Controller Performance Assessment For A Bioreactor Process

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Abstract: - The control of bioprocesses is an important problem attracting wide attention. The main motivation is to improve the operational stability and the production efficiency of such living processes. In the present work, a conventional PID controller is designed for controlling a bioreactor in which cell growth follows Monod kinetics. Design and analysis of model reference adaptive control systems based on MIT rule and Lyapunov rule are applied to a bioreactor first order process. The system is simulated using Mat lab simulink and it is investigated for various values of adaptation gain of the process. Performance of the adaptive controller is compared with the PI&PID controller for a step input.

Key-Words: PID controller, Adaptive control systems, Adaptation gain, MIT rule, Lyapunov rule, Model reference adaptive control

1 Introduction

The use of biological processes is growing rapidly due to the increasing demand in products such as pharmaceuticals, foods, alcoholic beverages, enzymes and others. Since bioprocesses involve living organisms, they often experience nonlinear behaviours which may include output multiplicity, bifurcations, chaos, unstable dynamic response to disturbances and changes in system parameters. All these phenomena can lead to instability and ultimately affect the yield of production. Mathematical modelling and various operating conditions are discussed in [1]. Adaptive control using different rules are explained in [2].Fed-batch bioreactor operation involving periodic addition of the substrate or nutrients is discussed [3]. An original Lyapunov based control design for the stabilization of CSTRs is proposed [4], in which a new Lyapunov function is proposed such as the control variable remains bounded. A classical absolute stability criterion is converted into nonlinear design procedures which employ efficient numerical tools, such as LMI's. An extended circle criterion is designed which eliminates the relative degree obstacle. There restrictions on the zero dynamics are relaxed by using the Popov multiplier, which also reduces controller complexity [5]. The problem of outer-approximating the region of feasible steady states, for processes described by uncertain nonlinear differential algebraic equations including discrete variables and discrete changes in the dynamics is addressed[6].Controller strategy is developed for a r eactor which h andles measured disturbances, manipulator constraints, dead time and nonlinearity[7]. Sliding-mode observers are proposed in [8], for the estimation of specific growth rate and substrate concentration from biomass measurements in fermentation processes in which global convergence is demonstrated using Lyapunov stability theory. For substrate estimation, an observer increases the convergence rate to a vicinity of the real substrate concentration to achieve asymptotic convergence despite kinetic model uncertainties are present. A process modification problem for a CSTR system [9], is worked out and solved to determine the minimal design parameter changes necessary to avoid input multiplicity. In [10], simple control system using Proportional-Integral (PI) controllers is designed for the implementation of regulatory control structures in the operation of a Simultaneous Saccharification and Co-Fermentation (SSCF). Many methods are employed for the modelling, analysis, and control of dynamical systems based on optimization schemes, e.g., parameter estimation and model predictive control. The parameter estimation problem for a model of an isothermal continuous tube reactor is illustrated [11] and an asymptotically stable reduction error estimator is derived and analyzed for optimization. Design of MRAC using Lyapunov's theory is explained in [12]&[13]. The purpose of this paper is to design and simulate a Model Reference Adaptive control (MRAC) u sing 2 different rules for a bio reactor. It includes the following parts: section 2 emphasizes mathematical modelling and ope rating conditions of a Bio chemical r eactor. Section 3 de scribes design methodology of PID controller using **Chine**– Hrones– Reswick tuning Algorithm. Section 4 describes an overview of model reference adaptive controller. In section 5, design of MRAC for a bioreactor process using MIT rule and Lyapunov rule is implemented and the simulation results of both the PID controller and adaptive controller methodologies are discussed followed by conclusions.

2. Mathematical Modelling of a Bioreactor

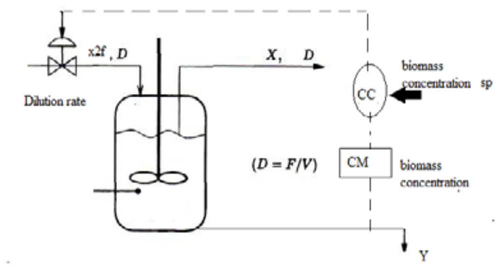

Fig 1: Schematic diagram of a continuous bioreactor

The Schematic diagram of a continuous bioreactor is shown in figure 1

The modelling equations of a bioreactor are given by equations (1)&(2)[1]:

$$\frac{dx_1}{dt} = (\mu - D)x_1 \quad (1)$$

$$\frac{dx_2}{dt} = \frac{D(x_{2f} - x_2) - \mu x_1}{Y} \quad (2)$$

Where the state variables are x_1, the biomass concentration and x_2, the substrate concentration. The manipulated input is D,dilution rate and the disturbance input is x_{2f},substrate feed concentration and Y is the yield rate and μ is the maximum growth rate.There are 2 possible solutions given by equations 3&4, which represents specific growth .They are Monod and substrate inhibition.

