Research on a Cournot-Bertrand Triopoly Game between the Upstream Firms and the Downstream Firm

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Abstract: In this paper, a new Cournot-Bertrand triopoly game between the upstream firms and the downstream firm is established, which is closer to the reality of the modern enterprises in the real economy. Using nonlinear dynamics and bifurcation theory, the local stable region of the Nash equilibrium point is obtained, and its complex dynamics are described by means of the bifurcation diagrams, the largest Lyapunov exponents and the phase portraits. The stability of the Nash equilibrium will change and even the complex dynamics such as doubling period bifurcation and chaos happen when the adjustment speed parameter exceeds a certain critical value. Furthermore, by using the straight-line stabilization method, the chaos can be eliminated. This paper has an important theoretical and practical significance to the enterprises under the background of globalization.

Key–Words: Nonlinear dynamic system; Complexity; Chaos; Cournot-Bertrand triopoly game; Bounded rationality players

1 Introduction

The economic system is whether a chaotic system is a very hot topic in the economic field. In recent years, a series of dynamic game models on the output decision (Cournot model) and price decision (Bertrand model) were studied in related references. Agiza [1] and Kopel [2] considered bounded rationality and established a duopoly Cournot model with linear demand function and cost functions. From then on, the model was extended to multi-oligopolistic market. Bischi et al. [3] studied a duopoly game model, in which the firms determined their output by the reaction functions, that is, all the two players toke naive expectation. Agiza and Elsadany [4] improved the model which contains two-typesbof heterogeneous players: one bounded rational player and one adaptive expectation player. Zhang et al. [5] further improved the model with nonlinear cost functions. Matsumoto and Nonaka [6] researched the complexity of the Cournot model with linear cost functions. Ma and Ji [7] considered a Cournot triopoly game model in electric power with square inverse demand, and the model was further studied by Ji [8] with heterogeneous players. Ma and Feng [9] studied the chaotic behavior in retailer's demand model. Xin et al. [10] researched the complexity of an adnascent-type game model. Yassen and Agiza [15] considered a Cournot duopoly game and the model with delayed rationality. Chen et al. [11] used Bertrand model with linear demand functions to study the competition in the Chinese telecommunications market. Sun and Ma [12] introduced a Bertrand model with nonlinear demand functions in Chinese cold rolled steel market, and researched the complexity and the control of the model. Xin and Chen [13] considered a master-slave Bertrand game model with boundedly rational players. In these literatures, adjustment speed or other parameters are taken as bifurcation parameters, and complex results such as period doubling bifurcation, unstable periodic orbits and chaos are discovered.

So far as we have known, Naimzada and Tramontana [14] are the first researchers who considered Cournot-Bertrand game model. Based on the master-slave Bertrand game model [13] and the triopoly Cournot game model [7], a new nonlinear Cournot-Bertrand triopoly game between upstream firms and downstream firm is built up. The Cournot-Bertrand triopoly model is closer to the real economy under the background of economic globalization. Suppose the upstream firms have nonlinear inverse demand function, and the downstream firm has linear inverse demand function. The cost functions are nonlinear. In this model, the bounded rational players regulate the output (or the price) speed according to marginal profit, and decide the output (or the price). By theoretical analysis and numerical simulations, the stable region about the output adjustment speed parameters is derived. The output adjustment speed parameters effects on the dynamics characteristics of the system are investigated. A further analysis of the current economic system has important theoretical and practical significance.

The paper is organized as follows. In Section 2, the dynamics of a nonlinear Cournot-Bertrand triopoly game model is presented the equilibrium points and their stability are analyzed. In Section 3, the dynamics characteristics of the system are studied, which is demonstrated by the bifurcation diagrams, the largest Lyapunov exponents and the sensitive dependence on initial conditions. In Section 4, The impact of price adjustment speed on average profit are studied.

In Section 5, chaos control of the system is considered with the straight-line stabilization method. Finally, some conclusions are made.

2 The model

For the economic globalization, the final product is not usually produced by a firm. In general, the upstream firms produce the primary products, but the downstream firms process and assemble the primary products. So, there is a cooperation between the upstream firms and the downstream firms. In the modern industry, the upstream companies may form an oligopoly market, and the downstream enterprise is a monopoly market for the intellectual property rights. So, suppose that there are two upstream firms X_1, X_2 in the oligopoly market, and one downstream firm Y in the monopoly market. Upstream firms X_1, X_2 make the optimal output decision, and suppose the t-output is $q_{x_i}(t), (i = 1, 2)$, respectively. However, the downstream firm Y make the optimal price decision, and suppose the t-price is p_y.

There is no difference between the products of the upstream companies. As the products of the upstream companies have many alternative uses, their demand is not affected by the demand of the downstream enterprise. At each period t, the price of the upstream oligopoly market p_X is determined by the total output $Q_X(t) = q_{x_1}(t)+q_{x_2}(t)$. Propose the inverse demand function is in the following nonlinear form as [7]:

$$p_X = p_X(Q_X) = a_1 - b_1 Q_X^2, \tag{1}$$

and the cost functions of firms X_1, X_2 are as follows:

$$C_{X_1} = c_1 q_{x_1}^2, \qquad C_{X_2} = c_2 q_{x_2}^2, \tag{2}$$

where a_1, b_1, c_1, c_2 are positive parameters.

As the products of the upstream companies are the complementary product for the downstream enterprise, the upstream market price p_x is a factor of the cost of the downstream firm Y. The downstream firm Y has the following linear form demand function and nonlinear cost function:

$$Q_Y = a_2 - b_2 p_y, \quad C_Y = c_3 Q_Y^2 + \gamma p_x Q_Y. \tag{3}$$

where a_2, b_2, c_3, γ are positive parameters.

