Computer Simulation of an Augmented Automatic Choosing Control Designed by Hamiltonian and Genetic Algorithm with Constrained Input

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Abstract: In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) using weighted gradient optimization automatic choosing functions for a class of nonlinear systems with constrained input. When designed the control, a constant term which arises from linearization of a given nonlinear system is treated as a coefficient of a stable zero dynamics. The controller is of a structure-specified type which has some parameters. Parameters of the control are suboptimally selected by minimizing the Hamiltonian with the aid of a genetic algorithm. This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the splendidness of the AACC. Simulation results show that the new controller can improve performance remarkably well.

Key–Words: augmented automatic choosing control, nonlinear control, genetic algorithm, zero dynamics

1 Introduction

It is generally easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[1]-[7]. One of most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion truncated at the first order and the linear optimal control method. This is only effective in a small region around steady state points or in almost linear systems[1]-[3].

As one of approaches to overcome these drawbacks, an augmented automatic choosing control(AACC) is proposed for nonlinear systems[7].Moreover, in many practical systems, there are physical constraints such as limitation and saturations of inputs, so a design of nonlinear control laws subject to constraints has been urgent but been studied a few[8]. In this paper we consider a design method of the AACC for nonlinear systems with constrained inputs. Its process is as follows.

Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls[2]. These LQ controls are smoothly united by weighted gradient optimization automatic choosing functions of sigmoid type to synthesize a single nonlinear feedback controller. This controller is then limited so as to hold a specified constraint.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, coefficients of the zero dynamics, and gradients of the automatic choosing function. These parameters must be selected optimally so as to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the genetic algorithm (GA) [9] suboptimally. In this paper the suboptimal values of these parameters are selected by minimizing a Hamiltonian function.

A power system in transient stability problem is one of the typical nonlinear systems with high nonlinearity, so the proposed method is successfully applied to it. Simulation results show that the new controller using the GA is able to improve performance remarkably well.

2 Augmented Automatic Choosing Control Using Zero Dynamics

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \;\; x \in \mathbf{D} \tag{1}$$

subject to

$$u_{j,min} \le u[j] \le u_{j,max} \quad (j = 1, \cdots, r) \tag{2}$$

where $\cdot = d/dt$, $x = [x[1], \cdots, x[n]]^T$ is an n-dimensional state vector, $u = [u[1], \cdots, u[j], \cdots, u[r]]^T$ is an r-dimensional bounded control vector, $u_{j,min}$: the minimum value of $u[j]$, $u_{j,max}$: the maximum value of $u[j]$, $f : \mathbf{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x)$ is an $n \times r$ driving matrix with $g(0) \neq 0$ and is continuously differentiable, $\mathbf{D} \subset R^n$ is a domain , and T denotes transpose.

Considering the nonlinearity of f, introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x[2]$ is the element which has the highest nonlinearity in f, then

$$C(x) = x[2] \in D \subset R \;\; (L = 1)$$

(see Section IV). The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \le i \le M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L} [a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \tag{3}$$

$$(x[n+1](0) \simeq 1, \quad 0 < \sigma_i < 1).$$

Equation(1) combines with (3) to form an augmented system

$$\dot{\mathbf{X}} = \bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u \tag{4}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \quad \in \mathbf{D} \times R$$

$$\bar{f}(\mathbf{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \bar{g}(\mathbf{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

We assume a cost function being

$$J = \frac{1}{2} \int_0^{\infty} \left(\mathbf{X}^T \mathbf{Q} \mathbf{X} + u^T R u \right) dt \tag{5}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0,\; R = R^T > 0$, and the values of these matrices are properly determined based on engineering experience.

