Design and Implementation of a Discrete Variable Gain Controller Based on a Numerical Approach

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Abstract:- this work presents a design and implementation of discrete variable gain controllers, which are used for regulating industrial systems. The design is derived from the classical discrete deadbeat response approach. The design is based simply on determining a variable gain for each discrete sample such to accomplish the regulation in finite steps equal the system order. The gain computation is performed numerically by solving a system of nonlinear coupled equations using one of the known evolutionary techniques, the genetic algorithm augmented with Newton-Raphson method. Two industrial control systems are considered to testify the designed controller for implementation. The efficacy of the proposed method for parameter variations is explored. Moreover, the results are compared with that based on a finite number of control steps.

Keywords-: Digital control, Discrete controllers, deadbeat responce, finite number of control steps, Genetic Algorithm.

1. Introduction

In industry practice, most useable machines are continuous systems that have to be regulated to fulfill certain jobs such as electro-hydraulic servomechanism. After the innovation of the digital elements, most of these systems became regulated by digital controllers, which replace the old analogue compensators. The first version of this replacement is copying the frequency approach, which was used with continuous systems. This brings the modeling of the sampled-data system, which utilize the z-transform tool.

For discrete systems, one interesting approach of design, which is not correlated to continuous design approaches, is that named deadbeat response. It is the response, which reaches the desired steady-state value in an infinite number of sampling intervals. Such response might be included with the time-optimal approaches, specifically, the H_2 linear quadratic regulating framework [1, 2]. However, analysis shows a crucial problem of demanding an excessively high control effort, and consequently, of an oscillated inter-sampling response. When constraints on the control action are imposed, the settling time of the controlled system will be prolonged for additional sampling periods. An alternative approach to the z-transform is to sample a continuous signal; it is the control of a continuous system by a discrete controller [3, 4].

Depending on the complexity of the control system, designers chose the methodology either based the transfer function or the state space. For complex industrial control system, it is found that the state space approach offers several methods of design, including optimum approaches [5].Of the known substantial state space advantages, it is also mentioned the finite number of control steps, or the finite sampling time. On the other hand, in industrial systems, not all states are available for control and some sort of state estimator has to be included [6]. Furthermore, for high-order systems, the computation burden of the finite control steps becomes large.

In a sampled data system, the computer function is to implement a control strategy, which is a control algorithm stored in its memory. For real time environments, it is recommended that the control algorithm is as simple as possible. Most design methods result controllers represented by pulse transfer functions, which in turn translated to difference equations for computer use.

In this paper, an alternative way is proposed based on assumption that the computer supplies a variable gain each sample. Then after, a pulse transfer function is obtained for simulation purposes. For this purpose, the concept of deadbeat response is invoked to determine this variable gain vector. Unlike the classical approach of the deadbeat response, the proposed approach is a ripple-free one.

2 Discrete Deadbeat Control

The deadbeat response receives a large amount of works for both continuous and discrete control systems [7, 8, 9]. In fact, this response is the ultimate of any design irrespective of the methodology used. However, the deadbeat response is often referred to a certain method of design in sampled-data control systems. On the other hand, for the discrete control systems, a theory of a finite number of control steps replaces the classical deadbeat response concepts. In this section, a glint review of the ripple-free deadbeat response for discrete control systems will be crossed over [8, 9].

Consider the unity negative feedback sampled-data control system shown in figure 1, where T is the sampling period. The continuous actuating signal $e(t) = r(t) - c(t)$ is sampled with a sampler T, such that $e^*(t)$ be an input to the discrete controller.

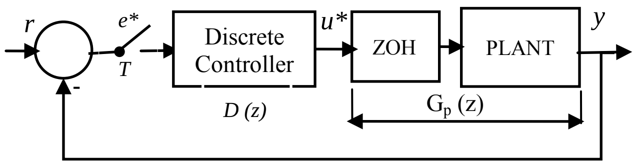

Figure 1 Sampled-data system

The z-transform of both e^* and u^* can be expressed by the following relations;

$$E(z) = e(0) + e(T)z^{-1} + \cdots + e(nT)z^{-n} \quad (1)$$

$$U(z) = u(0) + u(T)z^{-1} + \cdots + u(nT)z^{-n} \quad (2)$$

Then the pulse transfer function of the discrete controller and that of the closed-loop are given respectively by (3) and (4).

