Nonlinear Control of a Single-Link Flexible Joint Manipulator via Predictive Control


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Abstract: - In this paper, a NCGPC (Nonlinear Continuous Time Generalized Predictive Control) is proposed for a single-link flexible joint manipulator. This control is developed by using a property of NCGPC, the demonstration that the selection of particular design parameters, such as control order and predictor order leads to well-known feedback linearization. Simulations are presented in order to illustrate the effectiveness of the approach.

Key-Words: - Nonlinear control, Predictive Control, Feedback linearization, flexible joint manipulator.

1 Introduction
A very important feature in modern robot system is the flexibility, which is required in specific applications of industrial automation and space system. Meanwhile in order to increase the control performance, it is necessary to consider flexible joints instead rigid joints, then the mathematical model becomes more complex if the joint flexibility is taken into account. Many controllers have been designed for this kind of manipulator, as examples [1]-[4]. Model based predictive control has recently received much attention from researchers, as a popular control technique in linear and nonlinear systems. The NCGPC [5, 6] is an alternative nonlinear predictive controller; this controller was developed in a different way than conventional nonlinear predictive controllers. The NCGPC is based in the prediction of the system output and due to the fact that it was not derived with the objective of canceling nonlinearities, as feedback linearization techniques do, the NCGPC has three advantages: First, it can constrain the predicted control through $N_y$-additionally the response becomes slow and the control is not very active-; and second, when $N_y < N_u - r$, there is not zero dynamics cancellation and then the internal stability is preserved. Also, the NCGPC [7] provides a nice way of handling systems with unstable zero dynamics. And the last advantage is the control weight $\lambda$, it plays a very important role in the cost function. In [8], the non-regular nonlinear system is treated by using the last two advantages of the NCGPC. Another of the main advantages of NCGPC control schemes is that, when $N_y = N_u - r$ they do not require on-line optimization and asymptotic tracking of the smooth reference signal is guaranteed. This last advantage will be used in this paper in order to control a single link manipulator with flexible joint

Simulations are presented in this paper in order to show the effectiveness of the control strategy.

2 Review of NCGPC
This paper considers the nonlinear SISO systems with all system states assumed to be accessible, affine in the input with the following state-space representation:

$$\dot{x}(t) = f(x) + g(x)u$$
$$y(t) = h(x)$$

where $f$, $g$ and $h$ are differentiable $N_y$ times with respect to each argument. $x \in \mathbb{R}^n$ is the vector of the state variables, $u \in \mathbb{R}$ is the manipulated input and $y \in \mathbb{R}$ is the output to be controlled. It has a well-defined relative degree and its zero dynamics are stable.

The development of the NCGPC [5, 6] was carried out following the receding horizon strategy of its linear counterpart [9].

The output prediction is approximated for a Maclaurin series expansion of the system output as follows.

$$y^*(t,T) = y(t) + y^{(2)}(t) \frac{T^2}{2!} + \cdots + y^{(N_y)}(t) \frac{T^{N_y}}{N_y!}$$

or

$$\dot{y}(t,T) = T_{N_y} Y_{N_y}$$

where

$$Y_{N_y} = [y \ y^{(2)} \ \cdots \ y^{(N_y)}]^T$$
and

\[ T_{N_r} = [1 \ T \ T^2 \ \cdots \ T^{N_r}] \]  \hspace{1cm} (5)

The predictor order \( N_r \) is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in \( u(t) \).

The NCGPC is based in taking the derivatives of the output, which are obtained as follows

\[
\begin{align*}
y(t) &= L_0 h(x) \\
y^{(2)}(t) &= L_1^2 h(x) \\
& \vdots \\
y^{(r)}(t) &= L_I^r h(x) + L_{I_1} L_{I_2} \cdots L_{I_{r-1}} h(x) u(t) \\
y^{(r+1)}(t) &= S_0(x) + J_1(x) u(t) + L_I^r h(x) u(t) \\
& \vdots \\
y^{(n_r)}(t) &= S_{n_r}(x) + J_{n_r}(x) u(t) + L_I^r h(x) u(t) + L_{I_1} L_{I_2} \cdots L_{I_{n_r-1}} h(x) u(t) + \\
& \quad \quad + I_{(n_r-1),r}(x) u^{(n_r-r)}(t)
\end{align*}
\]  \hspace{1cm} (6)

