) = \hat{x}_N(t), \quad \text{otherwise} \quad (23b)$$

for some (large enough) integer N. Bearing in mind that $x(t)$ is $T-$periodic signal, it follows from (3c) and (23a) that (in steady-state):

$$\hat{x}_N(t) = x(t) + \frac{1}{N} \sum_{l=0}^{N-1} \xi(t + lT) \quad (24)$$

Accordingly, the noise $\xi(t)$ is presently supposed to be a zero-mean ergodic stochastic process featuring the $T-$periodic stationarity (on the set of T's of interest). The periodic stationarity means that $E\left(\xi(t + kT)\right) = E(\xi(t))$; for all t, k. This, together with zero-mean ergodicity, yields:

$$\frac{1}{N} \sum_{l=0}^{N-1} \xi(t + lT) \underset{N\to\infty}{\to} E(\xi(t + kT)) = 0 \;\; (w.p.1)\; \forall \;\; k \in \mathbf{N} \quad (25a)$$

Then, it follows from (21), (24) and (25a) that:

$$\hat{x}_N(t) \underset{N\to\infty}{\to} x(t) = s_0 + \sum_{k=0}^{\infty} s_k \sin\left(k\omega t + \psi_k\right) \;\; (w.p.1) \quad (25b)$$

That is, the s_k's and ψ_k's turn out to be $(w.p.1)$ the limits of Fourier-expansion parameters of $\hat{x}_N(t)$ as $N \to \infty$. The estimates of these parameters are given by the usual expressions:

$$\hat{s}_k(N) = \sqrt{\hat{a}_k(N)^2 + \hat{b}_k(N)^2}\,; \quad \text{for } k = 1, 2\ldots \quad (26a)$$

$$\hat{\psi}_k(N) = \tan^{-1}\left(\frac{\hat{a}_k(N)}{\hat{b}_k(N)}\right); \quad \text{for } k = 1, 2\ldots \quad (26b)$$

$$\hat{s}_0(N) = \frac{1}{T}\int_0^T \hat{x}_N(t)\,dt = \frac{\hat{a}_0(N)}{2} \quad (26c)$$

where:

$$\hat{a}_k(N) = \frac{2}{T}\int_0^T \hat{x}_N(t)\cos(k\omega t)\,dt\,; \text{ for } k = 1, 2\ldots \quad (26d)$$

$$\hat{b}_k(N) = \frac{2}{T}\int_0^T \hat{x}_N(t)\sin(k\omega t)\,dt\,; \text{ for } k = 1, 2\ldots \quad (26e)$$

Finally, the expression (20) holds whatever the sinusoidal input $u(t)$ provided this leads to a (steady-state) signal $x(t)$ that is T-periodic and it can be developed in Fourier series (21). Using the filtered version $\hat{x}_N(t)$ (given by (23a-b)) of $x(t)$, the Fourier-expansion parameters s_k's and ψ_k's are provided using (26a-e). Accordingly, the complex amplitudes $G(j\omega)$ can be obtained by focusing on a single frequency component of $x(t)$.

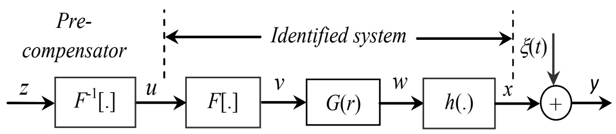

Fig.8. The system to be identified augmented with pre-compensator.

Remark 2.
Practically, it is judicious to limit the Fourier-expansion of $x(t)$ to those frequencies for which the Fourier-series coefficients are significant. Furthermore, it readily follows from (20) that:

$$\sum_{k=0}^{p} A_k\left(\left|G(j\omega)\right|\right)\sin\left(k\omega t + \beta_k\left(\varphi(\omega), \left|G(j\omega)\right|\right)\right) = s_0 + \sum_{k=0}^{\infty} s_k \sin\left(k\omega t + \psi_k\right) \quad (27)$$

One immediately gets from (27):

$$\sum_{k=p+1}^{\infty} \frac{s_k^2}{2} \approx 0 \quad (28)$$

Then it is reasonable to consider only the most significant frequency components, specifically, the coefficient list: $s_k \ (k = 0, 1, 2\ldots p)$). □