On each D_i, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$ (see Fig. 1):

$$f(x) + g(x)u \simeq A_i x + w_i + B_i u \quad \text{on} \;\; C^{-1}(D_i) \tag{6}$$

where

$$\begin{aligned} A_i &= \partial f(x)/\partial x^T|_{x=\hat{X}_i} \,, \; w_i = f(\hat{X}_i) - A_i \hat{X}_i \,, \\ B_i &= g(\hat{X}_i). \end{aligned}$$

Make an approximation of (4) by

$$\dot{\mathbf{X}} = \bar{A}_i \mathbf{X} + \bar{B}_i u \quad \text{on} \;\; C^{-1}(D_i) \times R \tag{7}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[2] to (5) and (7) yields

$$u_i(\mathbf{X}) = -R^{-1} \bar{B}_i^T \mathbf{P}_i \mathbf{X} \tag{8}$$

where the $(n+1) \times (n+1)$ matrix $\mathbf{P}_i$ satisfies the Riccati equation :

$$\mathbf{P}_i \bar{A}_i + \bar{A}_i^T \mathbf{P}_i + \mathbf{Q} - \mathbf{P}_i \bar{B}_i R^{-1} \bar{B}_i^T \mathbf{P}_i = 0. \tag{9}$$

Introduce a gradient optimization automatic choosing function of sigmoid type with weight d_i:

$$\begin{aligned} I_i(x) &= d_i \prod_{j=1}^{L} \Big\{ 1 - \frac{1}{1 + \exp\left(2N_i \left(C_j(x) - a_{ij}\right)\right)} \\ &\qquad - \frac{1}{1 + \exp\left(-2N_i \left(C_j(x) - b_{ij}\right)\right)} \Big\} \end{aligned} \tag{10}$$

where N_i:positive real value, $-\infty \le a_{ij}$, $b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero when $d_i = 1$(see Fig. 2).

Uniting $\{u_i(\mathbf{X})\}$ of (8) with $\{I_i(x)\}$ of (10) yields

$$\begin{aligned} \hat{u}(\mathbf{X}) &= [\hat{u}(\mathbf{X})[1], \cdots, \hat{u}(\mathbf{X})[j], \cdots, \hat{u}(\mathbf{X})[r]]^T \\ &= \sum_{i=0}^{M} u_i(\mathbf{X}) I_i(x). \end{aligned}$$

We finally have an augmented automatic choosing control which is satisfied with the constraint condition (2) by

$$u(\mathbf{X}) = [u(\mathbf{X})[1], \cdots, u(\mathbf{X})[j], \cdots, u(\mathbf{X})[r]]^T \tag{11}$$

where

$$u(\mathbf{X})[j] = \begin{cases} u_{j,max} & \text{if} \quad \hat{u}(\mathbf{X})[j] \geq u_{j,max} \\ u_{j,min} & \text{if} \quad \hat{u}(\mathbf{X})[j] \leq u_{j,min} \\ \hat{u}(\mathbf{X})[j] & \text{otherwise} \end{cases}$$

$$(1 \leq j \leq r).$$

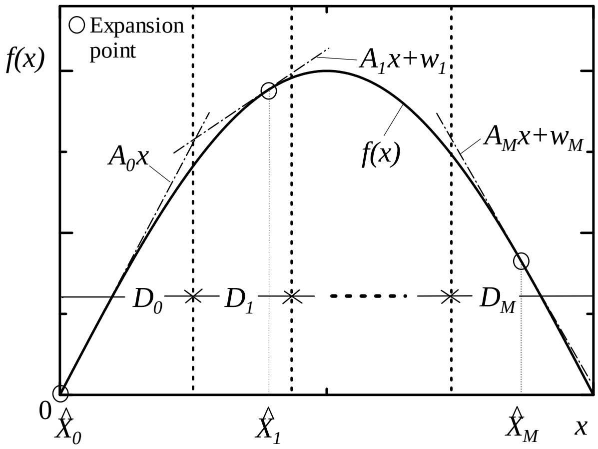

Fig. 1 Sectionwize linearization

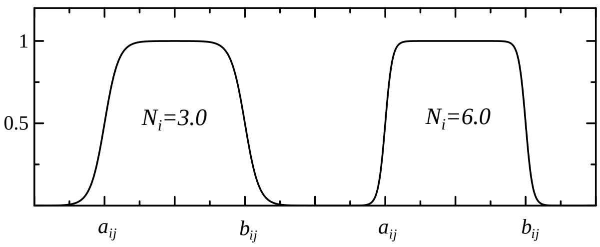

Fig. 2 Automatic Choosing Function(N_i=$3.0, 6.0$)

3 Parameter Selection by GA

The Hamiltonian for Eqs.(4) and (5) is given by

$$H(\mathbf{X}, u, \lambda) = \frac{1}{2}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T R u\right) + \lambda^T\left(\bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u\right). \quad (12)$$

Assume that the adjoint vector $\lambda \in R^{n+1}$ is

$$\lambda = \sum_{i=0}^{M} \mathbf{P}_i \mathbf{X} I_i(x). \quad (13)$$

The necessary condition of the optimality is $\partial H/\partial u = 0$ or $u = -R^{-1}\bar{g}(\mathbf{X})^T\lambda$, which derives Eq.(11) using Eq.(13) and

$$H(\mathbf{X}, u, \lambda) = \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u^T R u + \bar{f}^T(\mathbf{X})\lambda \quad (14)$$

using Eq.(12).
Thus we can define a performance

$$PI = \int_{\mathbf{D}} |H(\mathbf{X}, u, \lambda)|/\mathbf{X}^T\mathbf{X} d\mathbf{X}. \quad (15)$$

A set of parameters included in the control of Eq.(11) is

$$\bar{\Omega} = \{M, N_i, d_i, a_{ij}, b_{ij}, \hat{X}_i\} \quad (16)$$

which is suboptimally selected by minimizing PI with the aid of GA[9] as follows.

<**ALGORITHM**>

step1:Apriori: Set values $\bar{\Omega}_{apriori}$ appropriately.

step2:Parameter: Choose $\Omega \subset \bar{\Omega}$ to be improved and rewrite

$$\Omega = \{N_i, d_i, a_i, b_i \cdots\} = \{\alpha_k : k = 1, \cdots, K\}.$$

step3:Coding: Represent each α_k with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.

step4:Initialization: Randomly generate an initial population of $\widetilde{q}$ strings

$$\{\Omega_p : p = 1, \cdots, \widetilde{q}\}.$$

step5:Decoding: Decode each element α_k of Ω_p by

$$\alpha_k = (\alpha_{k,max} - \alpha_{k,min})A_k/(2^{\widetilde{L}} - 1) + \alpha_{k,min}$$

where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and A_k:decimal values of α_k.

step6:Control: Design $u = u(\mathbf{X})_p \quad (p = 1, \cdots, \widetilde{q})$ for Ω_p by using Eq.(11).

step7:Adjoint: Make $\lambda = \lambda(\mathbf{X})_p \quad (p = 1, \cdots, \widetilde{q})$ for Ω_p by using Eq.(13).

step8:Fitness value calculation: Calculate

$$PI_p = \int_{\mathbf{D}} \Big| \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u(\mathbf{X})_p^T R u(\mathbf{X})_p + \bar{f}^T(\mathbf{X})\lambda(\mathbf{X})_p \Big| / \mathbf{X}^T\mathbf{X} d\mathbf{X} \quad (17)$$

by Eqs.(14) and (15), or fitness $F_p = -PI_p$. Integration of (17) is approximated by a finite sum.

step9:Reproduction: Reproduce each of individual strings with the probability of

$$F_p / \textstyle\sum_{j=1}^{\widetilde{q}} F_j.$$

step10:Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability P_c.

step11:Mutation: Alter a bit of string (0 or1) by a mutation probability P_m.

step12:Repetition: Repeat step5$\sim$step11 until prespecified G-th generation. If unsatisfied, go to step2.

As a result, we have a suboptimal control $u(\mathbf{X})$ for the string with the best performance over all the past generations.