$$\frac{dx_2}{dt} = \frac{\mu_{max} x_2}{K_m + x_2} \quad (3)$$

$$\mu = \frac{\mu_{max} x_2}{K_m + x_2 + K_1 x_2^2} \quad (4)$$

Where K_1 is the substrate inhibition constant,K_m is the substrate saturation constant and μ_{max} is the maximum growth rate.

2.1 **Dynamic behaviour of a reactor**

Table 1 shows the parameters to find the steady state conditions for the model shown by equations 1&2.The steady state dilution rate is D=0.3 $hr^{-1}$and the feed substrate concentration is 4.0 g/ litre.

Table 1: parameters used for modelling of a bioreactor

S.No	Parameter Value
1	μ_{max}=0.5 hr^{-1}
2	k_m =0.12 g/litre
3	k_1=0.4545Litre/g
4	Y=0.4

S.No	Steady State	Biomass Concentration	Substrate Concentration	Stability
1	Equilibrium 1	X_{1s}=0	X_{2s}=4.0	stable
2	Equilibrium 2	X_{1s}=0.995	X_{2s}=1.5122	unstable
3	Equilibrium 3	X_{1s}=1.53	X_{2s}=0.175	stable

Table 2: Operating conditions of bioreactor

Table 2 shows the operating conditions for a dilution rate of 0.3hr^{-1}.Steady state condition 1 is a washout case since no reaction was occurred. (1) Substrate concentration is the same as feed concentration.

2.2 state space model of a reactor

The state space model matrices are

$$A=\begin{bmatrix} \mu - D_S & x_{1s}\mu_s' \\ \frac{-\mu_s}{Y} & -Dx - \frac{\mu_S x_{1s}}{Y} \end{bmatrix}$$

$$B= \frac{x_{1s}}{x_{2f} - x_{2s}}$$

C=
[1 0]
D=0

$\mu_s' = \frac{\partial \mu}{\partial x_{2s}} = \frac{\mu_{max} K_m}{(K_m + x_{2s})^2}$

2.3 Stable operating point

The following initial condition is used for simulation X(0)= $\begin{bmatrix} 1.53 \\ 0.175 \end{bmatrix}$ .The state space model for the corresponding to stable operating point is

$$A = \begin{bmatrix} 0 & 0.9056 \\ -0.7500 & -2.564 \end{bmatrix}$$

$$B = \begin{bmatrix} -1.5301 \\ 3.8255 \end{bmatrix}$$

C=[1 0]

D=0

Eigen values are determined for the above matrix and its values are -0.3,-2.264hr^{-1}, so the system is stable. The transfer function relating the dilution rate to the biomass concentration is determined using Matlab.

$$Gp(s) = \frac{-1.5302S - 0.4590}{S^2 + 2.564S + 0.6792} \quad (5)$$

after pole zero cancellation the above transfer function can be written as

$$Gp(s) = \frac{-0.6758}{0.4417S + 1} e^{-0.5s} \quad (6)$$

Where the delay time is assumed as 0.5 seconds

3. Design methodology of PID controller

A typical structure of a PID control system is shown in Fig.2,

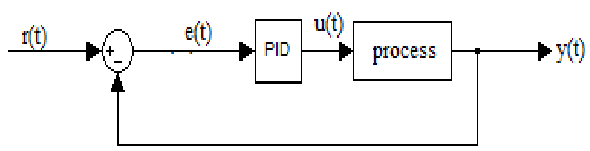

Figure 2: structure of PID controller system

Where it can be seen that in a PID controller, the error signal e *(t)* is used to generate the proportional, integral, and derivative actions, with the resulting signals weighted and summed to form the control signal *u(t)* applied to the plant model. A mathematical description of the PID controller is

$$u(t) = K_P \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right] \quad (7)$$

Where *u (t)* is the input signal to the plant model, the error signal *e (t)* is defined as *e (t)* =*r (t)* − *y (t)*, and *r (t)* is the reference input signal

3.1 Chien – Hrones –Reswick PID Tuning Algorithm

The first order transfer function of a bio reactor given by equation (6) is taken for analysis which has the standard form of

$$Gp(s) = \frac{K}{TS + 1} e^{-Ls} \quad (8)$$

The CHR PID controller tuning formulas are summarized in Table 3 for set-point regulation, in which a=KL/T, Ti=L/L+T. Simulated results are shown in figure 5.It is observed that PID controller has less settling time and rise time compared to PI controller output. However, in both the cases %overshoot is zero.