Where \( L_I h(x) \) represents the Lie derivative \( S_I, J_I \), and \( I_I \), are some functions of \( x \) (and not \( u \)). These output derivatives are obtained from the system of equation (1) and \( N_r \) is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in \( u(t) \). \( r \) is the relative degree. Output and its derivatives can be rewritten by

\[
Y_{N_r}(t) = O(x(t)) + H(x(t)) u_{N_r}
\]  \hspace{1cm} (7)

where

\[
Y_{N_r} = \begin{bmatrix} y \\ y \circ y^{(2)} \ \cdots \ y^{(N_r)} \end{bmatrix}
\]  \hspace{1cm} and

\[
u_{N_r} = \begin{bmatrix} u \\ u \circ u^{(2)} \ \cdots \ u^{(N_r-r)} \end{bmatrix}
\]  \hspace{1cm} (8)

In order to drive the predicted output along a desired smooth path (reference trajectory) to a set point. Two different reference trajectories are chosen in order to demonstrate the properties of NCGPC. The first reference trajectory is the output of the following reference model [9]

\[
W_r(t,s) = \frac{R_r(s)}{R_j(s)} \frac{w(t) - y(t)}{s}
\]  \hspace{1cm} (9)

Considering the following approximation

\[
\frac{R_r(s)}{R_j(s)} = \sum_{i=0}^{N_r} \frac{N_r!}{i!(N_r-i)!} s^i
\]  \hspace{1cm} (10)

The reference trajectory is given by

\[
w_r(t,T) = [r_0 + r_1 T + \cdots + r_{N_r} T^{N_r}] [w - y(t)] + y(t)
\]  \hspace{1cm} where

\[
w_r = [r_0 \ r_1 \ \cdots \ r_{N_r}]^T (w - y(t))
\]  \hspace{1cm} and

\[
T_{N_r} \] is given by (5)

The second reference trajectory \( y_r(t) \) is the output of the reference model represented by

\[
y_r(t) = A_r x_r(t) + B_r w(t)
\]  \hspace{1cm} (12)

\[
x_r \in R^n, A_r \in R^{n \times n_r}, B_r \in R^{n \times 1}, C_r \in R^{1 \times n_r}, w \in R
\]  \hspace{1cm} In order to define the predicted output of the reference trajectory \( y_r(t,T) \) a truncated Taylor series is used, obtaining:

\[
y_r(t,T) = y_r + y_r T + y_r T^2 + \cdots + y_r^{N_r} T^{N_r}! \]

where the derivatives are easy to obtain from the reference model simulation. Rewriting this equation

\[
w_r(t,T) = T_{N_r} w_r(t)
\]  \hspace{1cm} (13)

where

\[
w_r(t) = [y_r, y_r, y_r(2), \ldots, y_r^{(N_r)}]
\]  \hspace{1cm} and

\[
T_{N_r} \] is given by (5)

NCGPC calculates the future controls from a predicted output over a time frame. The first element \( u(t) \) of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. Thus predicted output depend on the input \( u(t) \) and its derivatives, and the future controls being function of \( u(t) \) and its \( N_r \)-derivatives. The cost function is:

\[
J(u_{N_r}) = \int_{T_f}^{T_i} [y^*(t,T) - w_r(T,t)]^2 dT
\]  \hspace{1cm} (14)

with the substitution of equations (7) and (12) or (13) the cost function the minimization results in

\[
u_{N_r} = K(w_r - O)
\]  \hspace{1cm} (15)

where

\[
T_f = \int_{T_i}^{T_f} T_{N_r} dT
\]  \hspace{1cm} (16)

The \( j \)th element of \( T_j \) is:

\[
T_{j} = \frac{T_{2j-1} - T_{2j-1}^{(i-1)}}{(i-1)!(j-1)!(i+j-1)!}
\]
and \( K = [H' T, H']^{-1} [H' T] \) \hspace{1cm} (17)
As explained above, just the first element of \( u_{\theta_0} \) is applied. The control law is given by
\[
u(t) = k [w_t - 0] \hspace{1cm} (18)
\]

3 Geometric Interpretation
In this section the input-output feedback linearization \[10\] and \[11\] is shown to be equivalent to NCGPC.