6 Simulation

Presently, the system (3a-c) is characterized by:

$$G(s) = \frac{0.2}{(s+0.5)(s+0.2)} \quad (29a)$$

$$h(x) = 0.2x^5 - 0.42x^3 + 0.25x \quad (29b)$$

and a Backlash operators $F[.]$ described by (1) and is shown by Fig.9. The input nonlinearity has the parameters: $S^a = 1$, $S^d = 1$, $d^+ = 0.5$ and $d^- = -0.5$. The noise $\xi(t)$ is a sequence of normally distributed (pseudo) random numbers, with zeromean and standard deviation $\sigma_\xi = 0.01$.

Following Section 3, we focus on the particular model $\left(\bar{F}[.], \bar{G}(s), \bar{h}(.)\right)$ defined by (4a-c), presently characterized by:

$$\bar{F}[u] = F[u] \quad (S^a = 1) \quad (30a)$$

$$\bar{G}(s) = \frac{0.1}{(s+0.5)(s+0.2)} \quad (30b)$$

$$\bar{h}(x) = 6.4x^5 - 3.36x^3 + 0.5x \quad (30c)$$

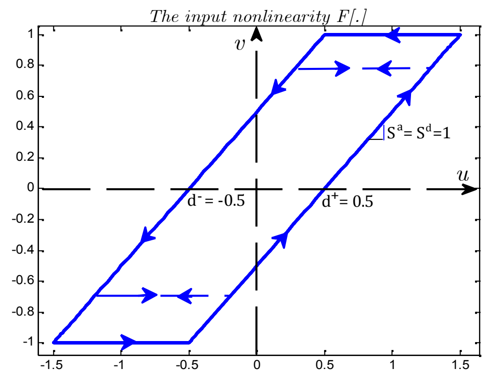

Fig.9. The Backlash operators considered in simulation.

6.1 Identification of system nonlinearities

According to the developed identification method, the first stage consists in identifying the input and output nonlinearities. Then, the Hammerstein-Wiener system is excited with a piecewise constant input (Fig.10) with $N = 10$. The resulting steady-state output signal $y(t)$ is shown by Fig.11. This shows that $j_a = 4$. Using the estimator (9) for $M = 100$, the undisturbed output estimates $\hat{X}_j(M)$ is given by Table 1. This shows that all estimates are quite close to their true values. Then, the estimate $\hat{h}_M(u - d^+)$ of $\bar{h}(u - d^+)$ is obtained using the set of points $\left\{\left(U_j, \hat{X}_j(M)\right); j = j_a + 1 \ldots N\right\}$.

Specifically, let $\hat{\bar{C}}_M = \left[\hat{\bar{c}}_0(M)\ldots\hat{\bar{c}}_5(M)\right]^T$ the coefficients vector of $\hat{h}_M(u-d^+)$. The vector $\hat{\bar{C}}_M$ is readily obtained as follows:

$$\hat{\bar{C}}_M = \begin{bmatrix} 1 & U_5 & U_5^2 & \cdots & U_5^5 \\ 1 & U_6 & U_6^2 & \cdots & U_6^5 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & U_{10} & U_{10}^2 & \cdots & U_{10}^5 \end{bmatrix}^{-1} \begin{bmatrix} \hat{X}_5(M) \\ \\ \vdots \\ \hat{X}_{10}(M) \end{bmatrix} = V^{-1} \begin{bmatrix} \hat{X}_5(M) \\ \vdots \\ \hat{X}_{10}(M) \end{bmatrix} \quad (31)$$

where V is Vandermonde matrix. Presently, $\hat{h}_M(u-d^+)$ is given as follows:

$$\hat{h}_M(u-d^+) = 6.39u^5 - 16.01u^4 + 12.66u^3 - 2.97u^2 - 0.022u - 0.029 \quad (32)$$

which is close to the true nonlinearity: $\bar{h}(u-d^+) = 6.4u^5 - 16u^4 + 12.64u^3 - 2.96u^2 - 0.02u - 0.03$. Then, one gets the estimate $\hat{d}^+(M) = 0.503$. It readily follows from (12) that, the estimate $\hat{h}_M(u)$ of the output nonlinearity is achieved. Specifically, the output nonlinearity estimate $\hat{h}_M(u)$ is $\hat{h}_M(u-d^+)$ horizontally shifted at the origin. For convenience, the nonlinearities $\bar{h}(u)$, $\bar{h}(u-d^+)$ and $\hat{h}_M(u)$, and the set of points $\left\{\left(U_j, \hat{X}_j(M)\right); j = 5 \ldots 10\right\}$ are plotted in Fig.12. These results show that, the output nonlinearity estimate $\hat{h}_M(.)$ is close to the true nonlinearity $\bar{h}(.)$, which clearly confirms satisfactory model accuracy.