Table 3: C HR tuning formulae for set-point regulation

Controller type	With 0% overshoot		
	Kp	Ti	Td
PI	0.35/a	1.2T	
PID	0.6/a	T	0.5L

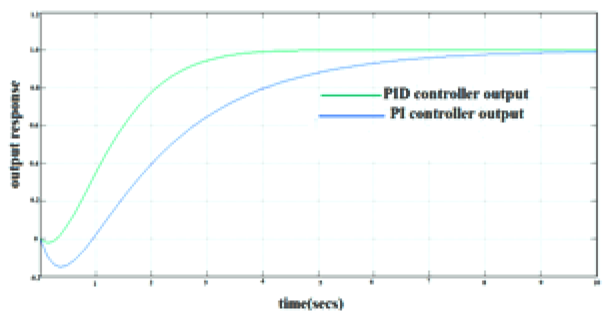

Fig 5: comparison of PI and PID controller output response for a bioreactor process using CHR tuning algorithm

4. Model Reference Adaptive Control

This technique of adaptive control comes under the category of Non-dual adaptive control. A reference model describes system performance. The adaptive controller is then designed to force the system or plant to behave like the reference model. Model output is compared to the actual output, and the difference is used to adjust feedback controller parameters.MRAS has two loops: an inner loop or regulator loop that is an ordinary control loop consisting of the plant and regulator, and an outer or adaptation loop that adjusts the parameters of the regulator in such a way as to drive the error between the model output and plant output to zero.

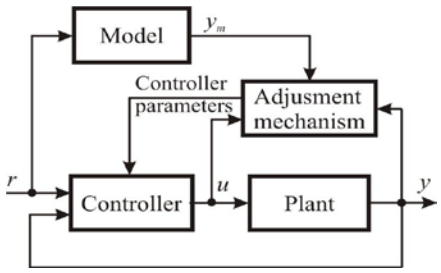

Figure 7: Parts of Model Reference Adaptive Controller

Reference Model: It is used to specify the ideal response of the adaptive control system to external command. It should reflect the performance specifications in control tasks. The ideal behaviour specified by the reference model should be achievable for the adaptive control system.

Controller: It is usually designed by a number of adjustable parameters. In this paper two parameters θ1 and θ2 are used to define the control law. The control law is linear in terms of the adjustable parameters (linear parameterization). Adaptive controller design normally requires linear parameterization in order to obtain adaptation mechanism with guaranteed stability and tracking convergence. The values of these control parameters are mainly dependent on adaptation gain which in turn changes the control algorithm of adaptation mechanism.

Adaptation Mechanism: It is used to adjust the parameters in the control law. Adaptation law searches for the parameters such that the response of the plant should be same as the reference model. It is designed to guarantee the stability of the control system as well as convergence of tracking error to zero.

Mathematical techniques like MIT rule, Lyapunov theory and theory of augmented error can be used to develop the adaptation mechanism. In this paper both MIT rule and Lyapunov rule are used for this purpose.

THE MIT rule

This rule was developed in Massachusetts Institute of Technology and is used to apply the MRAC approach to any practical system [2]. In this rule the cost function or loss function J is defined as

$$J(\theta)=\frac{e^2}{2} \tag{9}$$

Where, e is the output error and is the difference between the output of the reference model and the actual model, while θ is the adjustable parameter known as the control parameter.

In this rule the parameter θ is adjusted in such a way so that the loss function is minimized. Therefore, it is reasonable to change the parameter in the direction of the negative gradient of *J*, with the adaptation gain α ,so

$$\frac{d\theta}{dt} = -\alpha e \frac{\partial e}{\partial \theta} \tag{10}$$

The partial derivative term ∂e/ ∂θ, is called the sensitivity derivative of the system. This shows how the error is dependent on the adjustable parameter, θ. There are many alternatives to choose the loss function J. To develop the control law, equations (1), (2) are used based on MIT rule.