$$D(z) = \frac{U(z)}{E(z)} = \frac{u(0)+u(T)z^{-1}+\cdots+u(nT)z^{-n}}{e(0)+e(T)z^{-1}+\cdots+e(nT)z^{-n}} \quad (3)$$

$$M(z) = \frac{D(z)G_p(z)}{1+D(z)G_p(z)} \quad (4)$$

Thus,

$$D(z) = \frac{1}{G_p(z)}\frac{M(z)}{1-M(z)} \quad (5)$$

The necessary and sufficient conditions that the discrete-time system exhibits a deadbeat response to a polynomial time-domain inputs of degree m (for a step and a ramp input m=0 and m =1 respectively), are:

The $M(z)$ must be expressed as a finite polynomial in terms of powers in z^{-1}, i.e.

$$M(z) = \sum_{i=1}^{n} c_i z^{-1} \quad (6)$$

If and only if the impulse response coefficients c_i satisfy the following set of m +1 linear algebraic equations

$$\sum_{i=1}^{n} c_i i^j = \begin{cases} \delta_0 = 1 \\ \delta_{j\neq 0} = 0 \end{cases}, \; j = 0,1,2 \dots m \quad (7)$$

From condition 6, three important remarks can be concluded. The deadbeat response existence is independent on T, and a system which exhibits a deadbeat response to a time-domain input of degree m, will exhibit a deadbeat response to every time-domain input of lower degree. The third remark (from the solution of 6 for $n = m + 1$, the usual case for industrial plants) is that the deadbeat response has ripples of unaccepted magnitudes as the order n increases.

In [10], necessary and sufficient conditions for a ripple-freedeadbeat response were derived. In summary, it was shown that it is apparent that oscillation of the control sequence results only from zeros of the plant transfer function that are taking on negative values. However, these conditions consider only the response after the transient, and hence a non-minimum settling time response is obtained.

Parallel to the above analysis that uses the pulse transfer function approach, a finite number of control steps (minimum settling time) approach is also presented in literatures [11]. This alternative approach uses the state space theory of discrete linear control. Both cases of regulating and transient to steady state are considered for SISO and MIMO completely controllable systems. Unless the controlling signal is constrained, the time of transition from an initial to a final state is reduced proportionally to the reduction of the sampling interval T. The state feedback controller K is given by;

$$K = [0 \quad 0 \quad \cdots \quad 1]Q^{-1}F^n \quad (8)$$

Where, Q is the controllability matrix of the discrete-time system, F is the corresponding coefficient matrix and x is the state vector. However, for implementing the controller, all system states have to be available for control or measurement.

3 Variable Gain Controller

The discrete-time control system shown in figure 1 can be described by the state transition equation

$$x(mT + T) = Fx(mT) + Gu(mT) \quad (9)$$

Where $m = 0,1,2..$

$$y(mT) = Hx(mT) \quad (10)$$

where x is the state variable column vector and T is the sampling period, u is the control signal applied to the input of the ZOH device, and the matrix F and the vectors G and H are given by

$$F = e^{AT} = \mathcal{L}^{-1}(sI - A)^{-1}|_{t=T}, \tag{11}$$

$$G = \int_0^T [\mathcal{L}^{-1}(sI - A)^{-1}]Bd\tau \tag{12}$$

$$G = \begin{cases} A^{-1}(F - I_n)B, & \det(A) \neq 0 \\ T(I_n + \frac{AT}{2!} + \frac{(AT)^2}{3!} + \frac{(AT)^3}{4!} + \cdots)B, & \det(A) = 0 \end{cases} \quad \ldots\ldots. (13)$$

$$H = C \tag{14}$$

where n is the plant order, A, B, and C are the coefficient matrix, input vector and the output vector respectively of the continuous plant, s is the Laplace's operator, and I is a unit matrix.