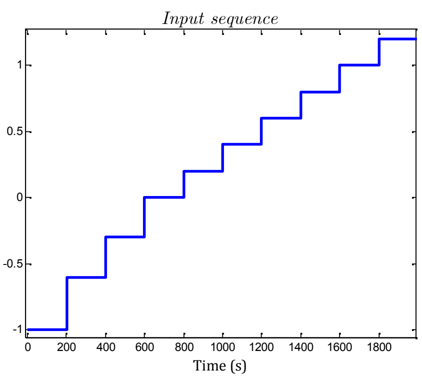

Fig.10. Input signal used for the identification of the system nonlinearities.

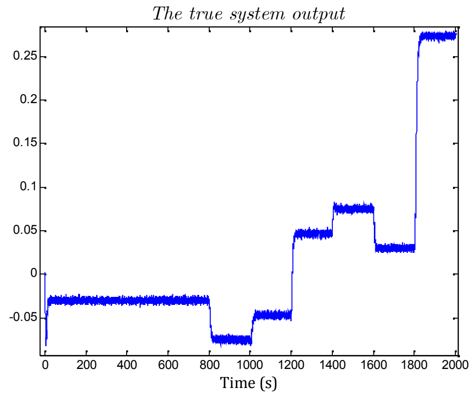

Fig.11. The measured system output signal *y*(*t*).

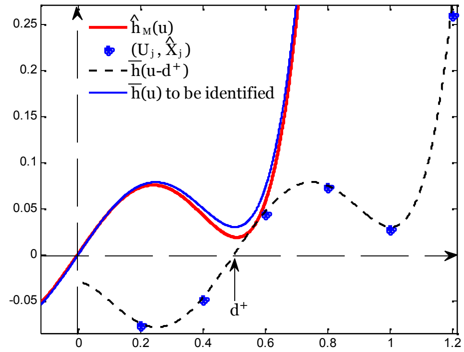

Fig.12. The nonlinearities $\bar{h}(u)$ (to be identified), $\bar{h}(u-d^+)$ and $\hat{h}_M(u)$, and the set of points $\left\{\left(U_j, \hat{X}_j(M)\right)\right\}$.

Similarly, using set of decreasing points $\left\{\left(U_j, \hat{X}_j(M)\right)\right\}$, the descendant border parameters can be accurately estimated. Specifically, apply the decreasing sequence as illustrated in Fig.13 to the above nonlinear system. The system output signal is displayed in Fig.14. Then, applying the estimator (9) (with $M = 100$), the undisturbed output estimates $\hat{X}_j(M)$ have been got and a sample of them is shown by Table 1, which is close to the true value. The value of j_d corresponding to the decreasing experiment is $j_d = 5$. It follows Section 4, the HW system boils down to the compound polynomial function $\bar{h} \circ \left(S^d u - S^d d^-\right)(U_j)$ for $j = N + j_d + 1 \ldots 2N + 1$ ($N - p \geq j_d + 1$). It turns out that this latter involves two unknown quantities, the slope of the descending border S^d, on the one hand, and the parameter d^-, on the other. Accordingly, using data acquisition (i.e. U_j and $\hat{X}_j(M)$), one gets the estimate:

$$\hat{h}_M \circ \left(\hat{S}^d(M)(u - \hat{d}^-(M))\right)(u) = 6.6u^5 + 16.7u^4 + 13.52u^3 + 3.36u^2 + 0.04u + 0.031$$

Consequently, using these data and knowing $\hat{h}_M(.)$, estimates of the parameters S^d and d^- are readily obtained. The estimate of S^d can be achieved using solely the leading coefficient of $\hat{h}_M \circ \left(\hat{S}^d(M)(u - \hat{d}^-(M))\right)$. $\hat{d}^-(M)$ can be given making use any other coefficient. The estimate of the descending border parameters can also be obtained making use the complete decompositions of $\hat{h}_M \circ \left(\hat{S}^d(M)(u - \hat{d}^-(M))\right)$ (Kozen and Landau, 1989). Then, one gets the estimates of the descending border parameters: $\hat{S}^d(M) = 1.01$ and $\hat{d}^-(M) = -0.49$.