We can see that the profit of the upstream firms and the downstream firm are

$$\begin{cases} \pi_{X_1}(t) = q_{x_1}(t)[a_1 - b_1 Q_X^2(t)] - c_1 q_{x_1}^2(t), \\ \pi_{X_2}(t) = q_{x_2}(t)[a_1 - b_1 Q_X^2(t)] - c_2 q_{x_2}^2(t), \\ \pi_Y(t) = p_y(t)(a_2 - b_2 p_y) - c_3 Q_Y^2 - \gamma p_X Q_Y. \end{cases} \tag{4}$$

As the game between the upstream firms and the downstream firm is a continuous and long-term repeated dynamic process, the dynamic adjustment of this repeated Cournot-Bertrand triopoly game with bounded rational players are as follows:

$$\begin{cases} q_{x_1}(t+1) = & q_{x_1}(t) + \alpha_1 q_{x_1}(t) \frac{\partial \pi_{X_1}}{\partial q_{x_1}}, \ 0 \le \alpha_1 \le 1, \\ q_{x_2}(t+1) = & q_{x_2}(t) + \alpha_2 q_{x_2}(t) \frac{\partial \pi_{X_2}}{\partial q_{x_2}}, \ 0 \le \alpha_2 \le 1, \\ p_y(t+1) = & p_y(t) + \alpha_3 p_y(t) \frac{\partial \pi_Y}{\partial p_y}, \quad 0 \le \alpha_3 \le 1. \end{cases} \tag{5}$$

where α_1, α_2 are output adjustment speed parameters, and α_3 is the price adjustment speed parameter.

Combining Eqs. (4), (5), a dynamic Cournot-Bertrand triopoly game between the upstream firms and the downstream firm with bounded rationality has the following form:

$$\begin{cases} q_{x_1}(t+1) & = q_{x_1}(t) + A, \\ q_{x_2}(t+1) & = q_{x_2}(t) + B, \\ p_y(t+1) & = p_y(t) + C. \end{cases} \tag{6}$$

$A = \alpha_1 q_{x_1}(t)[a_1 - 3b_1 q_{x_1}^2 - b_1 q_{x_2}^2 - 4b_1 q_{x_1} q_{x_2} - 2c_1 q_{x_1}]$,
$B = \alpha_2 q_{x_2}(t)[a_1 - 3b_1 q_{x_2}^2 - b_1 q_{x_1}^2 - 4b_1 q_{x_1} q_{x_2} - 2c_1 q_{x_2}]$,
$C = \alpha_3 p_y(t)[a_2 - 2b_2(1 + b_2 c_3) p_y(t) - \gamma b_1 b_2 (q_{x_1} + q_{x_2})^2 + 2a_2 b_2 c_3 + \gamma a_1 b_2]$.

2.1 The equilibrium point and stability analysis

The bifurcation parameter is $\alpha_i (i = 1, 2, 3)$, and other parameters of system (6) are as follows: $a_1 = 6, b_1 = 0.98, c_1 = 0.35, c_2 = 0.45, a_2 2 = 6.2, b_2 = 0.85 c_3 = 0.22, \gamma = 2$.

By solving the following equations, the fixed points of system (6) can be obtained

$$\begin{cases} q_{x_1}(t)A = 0, \\ q_{x_2}(t)B = 0, \\ p_y(t)C = 0. \end{cases} \tag{7}$$

$A = \alpha_1 q_{x_1}(t)[a_1 - 3b_1 q_{x_1}^2 - b_1 q_{x_2}^2 - 4b_1 q_{x_1} q_{x_2} - 2c_1 q_{x_1}]$,
$B = \alpha_2 q_{x_2}(t)[a_1 - 3b_1 q_{x_2}^2 - b_1 q_{x_1}^2 - 4b_1 q_{x_1} q_{x_2} - 2c_1 q_{x_2}]$,
$C = \alpha_3 p_y(t)[a_2 - 2b_2(1+b_2c_3)p_y(t) - \gamma b_1 b_2 (q_{x_1} + q_{x_2})^2 + 2a_2b_2c_3 + \gamma a_1 b_2]$.

The Eqs.(7) are solved and three meaningful fixed points $p_1(0.8459, 0.8050, 7.0263)$, $p_2(1.3145, 0, 7.8498)$, $p_3(0, 1.2837, 7.9159)$ are obtained. The stability of the Nash equilibrium point $p^*(q_{x_1}^* = 0.8459, q_{x_2}^* = 0.8050, p_y^* = 7.0263)$ is only considered here.

The Jacobian matrix of system (6) at the Nash equilibrium point p^* is

$$J = \begin{pmatrix} 1+j_{11} & j_{12} & 0 \\ j_{21} & 1+j_{22} & 0 \\ j_{31} & j_{32} & 1+j_{33} \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned} j_{11} &= \alpha_1 q_{x_1}^*(-6b_1 q_{x_1}^* - 4b_1 q_{x_2}^* - 2c_1), \\ j_{12} &= \alpha_1 q_{x_1}^*(-2b_1 q_{x_2}^* - 4b_1 q_{x_1}^*), \\ j_{21} &= \alpha_2 q_{x_2}^*(-2b_1 q_{x_1}^* - 4b_1 q_{x_2}^*), \\ j_{22} &= \alpha_2 q_{x_2}^*(-6b_1 q_{x_2}^* - 4b_1 q_{x_1}^* - 2c_2), \\ j_{31} &= j_{32} = -2\alpha_3 p_y^* \gamma b_1 b_2 (q_{x_1}^* + q_{x_2}^*), \\ j_{33} &= -2\alpha_3 b_2 (1 + b_2 c_3) p_y^*. \end{aligned} \tag{9}$$

The characteristic polynomial of system (6) is:

$$f(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0, \tag{10}$$

where

$$\begin{aligned} A_2 &= -(3 + j_{11} + j_{22} + j_{33}), \\ A_1 &= (1+j_{11})(1+j_{22}) - j_{21}j_{12} \\ &+ (2 + j_{11} + j_{22})(1 + j_{33}), \\ A_0 &= -(1+j_{33})[(1+j_{11})(1+j_{22}) - j_{21}j_{12}]. \end{aligned} \tag{11}$$