It is proposed that the controller is simply a forward gain varies every sample period, then c(mT) = y.

$$k(mT) = \frac{u(mT)}{e(mT)} = \frac{u(mT)}{r(mT) - c(mT)} \tag{15}$$

For simple writing, the variable gain is denoted k_m and the sampling period T is dropped. Therefore, it can be written

$$u(m) = k_m[r(m) - Hx(m)] \tag{16}$$

$$x(m+1) = Fx(m) + Gk_m[r(m) - Hx(m)] \tag{17}$$

To realize a deadbeat response, the system error must be zero for $t \geq nT$, where n is the smallest possible positive integer (order of the plant n=m+1).This condition is realized if the following two conditions are satisfied

$$x_1(nT) = r(nT) \tag{18}$$

$$x_2(nT) = r'(nT) \ldots x_n(nT) = r^{(n-1)}(nT) \tag{19}$$

For instance, for unit-step reference input, the above conditions are

$$x_1(nT) = 1 \tag{20}$$

$$x_2(nT) = x_3(nT) = \ldots\ x_n(nT) = 0 \tag{21}$$

Thus, the controller will be a vector defined as

$$k_v = [k_0 \quad k_1 \quad \cdots \quad k_{n-1}]^T$$

The above deadbeat response conditions arise a system of n algebraic nonlinear equations, which have the form

$$E_{n\times w}K_{w\times 1} = \begin{bmatrix} r(nT) \\ r'(nT) \\ \vdots \\ r^{(n-1)}(nT) \end{bmatrix} \tag{22}$$

Where the coefficient matrix E is derived from equation (17) and the dimension w is given by the mathematical expression;

$$w = \sum_{j=1}^{n} \frac{n!}{j!(n-j)!} \tag{23}$$

$$K_{w\times 1} = \begin{bmatrix} k_v \\ P_2 \\ P_3 \\ \vdots \\ P_{n-1} \\ \prod_{i=0}^{n-1} k_i \end{bmatrix} \tag{24}$$

Where P_q is a column vector of all combinations products of q gains; for example, for 3rd-order system

$$P_2 = [k_0k_1 \quad k_0k_2 \quad k_1k_2]^T$$

Furthermore, the following functions are defined

$$F_1 = f_1(k_0, k_1 \ldots k_{n-1}) - r(nT)$$

$$F_2 = f_2(k_0, k_1 \ldots k_{n-1}) - r'(nT)$$

$$\vdots$$

$$F_n = f_n(k_0, k_1 \ldots k_{n-1}) - r^{(n-1)}(nT) \tag{25}$$

It is clear that the nonlinear system (22) has to be solved numerically. For this purpose, Newton-Raphson or any other numerical method may be invoked; however, the issue of finding a correct initial start represents a problem by itself. An alternative way is to convert the problem to a minimum single-objective or a multi-objective unconstraint convex optimization one, and then to obtain the solution. Therefore, equivalently, for single-objective, the solution of the nonlinear system can be replaced by either of the followings;

$$\underbrace{min}_{k_v}\{J = (F_1^2 + F_2^2 + \cdots + F_n^2)\} \tag{26-a}$$

$$\underbrace{min}_{k_v}\{J = (|F_1| + |F_2| + \cdots + |F_n|)\} \tag{26-b}$$

The vector k_v that gives a zero minimum value is the solution of the nonlinear system. There are several optimization methods, including evolutionary techniques such as genetic algorithms to solve the problem. The used methods often give local solution, and special efforts have to be accomplished to reach global solution. However, such efforts are changed as the parameters of the problem change. When genetic algorithm is used, the setting of the genetic parameters plus the several runs of the algorithm with random initialization often accomplish the task. The solution corresponding to the nearest zero value of the objective function J is picked up. Furthermore, the genetic solution can be used as an initial point to start Newton-Raphson method.

For multi-objective criterion, the standard Pareto dominance relationship between solutions and an iterative strategy that evolves some random solutions during the search for optimal solution represents a logical selection of such efforts. This approach has been used efficiently to solve complex nonlinear systems [12].

4 Simulation and Results

In this section, the proposed approach of designing a discrete controller will be explored. Through simulation, a comparison with other designs is also presented. It is worth mentioning that an MATLAB code is developed to design discrete controllers for different system orders. The code is a general one in the sense that its input is only the continuous plant transfer function $G(s)$, the sampling period T, and the reference input $r(t)$.

Example 1: A controlled plant with an industrial integrating servomotor is given by the transfer function

$$G(s) = \frac{1}{s(s+0.5)^2}$$

It is to find a discrete controller such that the system will exhibit a deadbeat response for a unit step input.