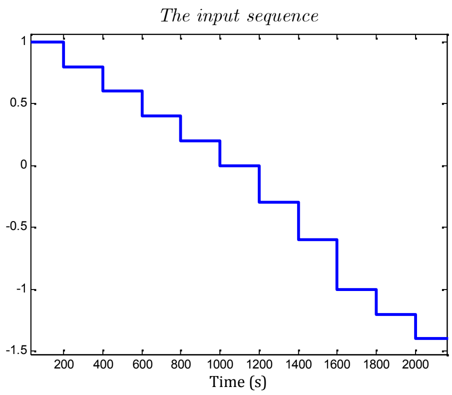

Fig.13. Input signal used for the identification of the Backlash descendant border.

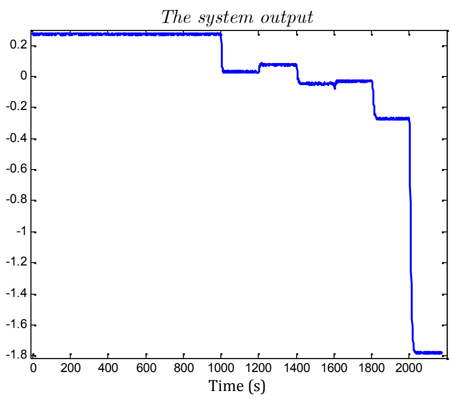

Fig.14. The measured system output corresponding to the decreasing input signal.

Table 1. The undisturbed output estimates $\hat{X}_j(M)$

j	5	6	7	8	9	10
U_j	0.2	0.4	0.6	0.8	1	1.2
$X_j(M)$	-0.075	-0.047	0.047	0.075	0.03	0.27
$\hat{X}_j(M)$	-0.076	-0.048	0.045	0.074	0.028	0.26
j	16	17	18	19	20	21
U_j	0	-0.3	-0.6	-1	-1.2	-1.4
$X_j(M)$	0.03	0.075	-0.047	-0.03	-0.27	-1.78
$\hat{X}_j(M)$	0.031	0.076	-0.046	-0.029	-0.265	-1.775

6.1 Identification of the linear transfer $G(s)$

The second stage consists in identifying the system transfer function $G(s)$. Following the procedure of Section 5, the first step consists in placing the Backlash-Inverse operator (Fig.5) as pre-compensator (Fig.8). The augmented system is excited with a sine input $z(t) = U_1 \sin(\omega_1 t)$. Presently, the choice $U_1 = 1$ and $\omega_1 = 0.01\, rd/s$ is made. The resulting steady-state output signal $y(t)$ is shown by Fig.15a and its filtered version $\hat{x}_N(t)$, obtained from $y(t)$ using (23a-b) with $N = 50$, is plotted in Fig.15b. The internal signal $w(t)$ turns out to be: $w(t) = U_1 |G(j\omega_1)| \sin(\omega_1 t - \varphi(\omega_1))$. Accordingly, knowing the estimate of the output nonlinearity, the undisturbed system output $x(t)$ can be expressed as follows:

$$x(t) \approx \hat{h}_M(w(t)) = \sum_{i=0}^{5} \hat{c}_i \left(U_1 |G(j\omega_1)|\right)^i \sin\left(\omega_1 t - \varphi(\omega_1)\right)^i \quad (33)$$

with $\hat{C}_M = [\hat{c}_0 \ldots \hat{c}_5]^T$ is the coefficients vector of $\hat{h}_M(.)$ and $p = 5$. Indeed, using the standard trigonometric linearization formulas, equation (33) rewrites:

$$x(t) = \sum_{k=0}^{5} A_k\left(|G(j\omega_1)|\right) \sin\left(k\omega_1 t + \beta_k\left(\varphi(\omega_1), |G(j\omega_1)|\right)\right) \quad (34)$$

where:

$$A_5 = \frac{1}{16}\hat{c}_5\left(U_1|G(j\omega_1)|\right)^5 \; ; \quad \beta_5 = -5\varphi(\omega_1) \quad (35a)$$

$$A_4 = \frac{1}{8}\hat{c}_4\left(U_1\left|G(j\omega_1)\right|\right)^4 \,;\; \beta_4 = -4\varphi(\omega_1) + \frac{\pi}{2} \qquad (35b)$$

$$A_3 = -\frac{5}{16}\hat{c}_5\left(U_1\left|G(j\omega_1)\right|\right)^5 - \frac{1}{4}\hat{c}_3\left(U_1\left|G(j\omega_1)\right|\right)^3 \,; \qquad (35c)$$
$$\beta_3 = -3\varphi(\omega_1)$$

$$A_2 = \frac{3}{8}\hat{c}_4\left(U_1\left|G(j\omega_1)\right|\right)^4 + \frac{1}{2}\hat{c}_2\left(U_1\left|G(j\omega_1)\right|\right)^2 \,; \qquad (35d)$$
$$\beta_2 = -2\varphi(\omega_1) - \frac{\pi}{2}$$

$$A_0 = \hat{c}_0 + \frac{1}{2}\hat{c}_2\left(U_1\left|G(j\omega_1)\right|\right)^2 + \frac{3}{8}\hat{c}_4\left(U_1\left|G(j\omega_1)\right|\right)^4 \qquad (35e)$$

$$A_1 = U_1\left|G(j\omega_1)\right|\sqrt{a^2+b^2} \,; \quad \beta_1 = -\varphi(\omega_1) + \tan^{-1}\left(\frac{b}{a}\right) \,; \qquad (35f)$$

with:

$$a = \hat{c}_1 + \frac{3}{4}\hat{c}_3\left(U_1\left|G(j\omega_1)\right|\right)^2 + \frac{5}{8}\hat{c}_5\left(U_1\left|G(j\omega_1)\right|\right)^4 \,; \qquad (35g)$$
$$b = -\frac{1}{8}\hat{c}_4\left(U_1\left|G(j\omega_1)\right|\right)^3$$

Note that, all (undisturbed) resulting signals $u(t)$, $v(t)$, $w(t)$ and $x(t)$ are $2\pi/\omega_1$-periodic in steady-state. Consider the Fourier transforms of the periodic steady-state signal $x(t)$ (see(21)):

$$x(t) = \sum_{k=0}^{5} A_k\left(\left|G(j\omega_1)\right|\right)\sin\left(k\omega_1 t + \beta_k\left(\varphi(\omega_1), \left|G(j\omega_1)\right|\right)\right)$$
$$= s_0 + \sum_{k=0}^{\infty} s_k \sin\left(k\omega t + \psi_k\right) \qquad (36)$$

This equality can be made beneficial if one can obtain accurate estimate of the (steady-state) signals $x(t)$. Fortunately, this is possible using the periodicity of undisturbed system output $x(t)$. First, operating the averaging (23a-b) on the resulting system output $y(t)$, one gets an estimate $\hat{x}_N(t)$ of $x(t)$. Then, estimates of the modulus $\left|G(j\omega)\right|$ and the phase $\varphi(\omega)$ are readily obtained using only one frequency component, e.g. the harmonic of 5th order:

$$\left|\hat{G}_N(j\omega_1)\right| = \frac{1}{U_1}\left(\frac{16\,\hat{s}_5}{\hat{c}_5}\right)^{1/5} \;;\; \hat{\varphi}_N(\omega_1) = -\frac{1}{5}\hat{\psi}_5 \qquad (37)$$

Then, for the couple $(U_1, \omega_1) = (1\,, 0.01\,rd/s)$, plotting the Fourier transform modulus and phase spectra of the undisturbed output estimate $\hat{x}_N(t)$ (Fig.16a-b), the frequency gain estimate $\hat{G}_N(j\omega_1)$ is obtained using (37). The above procedure can be repeated for as many times as necessary. Doing so, a number of estimates $\hat{G}_N(j\omega_i)$ have been got and a sample of them is shown by Table 2. This shows that all estimates are quite close to their true values.

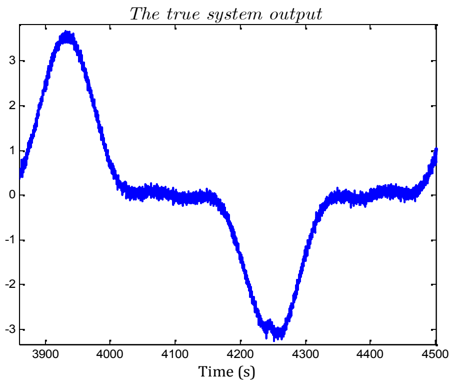

Fig.15a. Steady-state output $y(t)$ obtained with (U_1, ω_1) over one period.

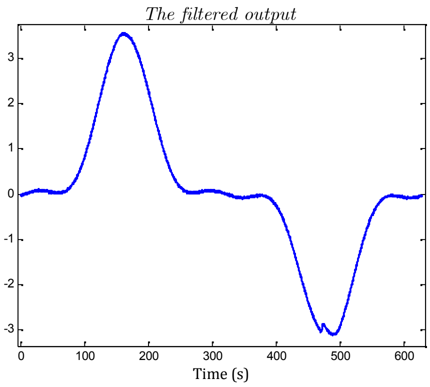

Fig.15b. Undisturbed output estimate $\hat{x}_N(t)$ over one period of time.

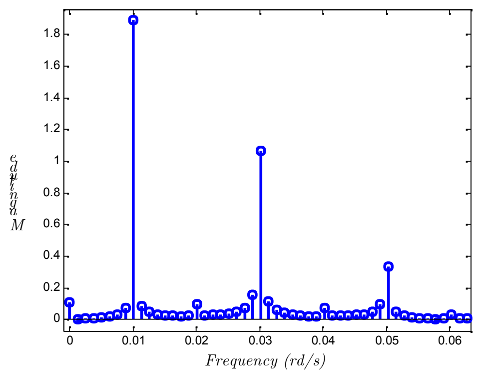

Fig.16a. Fourier transform modulus of $\hat{x}_N(t)$.

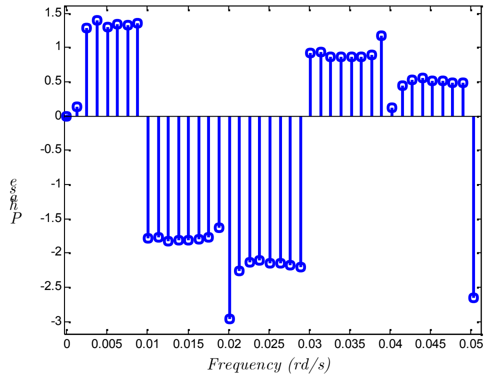

Fig. 16b. The phase spectra of $\hat{x}_N(t)$.

Table 2. Frequency gain estimates $\hat{G}_N(j\omega_i)$

i	1	2	3	4	5
U_i	1	1	1	1	1
$\omega_i\ (rd/s)$	0.01	0.02	0.05	0.1	0.2
$\varphi(\omega_i)\ (rd)$	0.07	0.14	0.34	0.66	1.16
$\hat{\varphi}_N(\omega_i)$	0.065	0.147	0.33	0.68	1.19
$\lvert G(j\omega_i)\rvert$	0.998	0.99	0.96	0.87	0.65
$\lvert \hat{G}_N(j\omega_i)\rvert$	1.01	0.97	0.95	0.89	0.68

7 Conclusion

The problem of system identification is addressed for Hammerstein-Wiener systems including memory operator of Backlash type bordered by straight lines. The proposed identification method is developed using frequency and Fourier analysis techniques. All involved estimators are shown to be consistent. Moreover, the input nonlinearity can be Backlash or Backlash-Inverse. The identification method also features the fact that the linear subsystem identification is made decoupled from the nonlinear elements identification.

The originality of the present study lies in the fact that the linear subsystem is of structure totally unknown. To the author knowledge the output nonlinearity may be noninvertible except at only one point. Another feature of the method is the fact that the exciting signals are easily generated and the estimation algorithms can be simply implemented.

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