4 Numerical Example

Consider a field excitation control problem of power system. Fig.3 is a diagram of Ozeki-Power-Plant of Kyushu Electric Power Company in Japan. This system is assumed to be described[6] by

$$
\begin{aligned}
&\widetilde{M}\frac{d^2\delta}{dt^2} + \widetilde{D}\frac{d\delta}{dt} + P_e = P_{in}\\
&P_e = E_I^2 Y_{11}\cos\theta_{11} + E_I\widetilde{V}Y_{12}\cos(\theta_{12}-\delta)\\
&E_I + T'_{d0}\frac{dE'_q}{dt} = E_{fd}\\
&E_I = E'_q + (X_d - X'_d)I_d\\
&I_d = -E_I Y_{11}\sin\theta_{11} - \widetilde{V}Y_{12}\sin(\theta_{12}-\delta)\\
&\widetilde{D} = \widetilde{V}^2\Big\{\frac{T''_{d0}(X'_d - X''_d)}{(X'_d + X_e)^2}\sin^2\delta\\
&\qquad\qquad + \frac{T''_{q0}(X_q - X''_q)}{(X_q + X_e)^2}\cos^2\delta\Big\},
\end{aligned}
$$

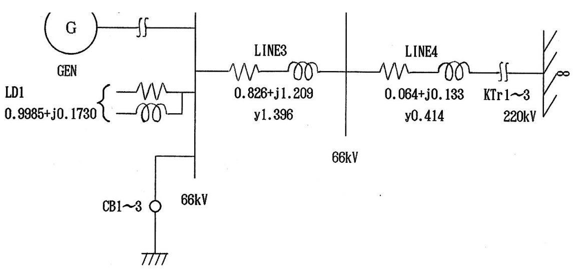

Fig. 3 Diagram of Ozeki-Power-Plant

where δ: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, P_{in}: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, E_I: open circuit voltage, E_{fd}: field excitation voltage, X_d: direct axis synchronous reactance, X'_d: direct axis transient reactance, X_e: external impedance, $Y_{11}\angle\theta_{11}$: self-admittance of the network, $Y_{12}\angle\theta_{12}$: mutual admittance of the network, and $I_d(\delta)$: direct axis current of the machine. Put x=$[x[1], x[2], x[3]]^T$=$[E_I - \hat{E}_I, \delta - \hat{\delta}_0, \dot{\delta}]^T$ and $u = E_{fd} - \hat{E}_{fd}$, so that

$$
\begin{bmatrix} \dot{x}[1]\\ \dot{x}[2]\\ \dot{x}[3]\end{bmatrix} = \begin{bmatrix} f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix} + \begin{bmatrix} g_1(x)\\ 0\\ 0\end{bmatrix} u \qquad (18)
$$

where

$$
\begin{aligned}
f_1(x) &= -\frac{1}{kT_{d0}}(x[1] + \hat{E}_I - \hat{E}_{fd})\\
&\quad + \frac{(X_d - X'_d)\widetilde{V}Y_{12}}{k}X_3\cos(\theta_{12} - x[2] - \hat{\delta}_0)\\
f_2(x) &= x[3]\\
f_3(x) &= -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}(x[1] + \hat{E}_I)\cos(\theta_{12} - x[2] - \hat{\delta}_0)\\
&\quad - \frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}(x[1] + \hat{E}_I)^2 - \frac{\widetilde{D}}{\widetilde{M}}x[3] + \frac{P_0}{\widetilde{M}}\\
g_1(x) &= \frac{1}{kT_{d0}}, \quad k = 1 + (X_d - X'_d)Y_{11}\sin\theta_{11}.
\end{aligned}
$$

Parameters are

$$
\begin{array}{llll}
\widetilde{M} = & 0.016095[pu] & T_{d0} = & 5.09907[sec]\\
\widetilde{V} = & 1.0[pu] & P_0 = & 1.2[pu]\\
X_d = & 0.875[pu] & X'_d = & 0.422[pu]\\
Y_{11} = & 1.04276[pu] & Y_{12} = & 1.03084[pu]\\
\theta_{11} = & -1.56495[pu] & \theta_{12} = & 1.56189[pu]\\
X_e = & 1.15[pu] & X''_d = & 0.238[pu]\\
X_q = & 0.6[pu] & X''_q = & 0.3[pu]\\
T''_{d0} = & 0.0299[pu] & T''_{q0} = & 0.02616[pu]\\
\hat{E}_I = & 1.52243[pu] & \hat{\delta}_0 = & 48.57^\circ\\
\hat{\dot{\delta}}_0 = & 0.0[deg/sec] & \hat{E}_{fd} = & 1.52243[pu].
\end{array}
$$

Set $\mathbf{X} = [x^T, x[4]]^T = [x[1], x[2], x[3], x[4]]^T$, $n = 3$, $\hat{X}_0 = \hat{\delta}_0 = 48.57^\circ$, $d_0 = 1$, $C(x)$=$x[2]$, $L = 1$, $\mathbf{Q}$=diag$(1,1,1,1)$, R=1, $\sigma_i = 0.33294 (0 \le i \le M)$ and $x[4](0) = 1$. Experiments are carried out for the new control(AACC), and the ordinary linear optimal control(LOC)[2].

1) AACC(New,umax=5):
M=1, $\hat{X}_1 = 80^\circ$, $D_0 = (-\infty, a - \hat{\delta}_0]$, D_1=$[a - \hat{\delta}_0, \infty)$. The parameters are suboptimally selected along the algorithm of section III. Ω=$\{N_1, N_2, d_1, a\}$,G=100, $\widetilde{q}$=100, $\widetilde{L}$=8, P_c=0.8, P_m=0.03. **D**=[0.0,2.0]$\times$[-0.5,2.0]$\times$[-5.0,5.0]$\times$[0.0,1.5]. The constrained input value is $u_{max} = -u_{min} = 5$. It results that N_1=2.27, N_2=0.22, d_1=0.10 and a=50.39°.

2) AACC(New,umax=10):
The parameters are suboptimally selected by using the same way of the AACC(New,umax=5). The constrained input value is $u_{max} = -u_{min} = 10$. It results that N_1=1.78, N_2=0.14, d_1=0.10 and a=50.00°.

Table 1: Performances

Method		$x^{\mathrm{T}}(0)$: initial point				
		$[0, 0.4, 0]$	$[0, 1.0, 0]$	$[0, 1.3, 0]$	$[0, 1.35, 0]$	$[0, 1.41, 0]$
umax=5	LOC	0.95375	$\times$	$\times$	$\times$	$\times$
	AACC(Old)	0.99825	2.19569	1.86642	$\times$	$\times$
	AACC(New)	1.16674	3.58263	3.34399	3.06732	2.79791
umax=10	LOC	0.95375	$\times$	$\times$	$\times$	$\times$
	AACC(Old)	0.93595	1.99870	2.18903	$\times$	$\times$
	AACC(New)	1.14259	3.39893	3.45338	3.27846	3.16581

$\times$: very large value

3) AACC(Old,umax=5):
The parameters are suboptimally selected by using the same way of the AACC(New,umax=5) which uses the fixed weight of the gradient optimization automatic choosing function [7]. $\Omega=\{N_1, N_2, a\}$. It results that N_1=7.42, N_2=0.10 and a=50.00°.

4) AACC(Old,umax=10):
The parameters are suboptimally selected by using the same way of the AACC(Old,umax=5). The constrained input value is $u_{max} = -u_{min} = 10$. It results that N_1=1.42, N_2=0.10 and a=50.10°.

Table 1 shows performances by the AACC(New), the AACC(Old) and the LOC. The cost function of Table 1 is

$$\widetilde{J} = \frac{1}{2}\int_0^{20} \left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T\mathbf{R}u\right) dt.$$

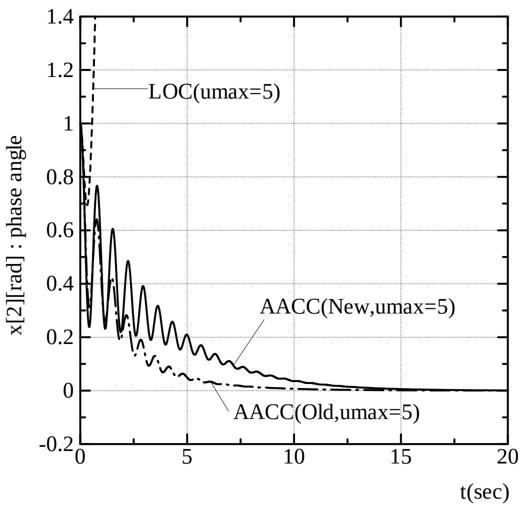

Fig. 5 Responses of LOC, AACC(Old), AACC(New) $(x^T(0) = [0, 1.0, 0])$

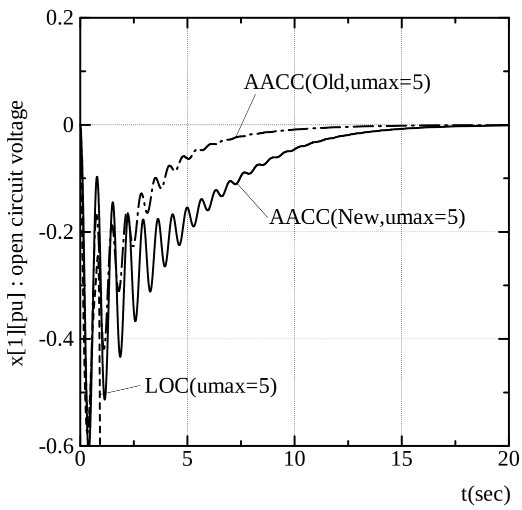

Fig. 4 Responses of LOC, AACC(Old), AACC(New) $(x^T(0) = [0, 1.0, 0])$

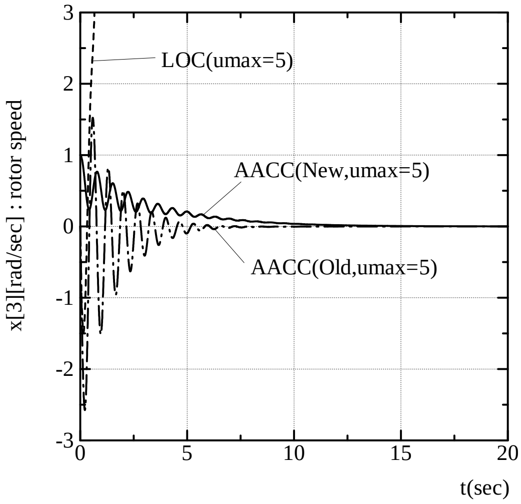

Fig. 6 Responses of LOC, AACC(Old), AACC(New) $(x^T(0) = [0, 1.0, 0])$

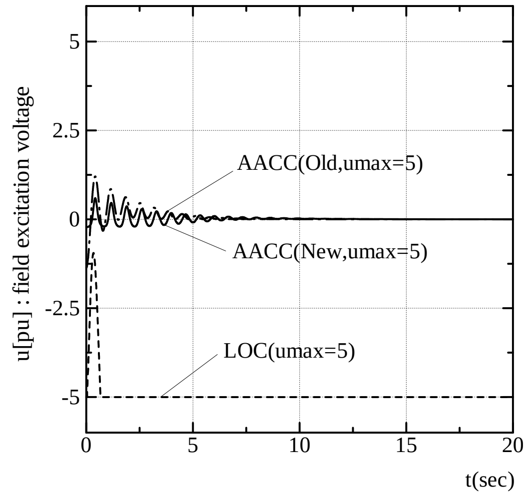

Fig. 7 Responses of LOC, AACC(Old), AACC(New) $(x^T(0) = [0, 1.0, 0])$

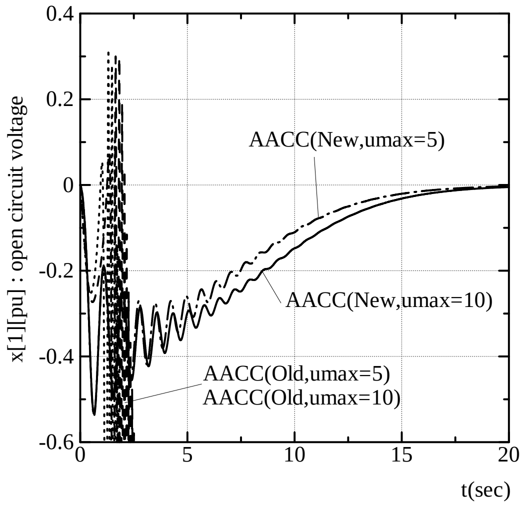

Fig. 8 Responses of AACC(Old), AACC(New) $(x^T(0) = [0, 1.35, 0])$

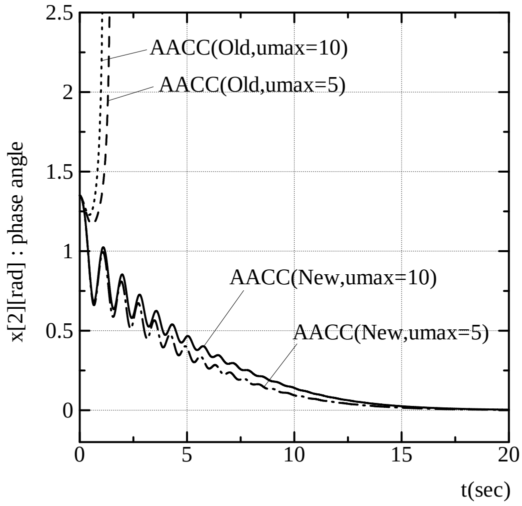

Fig. 9 Responses of AACC(Old), AACC(New) $(x^T(0) = [0, 1.35, 0])$

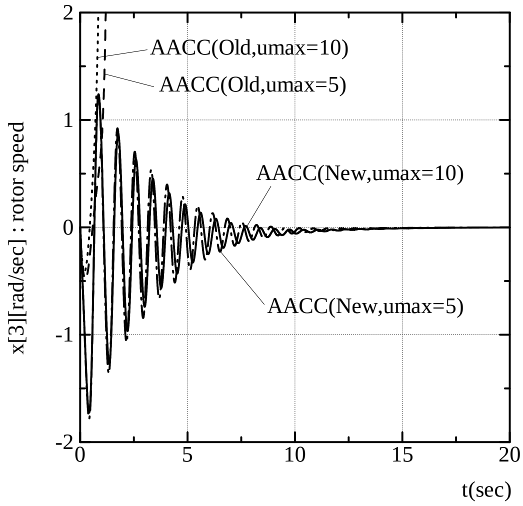

Fig. 10 Responses of AACC(Old), AACC(New) $(x^T(0) = [0, 1.35, 0])$