The sampling period $T = 1$. The plant has a pulse transfer function (?? a unit feedback)

$$G(z) = \frac{0.13061(z+2.928)(z+0.2072)}{(z-1)(z-0.6065)^2}$$

The corresponding discrete matrices F, G (as in equations 11 and 13 b) are

$$F = \begin{bmatrix} 1 & 0.9673 & 0.3608 \\ 0 & 0.9098 & 0.6065 \\ 0 & -0.1516 & 0.3033 \end{bmatrix}, \quad G = \begin{bmatrix} 0.1306 \\ 0.3608 \\ 0.6065 \end{bmatrix}$$

The nonlinear system is defined by

$$E_{3\times 7} K_{7\times 1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where the matrix E and the vector K are given respectively as

$$\begin{bmatrix} 1.4185 & 0.6985 & 0.1306 & -0.0912 & -0.0912 & -0.0171 & 0.0022 \\ 0.7117 & 0.6961 & 0.3608 & -0.0909 & -0.252 & -0.0471 & 0.0062 \\ -0.0664 & 0.1292 & 0.6065 & -0.0169 & -0.4237 & -0.0792 & 0.0103 \end{bmatrix}$$

$$K = [k_0 \quad k_1 \quad k_2 \quad k_0 k_1 \quad k_0 k_2 \quad k_1 k_2 \quad k_0 k_1 k_2]^T$$

The solution of the three nonlinear equations is converted to a single-objective optimization problem as it is suggested in equation (26-a). The genetic algorithm of 1000 generations, size of 300 populations, 10^{-16} tolerance function change, and 0.8 cross over fraction, is invoked 50 times with different random initialization. The results are the gain vector k_v, the minimum value of J, and the values of functions F_i, i = 1, 2, 3 after minimization; exactly they should all be zeros.

$$k_v = [1.6147 \quad -2.4825 \quad 4.6430]^T$$

$$|J_{min}| = 1.6 \times 10^{-4}$$

$$[F_1 \quad F_2 \quad F_3] = [-0.1146 \quad -0.0197 \quad 0.1096] \times 10^{-3}$$

The plot of the discrete output for unit step is shown in figure 2. The output achieves in three sampling periods the unity step input with very small steady state error (less than 0.001) due to the small residuals of F_i functions. Implementing this variable gain can be easily performed by a digital computer. It is only to output a constant value each period up to three periods and then to keep the final value to the control time end. Considering equations (1-3), a pulse transfer function of the controller can be obtained

$$D(z) = \frac{1.6147(z^2 - 1.213z + 0.368)}{(z+0.5609)(z+0.2282)}$$

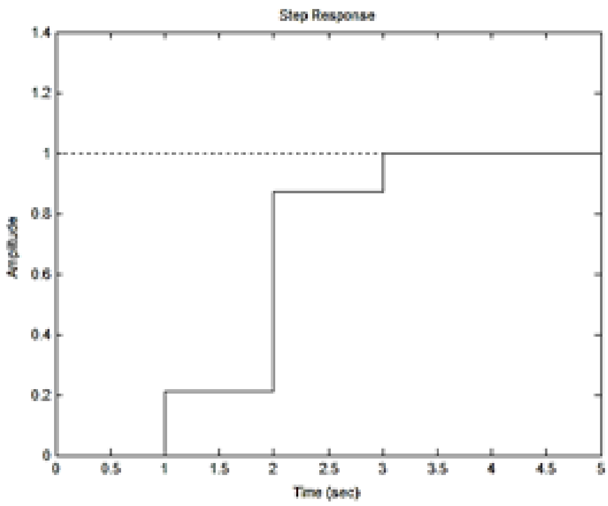

Figure 2. Discrete step response for example 1

The ripple-free continuous output can be obtained by simulating the system shown in figure 1; a SIMULINK modeling result is shown in figure 3.

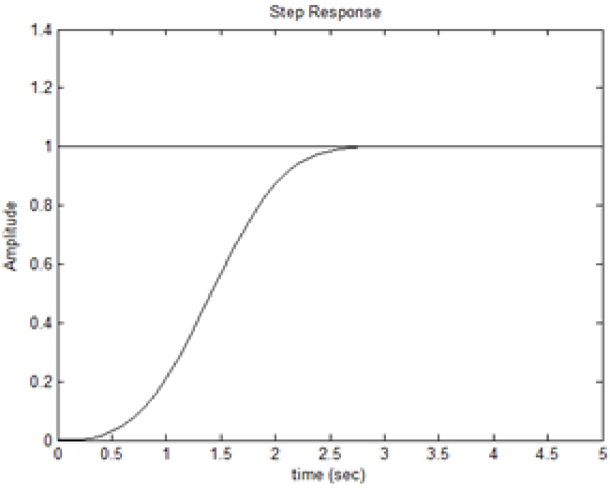

Figure 3 Continuous step-response for example 1

Example 2: The control of a position Ward-Leonard set servomechanism as a displacement machine tool is considered [13]. The linear part is described by the non-canonical state and equations