5.1 Design of MRAC using MIT rule

The plant process is given by the equation[14],

$$y_p(s) = G_P(S)U(S) = \frac{b}{s+a}U(S) \tag{11}$$

WhereGp(s) $=\frac{b}{s+a}$. (12)
is the transfer function of the process

and the reference model is given by

$$y_m(s) = G_m(s)R(s) = \frac{b_m}{s+a_m}R(s) \tag{13}$$

Where Gm(s) is the T.F of the process model

The process dynamics is given by

$$\dot{y}_p + ay_p = bu \tag{14}$$

Where a and b are known constants u is the control input.It should follow the reference dynamics

$$\dot{y}_m + a_m y_m = b_m r \tag{15}$$

The control law should also includes two parameters θ_1 and θ_2.Choosing the control law as

$$u = r\theta_1 - y_p\theta_2 \tag{16}$$

and substituting the above into equation (14) yields

$$\dot{y}_p + ay_p = b(r\theta_1 - y_p\theta_2) \tag{17}$$

The plant dynamics should match the reference model to minimize the error, so equating (15) and (17)

θ_1,θ_2 are calculated as

$$\theta_1 = \frac{b_m}{b} \tag{18}$$

$$\theta_2 = \frac{a_m - a}{b} \tag{19}$$

the system structure using this control law is illustrated in figure 8.

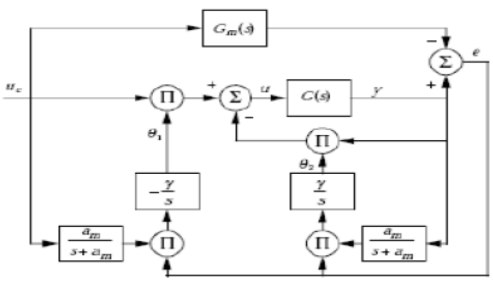

Figure 8: Block diagram of adaptive control system based on MIT rule for a first order process

The parameters θ_1,θ_2 are updated based on MIT rule

Equation (17) can be written as

$$\frac{d}{dt}(y_p) + ay_p + y_p b\theta_2 = br\theta_1 \tag{20}$$

$$y_p = \frac{b\theta_1}{s + a + b\theta_2} r \tag{21}$$

The sensitivity derivative $\frac{d}{d\theta}e(\theta)$ is given by

$$\frac{d}{d\theta_1}(e) = \frac{d}{d\theta_1}(y_p - y_m) \tag{22}$$

Where $e = y_p - y_m$

$$= \frac{d}{d\theta_1}\left(\frac{b\theta_1}{s + a + b\theta_2} r - y_m\right) \tag{23}$$

$$= \frac{b}{s + a + b\theta_2} r \tag{24}$$

Similarly

$$\frac{d}{d\theta_2}(e) = \frac{d}{d\theta_2}(y_p - y_m) \tag{25}$$

$$= \frac{d}{d\theta_2}\left(\frac{b\theta_1}{s + a + b\theta_2}\right) r \tag{26}$$

$$= -\frac{b^2\theta_1}{(s + a + b\theta_1)^2} r \tag{27}$$

$$\frac{d}{d\theta_2}(e) = \left(-\frac{b}{s + a + b\theta_2}\right)\left(\frac{b}{s + a + b\theta_2}\right) r \tag{28}$$

$$\frac{d}{d\theta_2}(e) = -\frac{b}{s + a + b\theta_2} y_p \tag{29}$$

Perfect modeling is achieved by choosing

$$\begin{aligned} \theta_1 b &= b_m \\ a + b\theta_2 &= a_m \end{aligned} \tag{30}$$

Substituting equation (30) in (24) yields,

$$\frac{d}{d\theta_1}(e) = \frac{b}{s + a_m} r \tag{31}$$

$$\frac{d}{d\theta_1}(e) = \frac{b}{a_m}\left(\frac{a_m}{s + a_m}\right) r \tag{32}$$

Similarly, equation (29) can be written as

$$\frac{d}{d\theta_2}(e) = -\left(\frac{b}{s + a_m}\right) y_p \tag{33}$$

Substituting equations (32) and (33) in equation (10) we get,

$$\frac{d}{dt}(\theta_1) = -\alpha e \frac{b}{a_m}\left(\frac{a_m}{s + a_m}\right) r \tag{34}$$

$$\frac{d}{dt}(\theta_1) = -\gamma e\left(\frac{a_m}{s + a_m}\right) r \tag{35}$$