The necessary and sufficient conditions for the local stability of Nash equilibrium can be obtained by Jury test [16]:

$$\begin{cases} i) \ \ f(1) = 1 + A_2 + A_1 + A_0 > 0, \\ ii) \ \ -f(-1) = 1 - A_2 + A_1 - A_0 > 0, \\ iii) \ 1 - A_0^2 > 0, \\ iv) \ \ (1 - A_0^2)^2 - (A_1 - A_2A_0)^2 > 0. \end{cases} \tag{12}$$

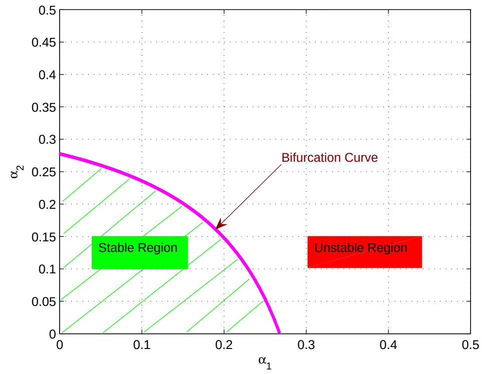

Figure 1: The stable region of Nash equilibrium point about adjustment speed (α_1, α_2) with $\alpha_3 = 0.1$

By solving the above equations, the local stable region of Nash equilibrium point can be got. The phase diagram of the stable region is shown in Fig. 1 with positive (α_1, α_2) and $\alpha_3 = 0.1$. Similarly, for the fixed α_1 and α_2, the other two stable region diagrams (α_1, α_3) and (α_2, α_3) can also be got, respectively, but they are omitted here. The Nash equilibrium is stable for the values (α_1, α_2) inside the stable region. The meaning of the stable region is that whatever initial output are chosen by the upstream firms and the downstream firm in the local stable region, they will eventually arrive at the Nash equilibrium output in a finite of games. It is valuable to analyze the enterprises on accelerating the adjustment speed for the expectation of getting more profits. However, the adjustment parameters have no matter with the Nash equilibrium point. Once one party is adjusting output speed too fast and pushing $\alpha_i, (i = 1, 2, 3)$ out of the stable region, the system tends to become unstable and even falls into chaotic state. Numerical simulation method is used to analyze the characteristics of the nonlinear dynamical system with the increasing of $\alpha_i, (i = 1, 2, 3)$. Numerical results such as the bifurcation diagrams, the largest Lyapunov exponents, the strange attractors and the sensitive dependence on initial conditions will be discussed in the following section.

3 Complex dynamics features of system

The bounded rational players make decision on the basis of the marginal profit of the last period. The companies decide to increase their output if it has a positive marginal profit, and decrease their output if the marginal profit is negative. Thus, adjustment speed parameter $\alpha_i, (i = 1, 2, 3)$ has an important effect on game results. In the following section, the effect of

$\alpha_i, (i = 1, 2, 3)$ on dynamical behaviors of system (6) will be investigated.

3.1 The output and price adjustment speed effect on the system

Once the upstream company X_1 accelerates output adjustment speed and pushes α_1 out of the stable region, the stability of Nash equilibrium point will change. For $\alpha_2 = 0.2$, Fig.2 illustrates that the output evolution of the upstream firms and the downstream firm start with equilibrium state, through period doubling, and end with chaotic state with output adjustment speed α_1 increasing. The diagrams of Bifurcation and the largest Lyapunov exponent with α_1 increasing are shown in Fig.2 when $(\alpha_2 = 0.09, \alpha_3 = 0.1)$. If the largest Lyapunov exponent λ_1 is positive, system (6) is in a chaotic state. We can see that system (6) is stable at Nash equilibrium point when $0 < \alpha_1 < 0.2362$. For $0.2362 < \alpha_1 < 0.3046$, system (6) occurs a 2-cycle. For $0.3046 < \alpha_1 < 0.3323$, system (6) has a 4-cycle. For $0.3323 < \alpha_1 < 0.3434$, system (6) is in a chaotic state, and the representative strange chaos attractor as shown in Fig. 3.

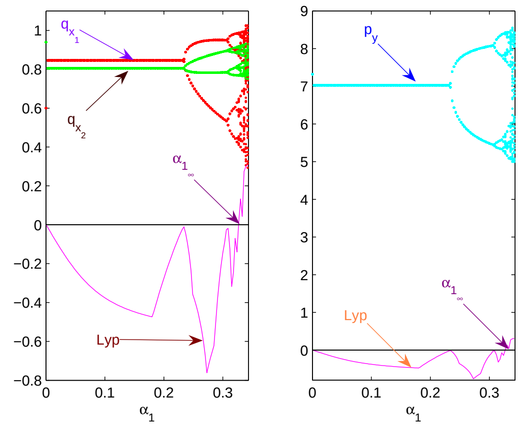

Figure 2: Bifurcation diagram and the largest Lyapunov exponent with $\alpha_1 \in (0, 0.3434]$, and $(\alpha_2 = 0.09, \alpha_3 = 0.1)$

Similarly, Fig. 4 shows an one-parameter bifurcation diagram and the maximal Lyapunov exponent with respect to α_2 and when $\alpha_1 = 0.07, \alpha_3 = 0.1$. We can see that Nash equilibrium point is stable for $0 < \alpha_2 < 0.2497$, which implies output of three the upstream firms and the downstream firm are in an equilibrium state. With α_2 increasing, the stability of equilibrium point changes, output undergo doubling period bifurcation and system (6) eventually falls into

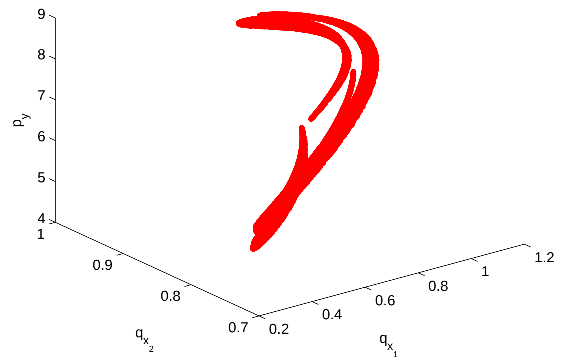

Figure 3: Chaos attractor of system (6) for $(\alpha_1 = 0.339, \alpha_2 = 0.09, \alpha_3 = 0.1)$, and initial point $(q^0_{x1} = 0.6, q^0_{x2} = 0.5, p^0_y = 0.2)$