$$x(t)' = \begin{bmatrix} -\frac{1}{T_D} & 0 & 0 & 0 \\ \frac{1}{L_{MD}} & -\frac{1}{T_{MD}} & -\frac{1}{L_{MD}K_M} & 0 \\ 0 & \frac{1}{J\,K_M} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{K_D K_a}{T_D} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$\varphi(t) = [0 \quad 0 \quad 0 \quad 1]x(t)$$

Where:

- T_D and K_D are the time constant and the linear part of the electromagnetic characteristic gain of the DC dynamo in seconds and volt/mA respectively.
- L_{MD} and T_{MD} are the induction and electromagnetic time constant of Ward-Leonard set circuit in Henry and seconds respectively.
- J is moment of inertia (kg m^2) of the DC motor
- K_M is the gain (rad/Wb) of the motor.
- K_a is the power amplifier gain (mA/volt)
- $u(t)$ is control signal (the output of the power amplifier in volt).
- x (t) isthe state vector, which is defined by the following four states

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \equiv \begin{bmatrix} \text{input voltage of DC dynamo (v)} \\ \text{Leonard anchors urrent(Amp)} \\ \text{motor angular velocity (rad/s} \\ \text{motor rotation angle } \varphi \text{ (rad)} \end{bmatrix}$$

For the values given in table 1 and a power amplifier of a gain K_a = 34 (mA/volt), the plant is sampled with T = 0.1 seconds to obtain the discrete matrices and vectors as

$$F = \begin{bmatrix} 1 & 0.0909 & 0.00313 & 0.000045 \\ 0 & 0.7274 & 0.04104 & 0.0008 \\ 0 & -4.7139 & -0.1359 & 0.0005 \\ 0 & -3.0950 & -5.2807 & -0.1625 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.6756 \\ 20.394 \\ 352.652 \\ 231.544 \end{bmatrix}$$

Table 1 Ward-Leonard set parameters/ value/ unit.

T_D	K_D	L_{MD}	T_{MD}	K_M	J
0.125	2.000	0.035	0.023	1.100	0.0315
sec	v/mA	Henry	sec	rad/Wb	kgm^2

For a step response of magnitude δ radian, the four nonlinear equations will have the form

$$E_{4\times15}K_{15\times1} = [\delta \quad 0 \quad 0 \quad 0]^T$$

For such complicated nonlinear equations, some additional efforts have to be done.

For δ of 0.2 radians, the genetic algorithm is parameterized by 3000 generations and a size of 200 populations and 10^{-32} tolerance function change. After 50 runs of the genetic algorithm, the minimum index value is 0.103, which is very high to consider that the optimization process is equivalent to the solution of the nonlinear equations. Therefore, another trail to achieve a deadbeat response is tried. Specifically, the obtained genetic solution is used as an initial vector to start the numerical Newton-Raphson method to reduce significantly the residues of the functions, F_i. The results are:

$$k_v = [0.23497 \quad -0.11982 \quad 0.00408 \quad -0.19993]^T$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} -0.0105 \\ 0.0049 \\ -0.0003 \\ 0.5962 \end{bmatrix} \times 10^{-14}$$

$$D(z) = \frac{0.23497(z - 0.4495)(z^2 + 0.02053z + 0.0129)}{(z + 0.03759)(z^2 + 0.8037z + 0.1813)}$$

The accuracy of implementing the variable gain controller by a digital computer is very high. However, it is not so with implementing the pulse transfer function by passive and active elements.

Therefore, the robustness of the controlled system depends only on the plant parameters.