Where $\gamma = \alpha b / a_m$ is the adaptation gain.

$$\frac{d}{dt}(\theta_2) = \gamma e\left(\frac{a_m}{s + a_m}\right) y_p \tag{36}$$

5.2. Design of MRAC Using the Lyapunov's Stability Theory

To derive an update law using Lyapunov theory, the following Lyapunov function is defined [10], [11]:

$$V = \frac{1}{2}\gamma e^2 + \frac{1}{2b}(b\theta_1 - b_m)^2 + \frac{1}{2b}(b\theta_2 + a - a_m)^2 \tag{37}$$

$$\dot{V} = \gamma e\dot{e} + \dot{\theta}_1(b\theta_1 - b_m) + \dot{\theta}_2(b\theta_2 + a - a_m) \tag{38}$$

$$\dot{V} = \gamma e(\dot{y}_p - \dot{y}_m) + \dot{\theta}_1(b\theta_1 - b_m) + \dot{\theta}_2(b\theta_2 + a - a_m) \tag{39}$$

$$\dot{V} = -\gamma a_m e^2 + (\gamma e r + \dot{\theta}_1)(b\theta_1 - b_m) + (\dot{\theta}_2 - \gamma e y_p)(b\theta_2 + a - a_m) \tag{40}$$

$$\frac{d\theta_1}{dt} = -\gamma e r \tag{41}$$

$$\frac{d\theta_2}{dt} = \gamma e y_p \quad (42)$$

Where γ is the adaptation gain, e is the error, r is the reference input and θ_1,θ_2 are the controller parameters and using the above ,adaptive control using Lyapunov rule is depicted in figure 9.

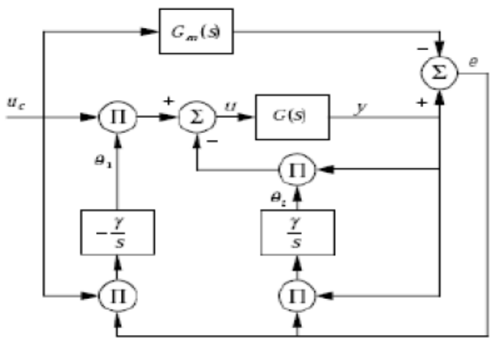

Figure 9: Block diagram representation of adaptive control structure based on Lyapunov rule for a first order process.

5.3 Performance evaluation of MIT rule and Lyapunov rule for a bioreactor process

The transfer function for a bioreactor process is given by $G_p(s) = \frac{-0.6758}{0.4417s+1}$ and can be written as

$$G_p(s) = \frac{-1.53}{s+2.264} \quad (43)$$

Which is of the form given by the equation (4), Where a=2.264 and b=1.53

The reference model is given by

$$G_m(s) = \frac{-2}{s+2.3} \quad (44)$$

Where a_m=2.3 and b_m=2.The controller parameters θ_1 and θ_2 are determined using the equations (18) and (19) respectively and are given by

θ_1=1.3071

θ_2=-0.1725.

The equations (35) and (36) are used in simulink diagrams represented in figure 8 and various results are shown in figures 10, 11, 12&13 using MIT rule with step input. Simulink diagram for Lyapunov rule is represented in figure 9 and corresponding results are shown in figures 14, 15, 16&17with step input using Matlab.Figure 10 r epresents the comparison of model reference output and output curves for different values ofγ. As gamma increases, the settling time decreases at the expense of overshoot and the rise time also decreases.Undershoot is 0% for all the values of γ for MIT rule. Figure 11 &12 shows the convergence of controller parameters θ_1&θ_2 respectively.The error converges quickly to zero as γ value increases as shown in figure 13. Figure 14 represents the comparative analysis of model output and process output for various values of γ using Lyapunov's criteria.It is observed that settling time and rise time is less compared to MIT rule.But the overshoot is highly increased as γ increases.and undershoot also increases. Figure 15 and 16 represents the controller parameters θ_1,θ_2 respectively, which shows the sluggish response as γ is very small.Figure 17 represents the tracking error between the reference model and the plant which converges quickly as the adaptation gain increases. Figure 18 s hows the comparative analysis of the adaptive controller using MIT rule and Lyapunov rule with the PI and PID controller outputs.Table 4 compares the results of two schemes of adaptive controller for different values of adaptation gain with PI and PID controller

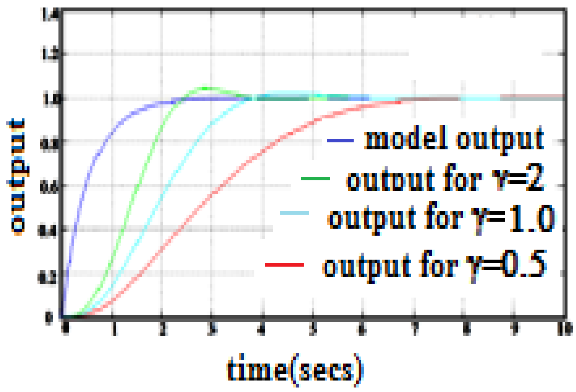

Figure 10: comparison of output responses for various values of γ

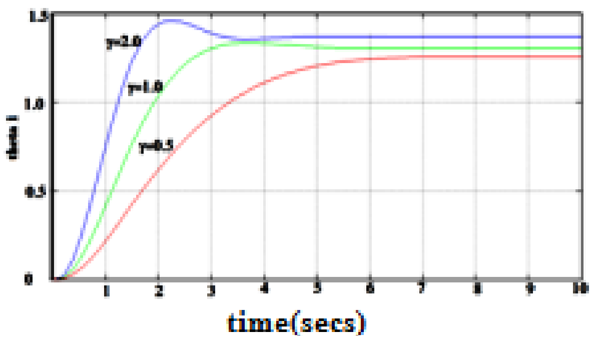

Figure 11: variation of controller parameter θ_1 for various values of γ

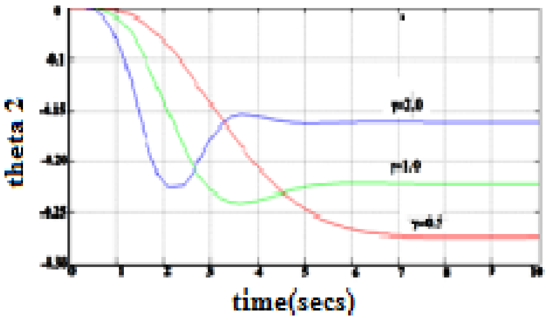

Figure 12: variation of controller parameterθ2 for various values of γ

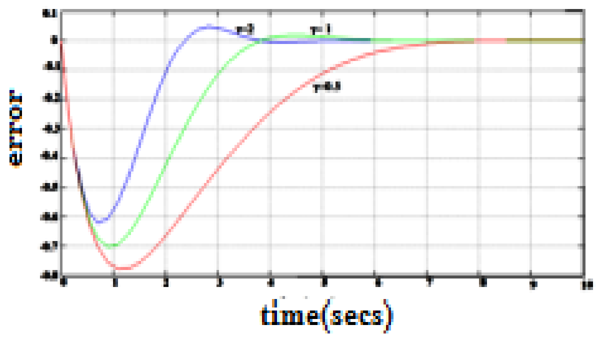

Fig13: tracking error for various values of γ

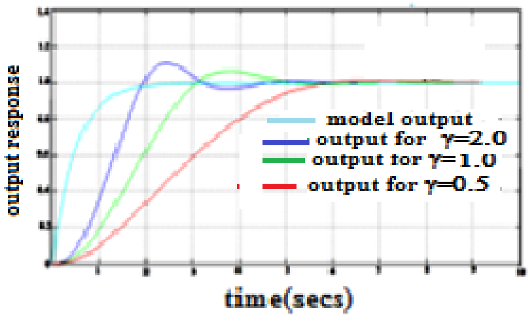

Fig14: comparison of output response for various valuesofγ

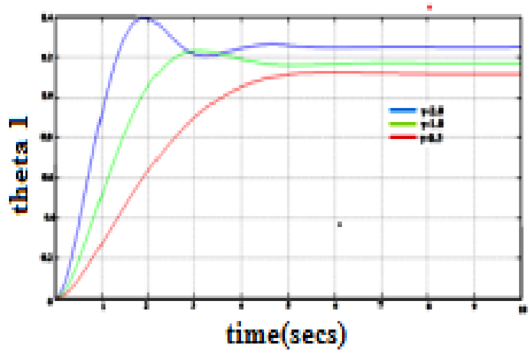

Figure 15: variation of controller parameter θ_1 for various values of γ

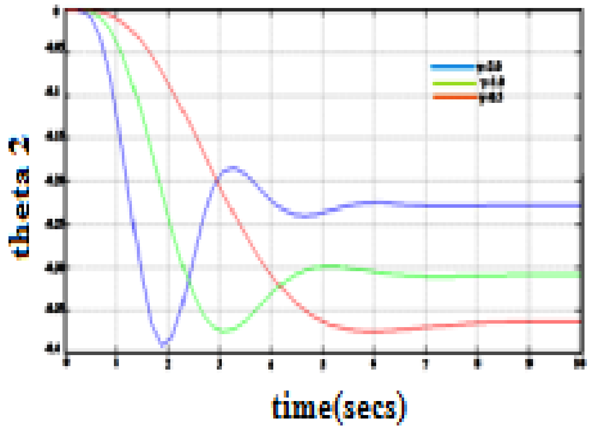

Figure 16: variation of controller parameter θ_2 for various values of γ

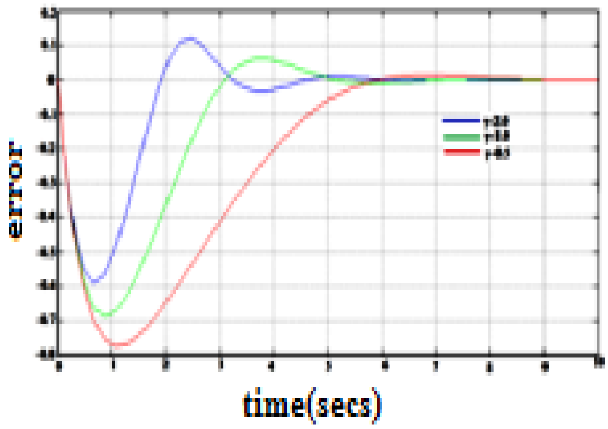

Fig 17: tracking error for various values of γ

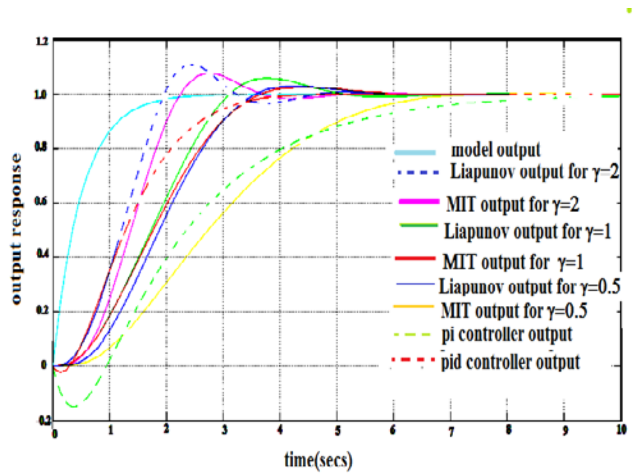

Figure 18: comparison of adaptive controller (using MIT rule and Lyapunov rule) with the PI and PID controller outputs

Table 4: comparison of transient parameters for different control strategies

Transient parameters	Model output	PID controller output		MIT rule			Lyapunov rule		
		PI controller	PID controller	γ=0.5	γ=1.0	γ=2.0	γ=0.5	γ=1.0	γ=2.0
Rise time(secs)	0.8	3.9	1.62	3.5	2.4	1.6	3.2	2	1.3
Settling time(secs)	3	7	3.46	7	6	4	6	5	4.7
%overshoot	0	0	0	0	4	6	0	5	11
%undershoot	0	0	0	0	0	0	0	1	2

6. Conclusion

A detailed comparison is done between two methods of model reference adaptive control system with the conventional controller results. Simulation analysis shows that PID controller gave good response compared to adaptive controller. But the mathematical modeling of system is simpler for MIT rule. The range of adaptation gain is selected as 0.5, 1.0, and 2.0. It can be observed easily that the performance of system for both the methods is improving with the increment in adaptation gain. But the rate of improvement is higher for Lyapunov theory. The system response does not have overshoot for least value of gamma, but the response is very sluggish. Now if the adaptation gain is increased slightly, response becomes oscillatory with reduction in settling time. Of all these, PID controller has less settling time, rise time and no overshoot, undershoot.

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