The control law given by equation (18) is analyzed; the matrix \( H \) equation (8) is decomposed as
\[
H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}
\]

\( H_1 \) is a zero matrix with dimension \( r \times (N_y - r + 1) \), and \( H_2 \) is a lower triangular matrix with dimension \( (N_y - r + 1) \times (N_y - r + 1) \) given by
\[
H_2 = \begin{bmatrix} L_y T_j h(x) & 0 & \cdots & 0 \\ J_1(x) & L_y T_j h(x) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J_{N_y-r_j}(x) & I_{N_y-r_j}(x) & \cdots & L_y T_j h(x) \end{bmatrix}
\]

The matrix \( T_j \) equation (5) is decomposed as
\[
T_j = \begin{bmatrix} T_{y11} & T_{y12} \\ T_{y21} & T_{y22} \end{bmatrix}
\]

Where
\( T_{y11} \) is \( r \times r \)
\( T_{y12} \) is \( r \times (N_y - r + 1) \)
\( T_{y21} \) is \( (N_y - r + 1) \times r \)
\( T_{y22} \) is \( (N_y - r + 1) \times (N_y - r + 1) \)

The equation (17) can be written as
\[
K = H_2^{-1} [T_{y22}^{-1} T_{y21}]
\]

The unitary matrix \( I \) has dimension \( (N_y - r + 1) \times (N_y - r + 1) \). The first row of the inverse of \( H_2 \) is given by
\[
h_2^{-1} = \begin{bmatrix} 1/L_y & 0 & \cdots & 0 \end{bmatrix}
\]

Then, the first row of \( K \), which will be called \( k \)
\[
k = \frac{1}{L_y} [L_y T_j h(x) t_1 \ t_2 \ \cdots \ t_r \ 1 \ 0 \ \cdots \ 0]
\]

where \( t_1, t_2, t_3, \ldots, t_r \) are elements of the first row of \( T_{y22}^{-1} T_{y21} \). If the reference trajectory is chosen as the equation (11), the control law is given by
\[
u(t) = \frac{(t_{f_0} + \cdots + t_{f_{r-1}})(w - y(t)) - \sum_{i=0}^{r-1} t_{i+1} L_y h(x) - L_y h(x)}{L_y T_j h(x)} \hspace{1cm} (25)
\]
\[
u(t) = \frac{(w - y(t)) - \sum_{i=0}^{r-1} \beta_i L_y h(x)}{\beta_i L_y T_j h(x)} \hspace{1cm} (26)
\]

where
\[
\beta_i = \frac{1}{t_{f_0} + t_{f_1} + \cdots + t_{f_{r-1}}}
\]
\[
\beta_i = t_{i+1} (t_{f_0} + t_{f_1} + \cdots + t_{f_{r-1}}) \hspace{1cm} i = 1, 2, \ldots, r - 1
\]

We can notice, that large \( N_r \) does not require a bigger computational effort, because as we can see from equation (26), the control depends just on the r-first derivatives, thus the rest of the derivatives only have influence in obtaining the parameters of \( t_f \), which just depends on T. Moreover, \( N_r \) can be chosen as the smallest predictor order, which is such that the predicted output depends on \( u(t) \). Because of this, the relative degree \( r \) will be the smallest predictor order \( N_r \). We can conclude if \( N_r = N_y - r \), the control law is independent of the last \( N_y - r \) derivatives. Then it is possible to calculate the parameters \( \beta_i \) considering the largest \( N_r \) without the use of the remaining derivatives. Substituting equation (26) into the rth derivative given by equation (6) leads to:
\[
y''(t) = \frac{1}{\beta_r} (w - y) - \sum_{i=r+1}^{r+1} \beta_i y''
\]

Rearranging and taking Laplace transforms, the resulting closed-loop transfer function is given by:
\[
Y(s) = G(s)W(s)
\]
\[
G(s) = \frac{1}{\beta_r s^r + \beta_{r-1} s^{r-1} + \cdots + \beta_1 s + 1}
\]

Note that, by using the Routh-Hurwitz criterion, we can show that the systems are stable only for systems with \( r \leq 4 \).