In practice, the parameters of industrial plants do changing during operation and life time. For exploring the design efficacy, changes of parameters will be assumed. In Ward-Leonard control system, parameter changes are mainly due to the variation of the power amplifier gain K_a, the electromagnetic characteristic gain K_D, and the motor gain K_M; consequently, T_{DM} ,since the electromagnetic time constant depends on K^2_M. As it can be noted from the state equation, variations in K_a and K_D, changes the non-zero element of the input vector. Figure 4 depicts the responses of the nominal and ± 30% variations in the gain product K_aK_D. The nominal response exhibits a deadbeat response to achieve 0.2 radians in 0.4 seconds (4T). All other responses exhibit expected behaviors; the controlled system has an overshoot for larger variation and sluggish behaviors for smaller value of the gain product K_aK_D. However, all responses show a stable and ripple-free performance.

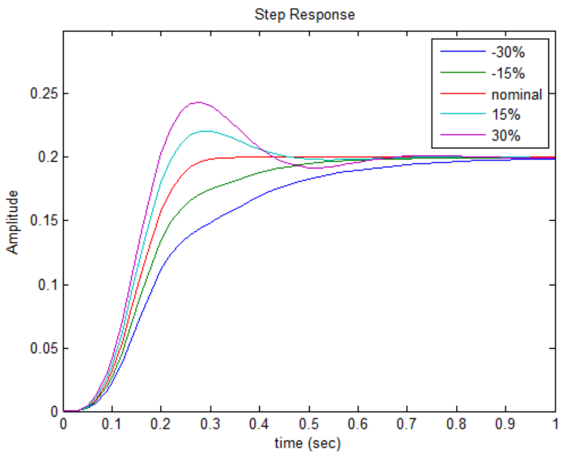

Figure 4 Time response for different variations of the gain K_aK_D.

Next, the effect of motor gain constant K_M variation will be studied; ± 30% changes in the nominal value are suggested. Figure 5 depicts all system responses, including the nominal one.Similar comments as with previous case of gain variation can be stated; however, the system exhibits larger overshoot and settling time for the same percentage of parameter changing. It can be noticed that the worst case when both the forward gain and the motor gain constant decrease or increase simultaneously, and the likely case when one increases while the other decreases. Figure 6 shows the nominal response and the responses of the two extreme cases. An excessive overshoot of 67%, and a sluggish settling time of two seconds are taken place when all K_a, K_D, and K_M are increased and decreased by 30% respectively.

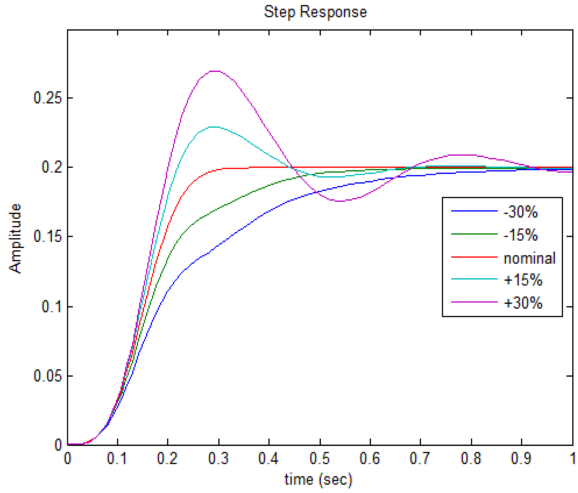

Figure 5 Time response for different variation of the gain K_M

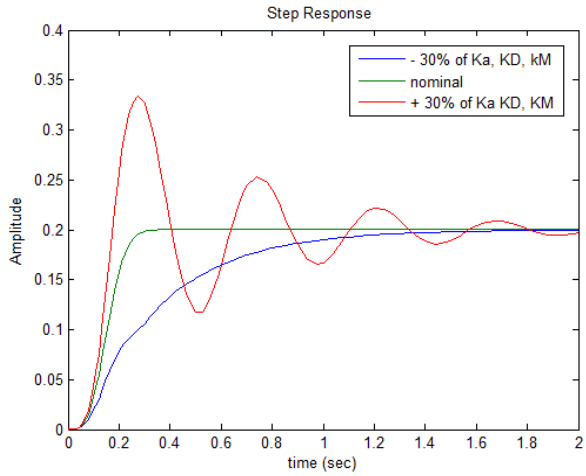

Figure 6 Time response for the two extreme cases of parameter variations.

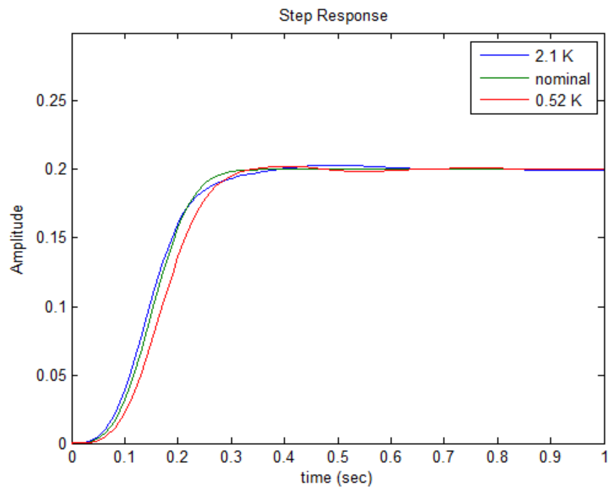

Figure 7 Adjusted responses to compensate 30% variations

Figure 7 shows again the three responses after multiplying the variable gain vector by 2.1 t o improve the sluggish response and by 0.52 to reduce the excessive overshot. In spite of the responses of the two extreme cases are not of deadbeat nature, they are good enough for practice implementation.

Finally, the proposed method is compared with the results of using the theory finite number of control steps. Applying equation 8, the state feedback gain vector is

$$K = [0.23497 \quad 0.03503 \quad 0.001156 \quad 0.00003]^T$$

Figure 8 shows unit step responses for both finite control steps and the proposed method. As it can be seen the output reaches the same values in each sample period, and both responses have deadbeat behaviors. In spite of this validation, one may argue that the proposed method has large computation burden. However, the proposed method has the advantages of lower hardware demands; most plants may require sensors to measure the plant states accurately.

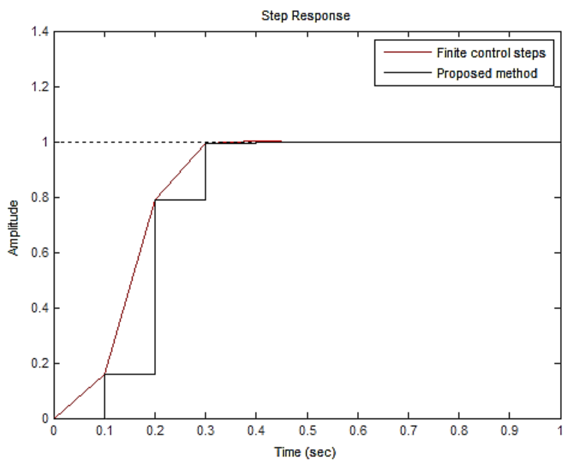

Figure 8 Responses of finite control steps and proposed method

5 Implementation

In this section the empirical considerations are discussed, it is focused on the practical design of the variable gain sample and hold unit. A digitally controlled variable gain amplifier (VGA) is injected before the sampler to get the proposed structure as in Figure 9. The gain and sampling circuit is implemented by a microcontroller. The gain values and sampling time are offline calculated and loaded in the microcontroller memory and its internal timer respectively.

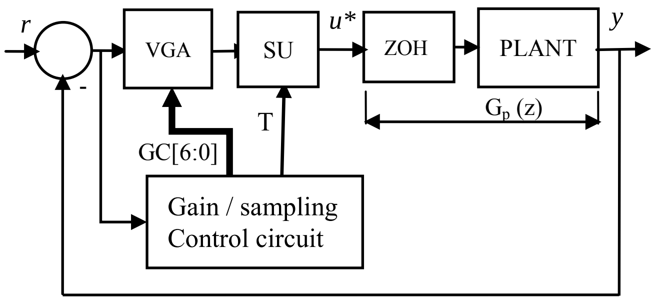

Figure 9 The proposed system control BD.

In the experimental simulation, the gain is selected from a l ookup table containing a p re calculated deadbeat gain values, and synchronized by a high precision timing circuit. The dead beat frequency is based on the sampling period (T) that is calculated from the process transfer function.

The essential properties for the VGA are the speed and gain resolution. The wide bandwidth, large dynamic range, and excellent noise-linearity HMC960LP4E Automatic Gain Control is selected to satisfy the required properties of variable gain values is selected. It supports discrete gain steps from dB in precise 0.5 dB steps with exceptionally smooth gain transitions, a gain of 0-40 dB and 0.5 dB step is applied through a parallel digital port. Gain can be controlled via either a parallel gain control interface (GC[6:0]) or via the read/write serial port (SPI) as shown in Figure 10.

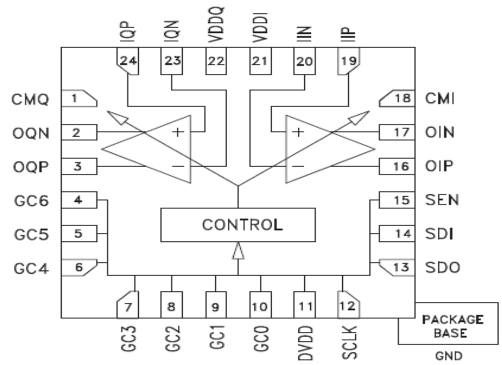

Figure 10 The HMC960LP4E architecture [14]

The gain is composed by three stages, second stage generates up to 10dB fully determined by the user, while first and third stages are adjusted to 0, 10, 20 dB values, automatically selected by the gain control code, as in Figure 11, or without using decode logic gain can be allocated arbitrarily.

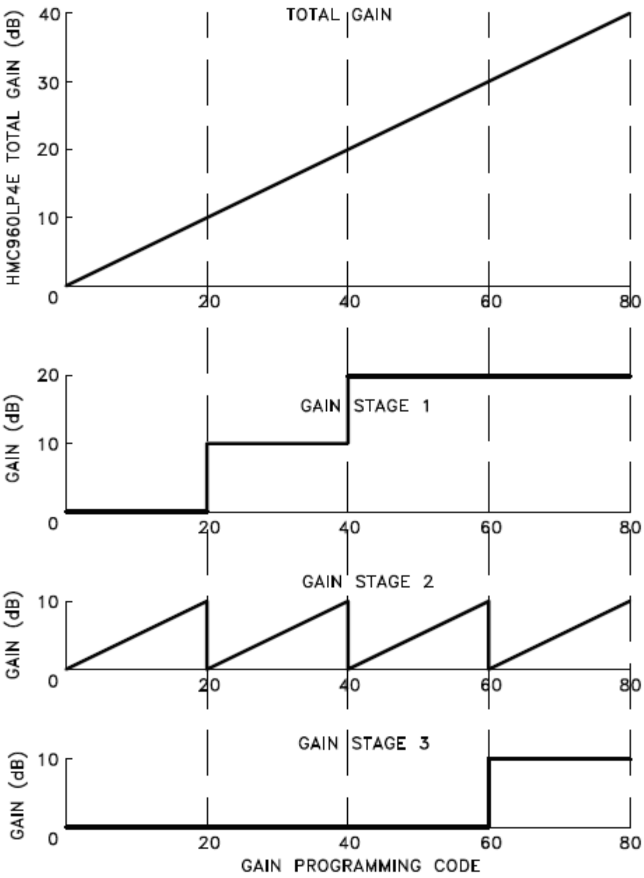

Figure 11 Gain allocations by programming code.

The main problem facing the practical implementation is the exact value justification of the gain. It is limited by the VGA resolution (0.5 dB). Hence a wide range, precise and linear varying gain is almost cannot be reached.

To overcome this problem we proposed a multi channels VGA algorithm to handle the problem by partitioning the gain into fractions < 9.5 dB belonging to the linear continuous ranges of the VGA stages. This will be investigated by truncating the gain digit with values ≤ (0.5+ß) dB resolution (ß is adjusting parameter), and apply them to the next VGA stage. The output of the parallel stages then gathered to form the high precision gain. The value of ß is selected to satisfy the accuracy criteria (k(ß)/ k) <0.0001.

6 Conclusions

In this paper, a method of design a discrete variable gain is introduced. In each sample, the controlled computer delivers a constant value in the forward of the closed-loop control system. The method is based on a numerical approach of solving nonlinear equations that are derived from the theory of achieving a d eadbeat response. The numerical approach is to convert the problem to a single or multi-objective unconstraint convex optimization. The genetic algorithm followed, if necessary, by Newton-Raphson method is proposed as a very high accurate solution of the nonlinear equations. The proposed method is testifying first with one illustrative example, and then it is used to control a Ward-Leonard servomechanism. Due to simplicity of design, a simple solution is suggested to resolve the problem of parameter variations that are usually taken place in industrial systems. Finally, it is also shown that the proposed method gives the same results as the design based on the theory of the finite number of control steps.

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