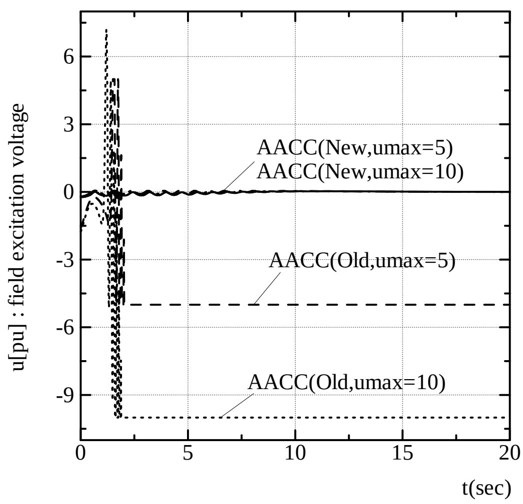

Fig. 11 Responses of AACC(Old), AACC(New) $(x^T(0) = [0, 1.35, 0])$

Figs. 4, 5, 6 and 7 show the responses in case of $x^T(0) = [0, 1.0, 0]$. Figs. 8, 9, 10 and 11 show the responses in case of $x^T(0) = [0, 1.35, 0]$. These results indicate that the stable region of AACC(New) is better than the AACC(Old) and LOC.

5 Conclusions

We have studied an augmented automatic choosing control with constrained input designed by Hamiltonian using the weighted gradient optimization automatic choosing functions for nonlinear systems. This approach was applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results have shown that this controller could improve performance remarkably well.

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