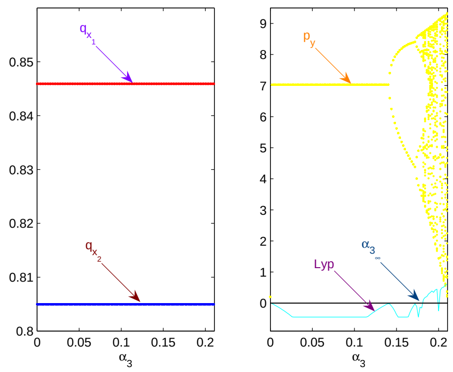

Figure 4: Bifurcation diagram and the largest Lyapunov exponent with $\alpha_3 \in (0, 0.2107]$, and $(\alpha_1 = 0.1, \alpha_2 = 0.12)$

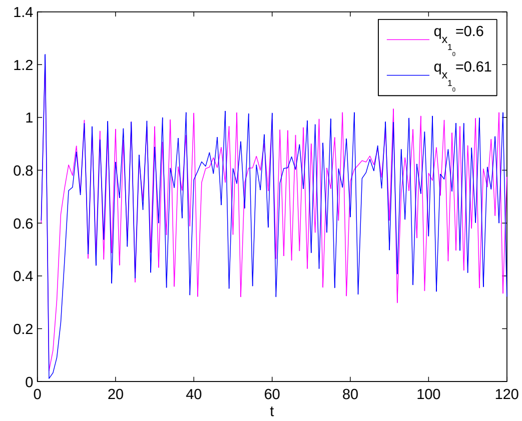

Figure 5: For the respective initial points (0.6, 0.5, 0.2) and (0.61, 0.5, 0.2), sensitive dependence on initial conditions: a) the two orbits of the output q_{x_1}; b)

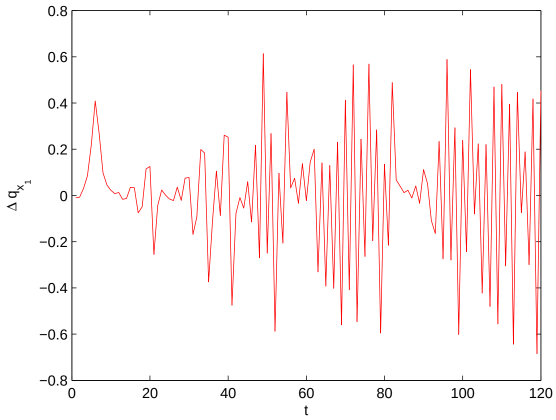

Figure 6: the difference between the two orbits of the output q_{x_1}

chaos. $\alpha_2 \in (0.2497, 0.3247]$ is the range of 2-cycle fluctuation. For $\alpha_2 > 0.3247$, output period doubling occurs again. $\alpha_2 \in (0.3247, 0.3496]$ is the domain of 4-cycle output fluctuation. $\alpha_2 \in (0.3496, 0.3871]$ is the domain of the system (6) in chaotic states, and the representative strange chaos attractor as shown in Fig. 15. likewise, Fig. 4 shows an one-parameter bifurcation diagram with respect to α_3 when $(\alpha_1 = 0.1, \alpha_2 = 0.12)$. From Fig. 4, we can see that the upstream firms output are all at Nash equilibrium point for $\alpha_3 \in (0, 0.2107]$ and $(\alpha_1 = 0.1, \alpha_2 = 0.12)$, but the downstream firm output through doubling period bifurcation to chaos.

The sensitive dependence on initial conditions is one of the important features of chaos. To verify whether system (6) depends on initial values sensitively, the relationships between output and time are

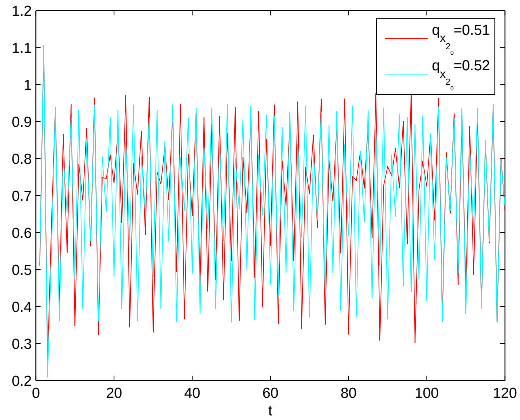

Figure 7: For the respective initial points (0.6, 0.51, 0.2) and (0.6, 0.52, 0.2), sensitive dependence on initial conditions: a) the two orbits of the output q_{x_2}; b)

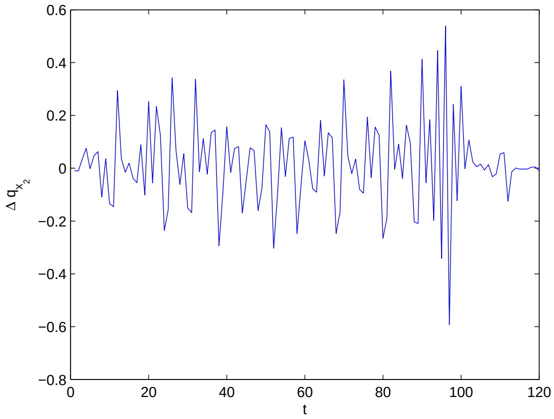

Figure 8: the difference between the two orbits of the output q_{x_2}

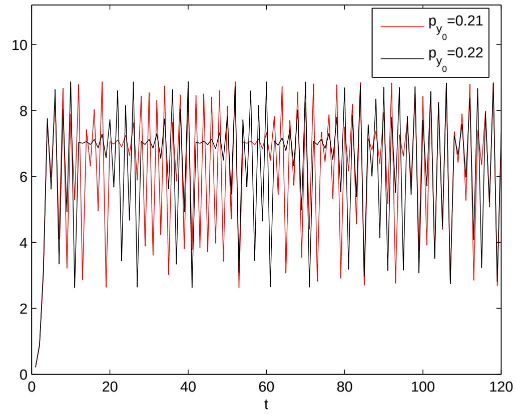

Figure 9: For the respective initial points (0.6, 0.5, 0.21) and (0.6, 0.5, 0.22), sensitive dependence on initial conditions: a) the two orbits of the output p_y; b)

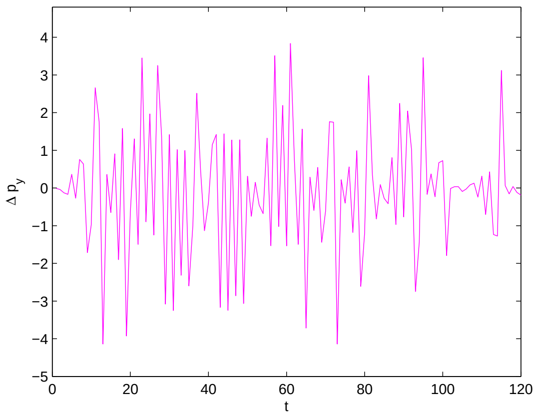

Figure 10: the difference between the two orbits of the output p_y

shown in Figs. 6, 8 and 10 when $(\alpha_1 = 0.339, \alpha_2 = 0.09, \alpha_3 = 0.1)$, $(\alpha_1 = 0.07, \alpha_2 = 0.357, \alpha_3 = 0.1)$ and $(\alpha_1 = 0.1, \alpha_2 = 0.12, \alpha_3 = 0.189)$, respectively. At first, the difference is indistinguishable, but with the number of the game increasing, the difference between them is great. This implies that only a little difference between initial data will have a great impact on the results of the game. It further proves that system (6) falls into a chaotic state when when $(\alpha_1 = 0.339, \alpha_2 = 0.09, \alpha_3 = 0.1)$, $(\alpha_1 = 0.07, \alpha_2 = 0.357, \alpha_3 = 0.1)$ and $(\alpha_1 = 0.1, \alpha_2 = 0.12, \alpha_3 = 0.189)$. The stability of the market will be destroyed, and it is difficult for the upstream firms and the downstream firm to plan long-term strategy. A slight adjustment of the initial data can have a great effect on the game results.

3.2 Evolution of attractors the system

Attractors are divided into ordinary attractors(called attractor for short), quasi periodic attractors, and chaotic attractors and so on. In this section, we study the evolution of attractors with $(\alpha_1, \alpha_2, \alpha_3)$, let $(\alpha_2 = 0.09, \alpha_3 = 0.1)$and let $\alpha_1$changes from 0.2 to 0.25,0.31,0.33,0.34,0.35, and we can get the attractors in the following figures: in Fig.11, attractor of system is the Nash equilibrium point of the system, that means after a series of games,the system will converge to the Nash equilibrium point, and as long as the system eventually converge to the Nash equilibrium point,the Nash equilibrium point of the system is uniquely identified.

in Fig.12 and 13 , with the increase of α_1, attractors of system are period-2 cycle and period-4 cycle , that means after a series of games, the system will converge to period-2 or period-4 , and as long as

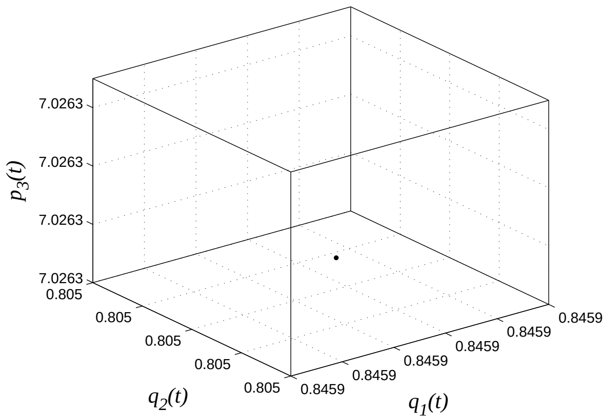

Figure 11: Attractor of system (6) for $(\alpha_1 = 0.2, \alpha_2 = 0.09, \alpha_3 = 0.1)$, and initial point $(q^0_{x_1} = 0.6, q^0_{x_2} = 0.5, p^0_y = 0.2)$

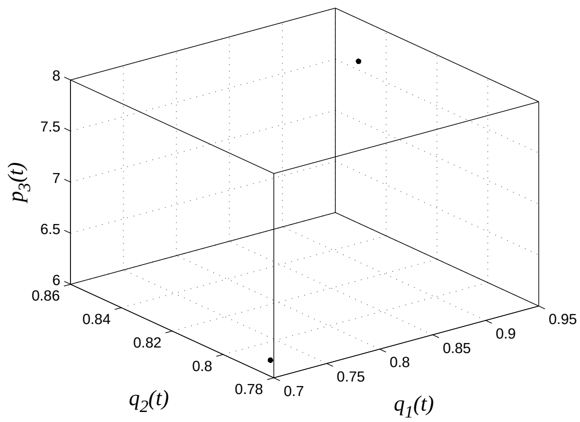

Figure 12: Attractor of system (6) for $(\alpha_1 = 0.25, \alpha_2 = 0.09, \alpha_3 = 0.1)$, and initial point $(q^0_{x_1} = 0.6, q^0_{x_2} = 0.5, p^0_y = 0.2)$

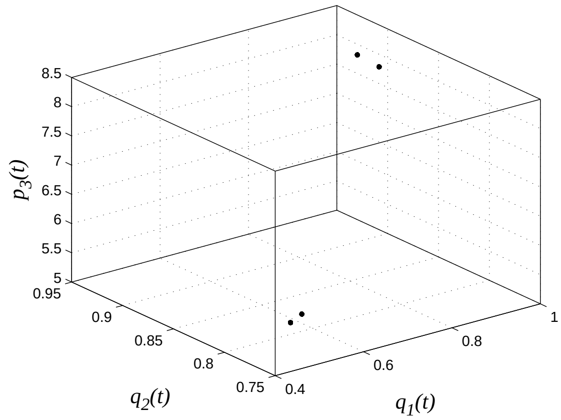

Figure 13: Attractor of system (6) for ($\alpha_1 = 0.31, \alpha_2 = 0.09, \alpha_3 = 0.1$), and initial point ($q^0_{x_1} = 0.6, q^0_{x_2} = 0.5, p^0_y = 0.2$)

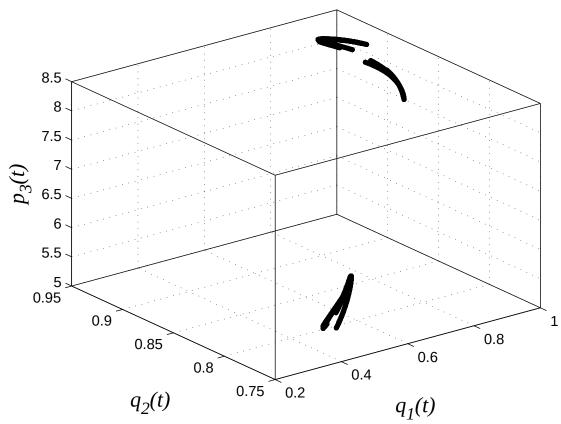

Figure 14: Chaos attractor of system (6) for ($\alpha_1 = 0.33, \alpha_2 = 0.09, \alpha_3 = 0.1$), and initial point ($q^0_{x_1} = 0.6, q^0_{x_2} = 0.5, p^0_y = 0.2$)

the system eventually converge toperiod-2 or period-4 point, the attractors of the system is uniquely identified.

in Fig.14,15and 16 , with the increase of α_1, attractors of system are all chaos attractor , that means after a series of games, the system will converge to chaos, the biggest difference between chaotic region and periodic oscillation region is that chaos attractor is changed. For example, in the stable region, the attractor is not changed with the α_1, but in the chaos region, that chaos attractor is changed as can be seen in in Fig.14,15and 16.

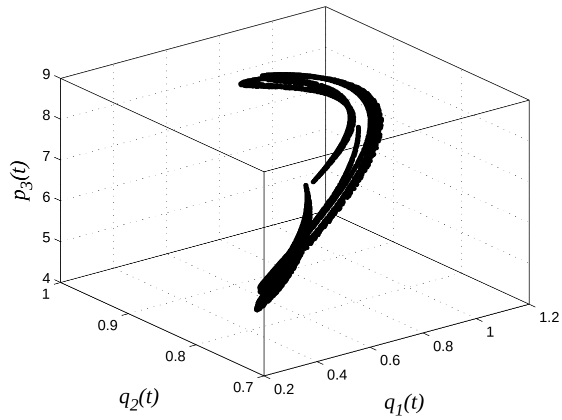

Figure 15: Chaos attractor of system (6) for ($\alpha_1 = 0.34, \alpha_2 = 0.09, \alpha_3 = 0.1$), and initial point ($q^0_{x_1} = 0.6, q^0_{x_2} = 0.5, p^0_y = 0.2$)

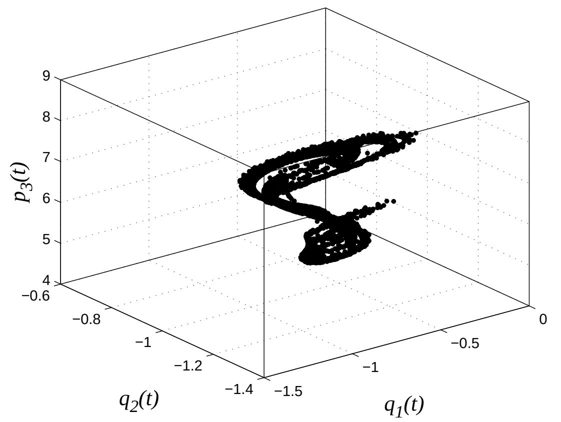

Figure 16: Chaos attractor of system (6) for ($\alpha_1 = 0.35, \alpha_2 = 0.09, \alpha_3 = 0.1$), and initial point ($q^0_{x_1} = 0.6, q^0_{x_2} = 0.5, p^0_y = 0.2$)

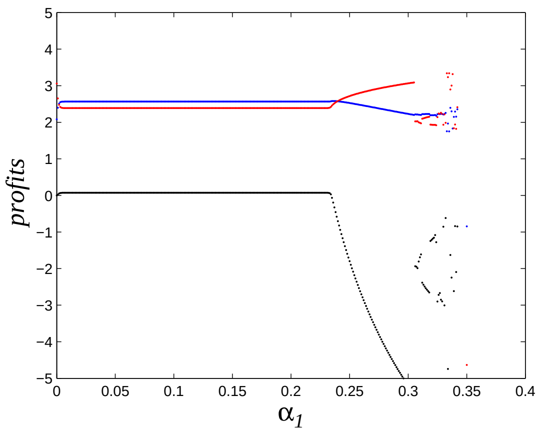

Figure 17: Bifurcation diagram of profit with α_1

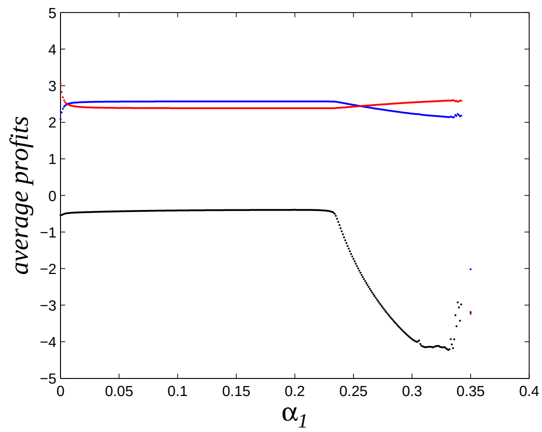

Figure 18: Effect of α_1 on average profit

4 The impact of price adjustment speed on average profit

As the game between the upstream firms and the downstream firm is a continuous and long-term repeated dynamic process, the dynamic adjustment of this repeated Cournot-Bertrand triopoly profit game with bounded rational players are as follows:

$$\begin{cases} q_{x_1}(t+1) = q_{x_1}(t) + \alpha_1 q_{x_1}(t)\frac{\partial \pi_{X_1}(t)}{\partial q_{x_1}(t)}, \\ q_{x_2}(t+1) = q_{x_2}(t) + \alpha_2 q_{x_2}(t)\frac{\partial \pi_{X_2}(t)}{\partial q_{x_2}(t)}, \\ p_y(t+1) = p_y(t) + \alpha_3 p_y(t)\frac{\partial \pi_Y(t)}{\partial p_y(t)}, \\ \pi_{X_1}(t+1) = q_{x_1}(t+1)[a_1 - b_1 Q_X^2(t+1)] \\ -c_1 q_{x_1}^2(t+1), \\ \pi_{X_2}(t+1) = q_{x_2}(t+1)[a_1 - b_1 Q_X^2(t+1)] \\ -c_2 q_{x_2}^2(t+1), \\ \pi_Y(t+1) = p_y(t+1)(a_2 - b_2 p_y(t+1)) \\ -c_3 Q_Y(t+1)^2 - \gamma p_X(t+1) Q_Y(t+1). \end{cases} \tag{13}$$

The bifurcation parameter is α_1, and other parameters of system (13) are as follows: $a_1 = 6, b_1 = 0.98, c_1 = 0.35, c_2 = 0.45, a_2 2 = 6.2, b_2 = 0.85 c_3 = 0.22, \gamma = 2, \alpha_2 = 0.09, \alpha_3 = 0.1$.

Fig.17 illustrates that the profits evolution of the upstream firms and the downstream firm start with equilibrium state, through period doubling, and end with chaotic state with output adjustment speed α_1 increasing. The diagrams of Bifurcation with α_1 increasing are shown in Fig.18 then Fig.18 shows the evolution of average profits of all the firms through equilibrium state, period doubling, and end with chaotic state with output adjustment speed α_1 increasing.

In Fig.17 , blue,red and black region denote profits of firm 1, firm 2 and firm 3 respectively, and in

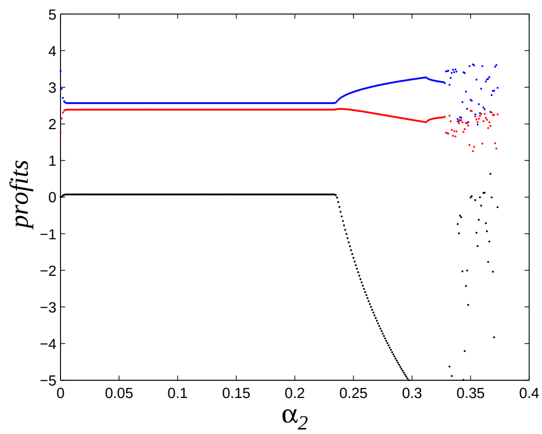

Figure 19: Bifurcation diagram of profit with α_2

Fig.18,blue,red and black region denote average profits of firm 1, firm 2 and firm 3 respectively.

As can be seen in Fig.17, The diagrams of Bifurcation of profits has the same flip Bifurcation point as Fig.2. and in Fig.18, As can be seen, if the system lose stable and donot enter chaos, then with the increase of α_1, profit of firm 1 and firm 3 decrease and profit of firm 2 increase, when the system enter chaos, then with the increase of α_1, profit of firm 3 increase. As can be seen in Fig.20, if the system lose stable and donot enter chaos, then with the increase of α_2, profit of firm 2 and firm 3 decrease and profit of firm 1 increase, when the system enter chaos, then with the increase of α_2, profit of firm 3 increase.

As can be seen in Fig.22, if the system lose stable and donot enter chaos, then with the increase of α_3, profit of firm 1 and firm 3 donot change and profit of firm 3 decrease, when the system enter chaos, then with the increase of α_3, profit of firm 3 increase.

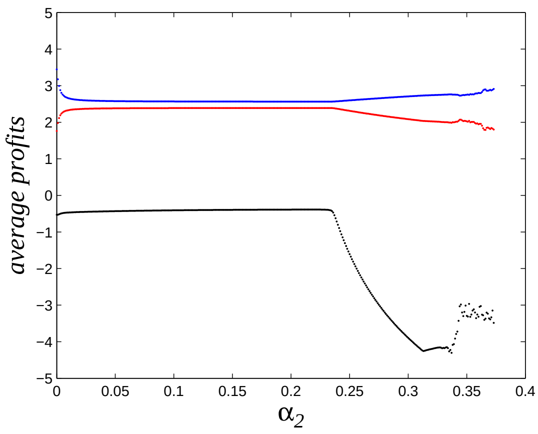

Figure 20: Effect of α_2 on average profit

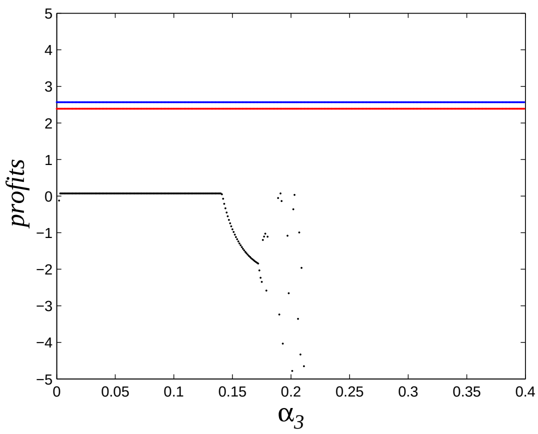

Figure 21: Bifurcation diagram of profit with α_3

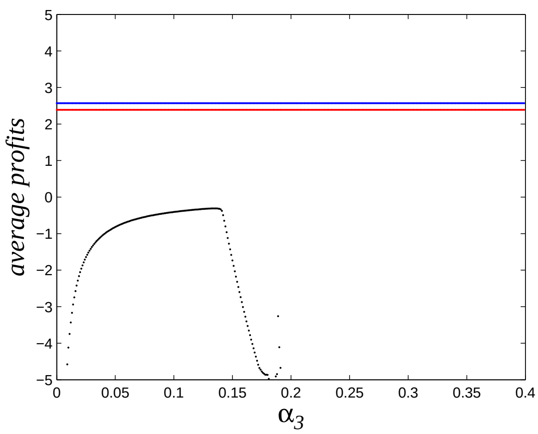

Figure 22: Effect of α_3 on average profit

5 Chaos control

Through the above analysis, we can find that system (6) will become unstable and even fall into chaos if the adjustment speed parameters out of the stable region. It will harm all the upstream firms and the downstream firm and make the markets irregular when the system in a chaotic state. Therefore, nobody is able to make good strategies and decide reasonable output. To avert the risk, it is a good ideal for the triopoly to maintain at Nash equilibrium output.

Perturbation feedback and non-feedback are two methods for the chaos control. Recently, Yang and Xu et al. [17, 18] proposed a new control method which is called as the straight-line stabilization method. This method is adopted to control the chaos in this paper. Denote

$$\delta = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix} \begin{pmatrix} A & -j_{12} & 0 \\ -j_{21} & B & 0 \\ -j_{31} & -j_{32} & C \end{pmatrix} \begin{pmatrix} D \\ E \\ F \end{pmatrix} = \begin{pmatrix} [\mu - (1+j_{11})]D - j_{12}E \\ -j_{21}D + [\mu - (1+j_{22})]E \\ -j_{31}D - j_{32}E + [\mu - (1+j_{33})]F. \end{pmatrix}. \tag{14}$$

$A = \mu - (1 + j_{11})$,
$B = \mu - (1 + j_{22})$,
$C = \mu - (1 + j_{33}$,
$D = q_{x_1}(t-1) - q^*_{x_1}$,
$E = q_{x_2}(t-1) - q^*_{x_2}$,
$F = p_y(t) - p^*_y$.
Where $|\mu| < 1$ is the feedback control parameter and other parameters are the same as above.

Adding the external control signal (14) to system (6), the controlled system is as follows

$$\begin{cases} q_{x_1}(t+1) &= q_{x_1}(t) + A, \\ q_{x_2}(t+1) &= q_{x_2}(t) + B, \\ p_y(t+1) &= p_y(t) + C. \end{cases} \tag{15}$$

$A = \alpha_1 q_{x_1}(t)[a_1 - 3b_1 q^2_{x_1} - b_1 q^2_{x_2} - 4b_1 q_{x_1} q_{x_2} - 2c_1 q_{x_1}] + \delta_1$,
$B = \alpha_2 q_{x_2}(t)[a_1 - 3b_1 q^2_{x_2} - b_1 q^2_{x_1} - 4b_1 q_{x_1} q_{x_2} - 2c_1 q_{x_2}] + \delta_2$,
$C = \alpha_3 p_y(t)[a_2 - 2b_2(1 + b_2 c_3) p_y(t) - \gamma b_1 b_2 (q_{x_1} + q_{x_2})^2 + 2a_2 b_2 c_3 + \gamma a_1 b_2] + \delta_3$.

It can be seen from Fig. 23, at ($\alpha_1 = 0.339, \alpha_2 = 0.09, \alpha_3 = 0.1$), controlled system (15) stabilized at Nash equilibrium point when $-1 < \mu < -0.2903$ in Fig. 23 (a) and $-1 < \mu < -0.5968$ in Fig. 23 (b). It demonstrates that the chaos control can be realized even if the perturbation is very small.

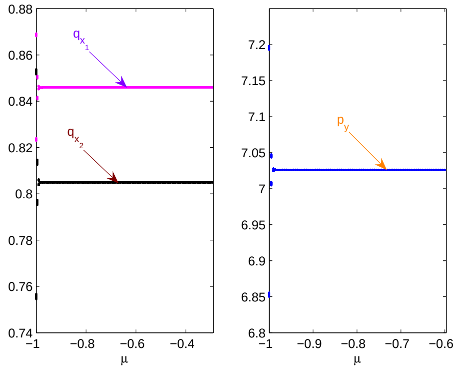

Figure 23: Bifurcation diagram with $\alpha_1 = 0.339, \alpha_2 = 0.09, \alpha_3 = 0.1$, and : (a) $k \in [-1, -0.2903]$, (b) $k \in [-1, -0.5968]$

6 Conclusions

A new dynamic nonlinear Cournot-Bertrand triopoly game between between the upstream firms and the downstream firm is established in this paper.

The adjustment speed parameter $\alpha_i, (i = 1, 2, 3)$ has an important effect on game results, if (α_1, α_2) is in the stable region, the system will eventually arrive at the Nash equilibrium output in a finite of games.It is shown that bifurcation, chaos and other complex phenomena occur when the speed adjustment parameters change. It is well-known that the occurrence of chaos depends on the values of bifurcation parameters.

The impact of price adjustment speed on average profit is discussed we can see that chaos is harmful to all the players. Perturbation feedback and nonfeedback are two methods for the chaos control. The straight-line stabilization method is adopted to control the chaos in this paper. The straight-line stabilization method is used to control the period-doubling bifurcation, unstable periodic orbits and chaos. The system quickly comes back to the Nash equilibrium point when a small perturbation is added to the system.

This paper shows a guide for the upstream firms and the downstream firm to formulate strategies of output and price, respectively. The research results also have an important theoretical and practical significance to the enterprises under the background of globalization.

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