If the reference trajectory is chosen as the equation (12), following the same procedure the control law is given by
\[
u(t) = \frac{\sum_{i=0}^{r} t_{i+1} [y_i L_y h(x) - L_y h(x) + y_j]}{L_y T_j h(x)} \hspace{1cm} (30)
\]

We can see that the control law is identical to the control law presented by Isidori \[10\], which solves the problem known as asymptotic model matching.
4 Control law of a Single-Link Flexible Joint Manipulator

In this section the predictive control (30) of the single-link flexible joint manipulator is obtained, the derivatives are required for this controller, which are obtained of the single-link flexible joint manipulator model. The single-link flexible joint manipulator is shown in Fig. 1, which has a difference between the angular position of the motor and that of the driven link, i.e. joint flexibility exists. The mathematical model is given by [4].

\[
x_i = x_i
\]

\[
\dot{x}_i = -\frac{MgL}{I} \sin(x_i) - \frac{K}{I} (x_i - x_i)
\]

(31)

\[
\dot{x}_i = x_i
\]

\[
x_i = \frac{K}{J} (x_i - x_i) + \frac{1}{J} u
\]

where

\[I\] Inertia of flexible manipulator 0.03 kgm^2

\[J\] Inertia of rotational platform 0.004 kgm^2

\[g\] Gravitational acceleration 9.81 N/m

\[L\] Distance to center of gravity of rotational platform 0.135m

\[M\] Mass of the flexible joint 0.6 Kg

\[k\] Flexibility coefficient joint 31.0 Nm/rad

The values are considered from [4], the output is the link angular displacement \(x_i\) and the control \(u\) is the torque given by the motor.

![Figure 1. Single-link flexible joint manipulator](image)

The reference trajectory \(y_r(t)\) is the output of the reference model represented by

\[
\dot{y}_r = y_{r1}
\]

\[
\dot{y}_{r1} = y_{r2}
\]

\[
\dot{y}_{r2} = y_{r3}
\]

\[
\dot{y}_{r3} = -16y_{r3} - 96y_{r2} - 256y_{r1} - 256y_r + 256w
\]

(32)

The derivatives are obtained as follows

\[h(x) = x_1\]

\[L_j h(x) = x_2\]

\[L_j^2 h(x) = -\frac{MgL}{I} \sin(x_1) - \frac{K}{I} (x_1 - x_1)\]

(33)

\[L_j^3 h(x) = -\frac{MgL}{I} \cos(x_1) - \frac{K}{I} (x_2 - x_1)\]

\[L_j^4 h(x) = \frac{MgL}{I} \sin(x_1) \left[ x_2^2 + \frac{MgL}{I} x_2 \cos(x_1) + \frac{K}{I} \right] + \frac{K}{I} (x_1 - x_1) \left[ \frac{K}{J} + \frac{K}{J} + \frac{MgL}{I} x_2 \cos(x_1) \right]
\]

\[L_j^5 h(x) = \frac{K}{JJ}\]

5 Simulations

In order to show the effectiveness of the proposed controller simulation will be presented. In this simulation design parameters are chosen as: setpoint equal \(\pi/2, r=4, N_r=4, N_c=N_r-r=0, T_2=1\) and \(T_1=0\). The output system and the reference are shown Fig. 2, Fig. 3 show the error tracking and Fig. 4 the systems states.

![Figure 2. System Output and reference trajectory](image)

![Figure 3. System Tracking error](image)
4 Conclusion

The selection of particular design parameters NCGPC (Nonlinear Continuous Time Generalized Predictive Control), such as control order and predictor order leads to well-known feedback linearization. The response of closed loop is influenced by the prediction horizon and the reference model. Simulations are presented in order to demonstrate the effectiveness of NCGPC.

Another of the main advantages of NCGPC control schemes is that, when $N_u = N_y - r$, they do not require on-line optimization and asymptotic tracking of the smooth reference signal is guaranteed. We can show, that incredible as it may seem, large $N_y$ does not require a bigger computational effort, because the control depends just on the r-first derivatives, thus the rest of the derivatives only have influence in obtaining the parameters of $t_i$, which just depends on T. Then it is possible to calculate the parameters $\beta_i$ considering the largest $N_y$, without the use of the remaining derivatives. Additionally, it is shown that the control law is another feedback linearization, thus closed-loop stability is ensured.

A closed-loop transfer function was found, it is possible to infer that, by using the Routh-Horwitz criterion, systems are stable only for systems with $r \leq 4$.

References:


