Robustness and Fragility of Evolving Networks with a New Local Synchronization Preferential Mechanism

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Abstract: In complex network, the preferential attachment mechanism which originated from the BA model is one of the essentials to improve the synchronizability. First of all, the synchronizability of a class of continuous-time dynamical networks is investigated in the paper. Then some local-world network models, such as the local-world networks, the local-world synchronization-optimal networks and the local-world node deleting evolving network, are introduced. And we have proposed a local-world synchronization-preferential growth topology model. The view have been validated that synchronizability is always improved as the maximum betweenness centrality is reduced. Furthermore, it has been found that the synchronizability of the dynamical network with the local-world synchronization-preferential mechanism is robust against not only the random removal of vertices but also the specific removal of those most connected vertices.

Key–Words: Complex network, local-world, synchronization-preferential, robustness, fragility.

1 Introduction

In the past ten years, in order to understand the generic features that characterize the formation and topology of complex networks, a lot of research work has been devoted to the study of a large-scale complex system described by a network or a graph with complex topology, whose nodes are the elements of the system and whose edges represent the interactions among them. Examples of all kinds of complex networks contain the Internet, the World Wide Web, food webs, electric power grids, cellular and metabolic networks, etc. [1-7]. There are always better cooperative or synchronous behaviors among their constituents as shown in these good-sized complex networks.

In real-world complex networks, community structure is an important characteristic, including biological networks composed of functional modules, and social networks. And these networks are often composed of groups of similar individuals. Researchers proposed a lot of methods that examine community topology, or connectivity between its nodes, to identify interesting structures. We claim, however, that the network structure of complex networks is the product of both their topology and dynamical processes taking place on them. Conventionally complex networks were researched by graph theory, for which a complex network was described by a random graph, where the radical theory was introduced by Erdös and Rényi [8]. Currently, Watts and Strogatz (WS) [9] proposed the conception of small-world networks to describe a transition from a regular lattice to a random graph. The WS network exhibits two properties, which are a high degree of clustering as in the regular networks and a small average distance between two nodes as in the random networks. In addition, the random graph model and the WS model are both homogeneous in substance. Nevertheless, based on Barabási and Albert [10], empirical results display that many large-scale complex networks are scale-free. They have addressed that two key mechanisms are indispensable for explaining the scale-free feature in complex networks. And the two key mechanisms are growth and preferential attachment.

The BA scale-free network model captures the basic mechanism which is known as the power-law degree distribution, but the model still has several limitations: it only predicts a fixed exponent in a power-law degree distribution, while the measured real networks' exponents actually vary mostly from 1 to 3. To overcome these limitations and further understand various microscopic processes under the influence of the network topology and evolution, the researcher have done many valuable work about the aspect. The evolution factors may include a kind of sides. To some extent, some researchers discussed a nonlinear preferential attachment scheme with the degree probability; some researchers considered the accelerated growth in a directed network; the others also investigated the competition aspect and the distance preference. For details, please refer to the relevant literature [11-13]. Synchronization of complex networks has been a subject of intensive research with potential applications. On the one hand, some novel control laws were proposed to study the synchronization of complex networks. In Ref. [14], a novel impulsive control law was proposed for synchronization of stochastic discrete complex networks. A simple but effective pinning algorithm for reaching synchronization on a general complex dynamical network was proposed [15]. Recently, researchers discussed the consensus in multi-agent dynamical systems [16-18]. On the other hand, network topology structure provides a powerful metaphor for describing sophisticated collaborative dynamics of many practical systems in essence. Thus some researchers proposed some new network models to study the synchronization of complex networks. Local-world evolving network model was proposed in [19]. It captured an important feature in evolution of many real-world complex networks: preferential attachment mechanism works only within local world instead of whole network-wide. According to the local-world evolving model, S. W. Sun *et al.* [20] studied the statistical properties of networks constructed and found that local world size M had great effect on network's connectivity: bigger M made the networks more heterogeneous in connectivity. A comprehensive multi-local-world model was proposed in [21]. In the paper, we have proposed a local-world synchronization-preferential growth topology model. Furthermore, it has been found that the synchronizability of the dynamical network with the local-world synchronization-preferential mechanism is robust against not only the random removal of vertices but also the specific removal of those most connected vertices. The rest of the paper is organized as follows: In the preliminary Section II, a synchronization stability criterion as well as the local-world evolving network model and the local-world node deleting evolving network model are described. Then a local-world synchronization-preferential growth topology model is presented in the Section III. In the Section IV, the synchronization is introduced and studied, followed by some discussions on its synchronization robustness and fragility. Finally, Section V concludes the investigation.

2 Preliminaries

2.1 Synchronization stability criterion of complex dynamical networks

Consider a dynamical network consisting of N identical linearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system. The state equations of the network can be written as[11, 22]

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j, i = 1, 2, ..., N \qquad (1)$$

where $x_i = (x_{i1}, x_{i2}, ..., x_{in})^T \in R^n$ is a state vector representing the state variables of node i, the constant $c > 0$ is the coupling strength. For simplicity, we take $\Gamma = diag\{1, 0, ..., 0\} \in R^{n\times n}$. $A = (a_{ij})$is the coupling configuration matrix representing topological structure of the network , in which a_{ij} is defined as follows: if there is a connection from node i to node $j(i \neq j)$, then $a_{ij} = a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0(i \neq j)$, and the diagonal elements of matrix A are defined by

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -k_i, i = 1, 2, ..., N \qquad (2)$$

where the degree k_i of node i is defined to be the number of connection incidents on node i.

The coupling matrix A represents the coupling configuration of the network. Suppose that the network is connected in the sense that there are no isolate clusters. A is a symmetric and irreducible matrix. In this case, it can be shown that zero is an eigenvalue of A with multiplicity 1 and all the other eigenvalues of A are strictly negative[23].

Dynamical network (1) is said to be (asymptotically) synchronized if

$$x_1(t) = x_2(t) = ... = x_N(t) = s(t), \quad \text{as} \quad t \to \infty \qquad (3)$$

where $s(t) \in R^n$ is a solution of an isolate node, namely, $\dot{s}(t) = f(s(t))$, which can be an equilibrium point, a periodic orbit, or a chaotic attractor, depending on the interest of study.

Let $0 = \lambda_1 \geq \lambda_2 \geq ...\lambda_N$ be the eigenvalues of the coupling matrix A. Suppose that there exists an $n \times n$ diagonal matrix Λ and two constants $\bar{d}$ and τ , such that

$$[Df(s(t)+d\Gamma)]^T\Lambda+\Lambda[Df(s(t)+d\Gamma)] \leq -\tau I_n \qquad (4)$$

For all $d \leq \bar{d}$, where $I_n \in R^{n\times n}$ is an unit matrix and $Df(s(t))$ is the Jacobian of f at $s(t)$. It has been shown that the synchronized state (3) is exponentially stable if[12]

$$c \geq |\bar{d}/\lambda_2| \tag{5}$$

Note that criterion (5) may not hold if the dynamical equations of a network cannot be written in the form of (1)[24].

Given the dynamics of an isolated node, the synchronizability of network (1) with respect to a specific coupling configuration A is said to be strong if the network can synchronize with a small coupling strength c. Inequality (5) implies that the synchronizability of network (1) can be characterized by the second-largest eigenvalue of its coupling matrix, i.e., the smaller the second-largest eigenvalue, the stronger the synchronizability of a network.

2.2 The local-world evolving network model

In many real-life networks, owing to the existence of the local-world connectivity discussed above, each node in a network only has local connections therefore only owns local information about the entire network. To model such a local-world effect, a local-world evolving network model is proposed, to be generated by the following algorithm [19]:

(i) Start with a small number m_0 of nodes and a small number e_0 of edges.

(ii) Select M nodes randomly from the existing network, referred to as the "local world" of the new coming node.

(iii) Add a new node with m edges, linking to m nodes in its local world determined in (ii), using a preferential attachment with probability $\Pi(k_i)$ defined at every time step t by

$$\Pi(k_i) = \frac{M}{m_0 + t} \frac{k_i}{\sum_{j \in Local} k_j} \tag{6}$$

After t time steps, this procedure results in a network with $N = t + m_0$ nodes and $E = e_0 + mt$ edges. In the following. For simplicity and no loss of generality, we assume $M = m_0$.

In order to obtain a dynamical network model whose synchronizability is stronger than the local-world model, we have constructed a network growth model with optimal synchronizability[25]. When a new vertex is added to the network, the criterion for choosing the m vertices to which the new vertex connects is to optimize the synchronizability of the obtained network, that is, to minimize the second-largest eigenvalue of the corresponding coupling matrix. After $t \gg m_0$ time steps, we obtain a local-world synchronization-optimal growing network with $N = t + m_0$ vertices.

2.3 The local-world node deleting evolving network model

In the local-world node deleting evolving network (LWD network), an undirected and unweighted network is initialized with a small number m_0 isolated nodes. The network is evolved with the following scheme [26].

At each time step t, either we act (i) with probability p_a or we act (ii) with probability $1 - p_a$.

(i) Node adding. The addition is achieved as follows:

(1) Growth: add a new node with $m(m \leq m_0)$ edges connected to the network;

(2) Local-world establishment: randomly select M nodes from the whole network as the local world;

(3) Preferential attachment: add m edges between the new coming node and M existing nodes in the local-world, the probability for node i selected in the local world is:

$$\Pi(k_i) = \frac{M}{N(t)} \frac{k_i}{\sum_{j \in Local} k_j}$$

where $N(t)$ is the total number of nodes after t time steps.

(ii) Node deleting: delete a node from the network randomly and remove all the edges once attached to the deleting node.

3 The local-world synchronization-preferential dynamical network model

The synchronizability of the newly generated network is optimal when each new edge is added into the above local-world synchronization-optimal network. However, in most real networks the preferential attachment rule exists. We assume that a new vertex is more likely to link to the vertices to form the network with strong synchronizability. Hereby, a novel local-world dynamical network model called the local-world synchronization-preferential dynamical network model is proposed in this paper. Then the new network generation algorithm of the model is as follows:

(i) Start with a small number m_0 of nodes and a small number e_0 of edges.

(ii) Select M nodes randomly from the existing network, referred to as the "local world" of the new coming node.

(iii)Add a new node with m edges, linking to m nodes in its local world determined in (ii), using a preferential attachment with probability Π_i defined at ev-

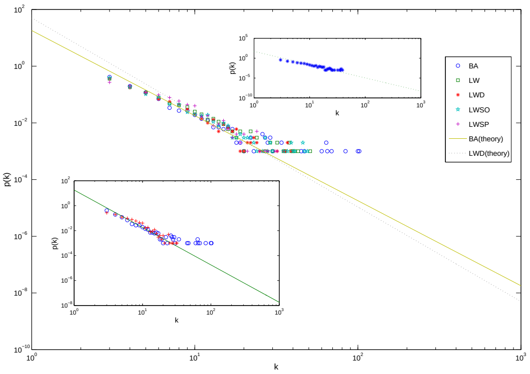

Figure 1: (color online)The degree distribution of nodes p(k) of the BA networks (circles), the LW networks (squares), the LWD networks (diamonds), LWSP (plus) and the LWSO networks (pentagrams). In addition, the real line is p(k) of BA model and the dash line is the theoretical prediction of p(k) of LWD network. One inset shows the degree distribution of the BA networks, the LWSP networks and the theoretical prediction(BA); the other shows the LWD part.

ery time step t by

$$\Pi_i = \frac{M}{m_0 + t} \frac{\lambda_{2i}}{\sum_{j \in Local} \lambda_{2j}} \tag{7}$$

This procedure results in a network with $N = t + m_0$ nodes and $E = e_0 + mt$ edges after t time steps.

The network topology has significant effects on its traffic protocols, searching algorithms, and even virus propagation, therefore modeling the network topology is extremely important. Currently, there are quite some models proposed and applied to describe the network topological features and properties, such as the BA[10], LW[19] and LWD[26] models. For simplify, we take $N = 1000$, $m = 3$, $m_0 = 10$, $M = 10$ and $p_a = 0.7$. The details are as shown in Fig.1.

We have known that the connectivity of the BA scale-free network is heterogeneous: most vertices have few connections and a small number of vertices have many connections. We have also found that local-world networks are topological quasimulticenter networks. There is a number of the "hubs" which are almost connected with all of vertices, but most of the vertices have very few connections. However, how about the topological structure of this local-world synchronization-optimal network model? It has been pointed out that the eigenvalues spectrum of complex networks provides information about their structural properties and the quantity measures the distance of

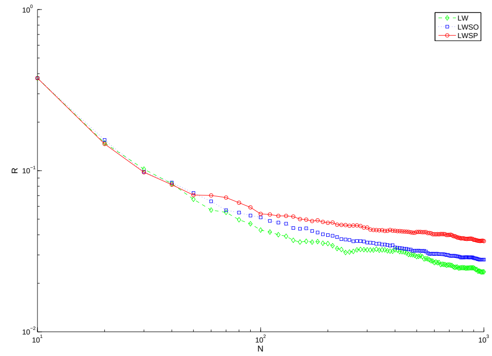

Figure 2: (color online)The values of R for the local-world networks(solid line with circles), the local-world synchronization-optimal networks(dotted line with diamonds) and the local-world synchronization-preferential dynamical network(dash-dotted line with squares). Each curve in the figure is the average result of 5 groups of networks.

the first eigenvalue from the rest part of the spectral density normalized by the extension of the rest part[27].

As shown in Fig.2, the figure represents the values of R of the local-world networks (LW), the local-world synchronization-optimal networks (LWSO) and the local-world synchronization-preferential dynamical network (LWSP) with $M = 10$ and $m = 3$, while the size N of networks ranges from 10 to 1000. Then those symbols stand for the same meaning in the following figures. It can be observed that as the network size N increases, values of R of the three categories of networks decay to converge to a power law as M increases. However, the value of R of the local-world synchronization-preferential dynamical network model changes the most slowly. It explains that R spans widest for local-world topology than for the networks from the other two dynamical network models.

4 Synchronization robustness and fragility in dynamical networks

4.1 Synchronization in dynamical networks

For clarity, we take $M = m_0$ in the construction of the three models. Then A_{lw} , A_{lwso} and A_{lwsp} represent the coupling matrices of the dynamical network (1) with the local-world evolving network model, the local-world synchronization-optimal dynamical network model and the local-world synchronization-preferential dynamical network, respectively, which has N nodes and $m(N - M) + e_0$ connections. Let

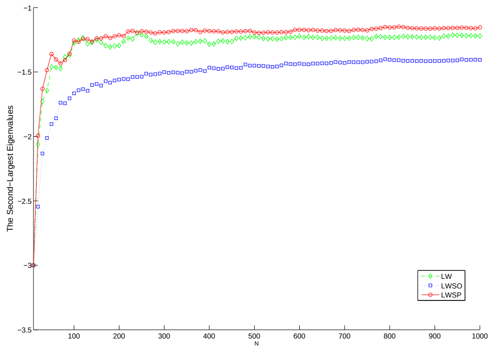

Figure 3: (color online)The second-largest eigenvalues of the coupling matrixes for the local-world networks(solid line with circles), the local-world synchronization-optimal networks(dotted line with diamonds) and the local-world synchronization-preferential dynamical network(dash-dotted line with squares). Each curve in the figure is the average result of 5 groups of networks.

λ_{2lw}, λ_{2lwso} and λ_{2lwsp} be the second-largest eigenvalue of A_{lw} , A_{lwso} and A_{lwsp}, respectively. In numerical computation, the eigenvalues are obtained by averaging the results of 5 runs. For a fixed value of m and M, the phenomenon is found out that λ_{2lw} decreases to a negative constant $\bar{\lambda}_{2lw}$ as N increases. At the same time, λ_{2lwso} decreases to a negative constant $\bar{\lambda}_{2lwso}$ and λ_{2lwsp} decreases to a negative constant $\bar{\lambda}_{2lwsp}$. For simplify, we take $N = 1000$, $m = 3$ and $M = 10$, respectively. The details are as shown in Table I and in Fig. 3.

Table 1: A comparison of the second-largest eigenvalues of different coupling topologies

N	m	M	$\bar{\lambda}_{2lw}$	$\bar{\lambda}_{2lwso}$	$\bar{\lambda}_{2lwsp}$
1000	3	10	-1.2205	-1.4052	-1.1541

As shown in Fig.3, the three second-largest eigenvalues have converges to the three negative constants as N increases. It has been observed that the second-largest eigenvalue of the local-world synchronization-optimal networks is smallest in the three dynamical network models, which indicates that the synchronizability of the local-world synchronization-optimal network model is the stronger among the three models.

Recently, it has been paid more and more attention to that the relation between the complex dynamic network topology characteristic and the network synchronizability by study scholars. The recently research work have discovered that many factors have had the different influence on the network synchronizability, such as the maximum degree, the average way length, the degree distribution. Generally speaking, networks can be divided into two categories, that is, homogeneous networks and heterogeneous networks. For homogeneous networks, shorter average distance will lead to better synchronizability[29]. Some numerical studies have been done to check if the maximal betweenness B_{max} is a proper quantity to estimate network synchronizability for heterogeneous networks[30, 31]. The effects of the maximum betweenness centrality B_{max} on the network synchronizability appear to be as follow[29, 30]: Synchronizability is always improved as B_{max} is reduced. Therefore, the betweenness centrality is proposed as a suitable indicator for predicting synchronizability on complex networks. On the one hand, we can see that the second-largest eigenvalue of the 'LWSP' is largest as shown in Fig. 3. Inequality (5) implies that the synchronizability of network (1) can be characterized by the second-largest eigenvalue of its coupling matrix, i.e., the smaller the second-largest eigenvalue, the stronger the synchronizability of a network. Then it is clear that the synchronizability of the 'LWSP' is weakest. On the other hand, we also notice in Fig. 4 that the maximum betweenness centrality of the 'LWSP' is largest among the three models. At the same time, the local-world node deleting evolving network is one of heterogeneous networks as shown in Fig.1. Accordingly, the view, namely, synchronizability is always improved as B_{max} is reduced, has been validated through the foregoing two simulations. Besides, the computation of the betweenness is not an easy job, and is especially impossible when the information of the network is incomplete. For heterogeneous networks, Chen *et al.*[32] provided some clues to mathematically solve the relation between the synchronizability and the network degree. They pointed out that the maximal degree was a proper quantity to predict network synchronizability.

Here it is worthwhile to emphasize that we have found some evidence indicating there may exist some common features between synchronization and network traffic on a dynamical level [28, 33-38]. Many previous works focus on the relationship between the distribution of BC and the capability of communication networks, with a latent assumption that the information packets go along the shortest paths from source to destination. Hence, the BC is always considered as a static topological measure of networks. Here we discover that this quantity is determined both by the routing algorithm and network topology, thus one should pay more attention to the design of network topology. We believe this work may be helpful

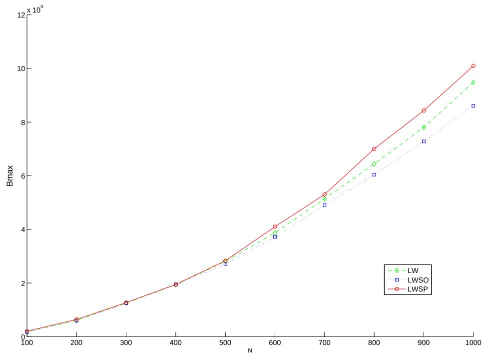

Figure 4: (color online)The values of B_{max} for the local-world networks(solid line with circles), the local-world synchronization-optimal networks(dotted line with diamonds) and the local-world synchronization-preferential dynamical network(dash-dotted line with squares). Each curve in the figure is the average result of 5 groups of networks.

for understanding the intrinsic mechanism and the capability of network traffic.

4.2 Robustness and fragility

Now we consider the robustness of synchronization in dynamical network (1) against either random or specific removal of a small fraction f ($0 < f < 1$) of nodes in the network. Clearly, the removal of some nodes in a network (1) will change its coupling matrix. However, if the second-largest eigenvalue of the coupling matrix remains unchanged, then the synchronizability of the network will also remain unchanged after the removal of some of its nodes.

Let $A \in R^{N\times N}$ and $\tilde{A} \in R^{(N-[fN])\times(N-[fN])}$ be the coupling matrices of the original network with N nodes and the new network after removal of $[fN]$ nodes, respectively. Denote λ_2 and $\tilde{\lambda}_2$ as the second-largest eigenvalues of A and $\tilde{A}$, respectively. Suppose that nodes $i_1, i_2, ..., i_{[fN]}$ have been removed from the network. One can construct the new coupling matrix $\tilde{A}$ from the original coupling matrix A as follows[11]:

(i) Get the minor matrix $\bar{A}$ of A by removing the $i_1th, i_2th, ..., i_{[fN]}th$ row-column pairs of A.

(ii)Obtain $\tilde{A} = (\tilde{a}_{ij})$ by re-computing the diagonal elements of the minor matrix $\bar{A} = (\bar{a}_{ij})$ as following:

$$\begin{cases} \tilde{a}_{ij} = \bar{a}_{ij}, i \neq j \\ \tilde{a}_{ij} = -\sum\limits_{j=1,j\neq i}^{N-[fN]} \bar{a}_{ij}, i = 1, 2, ..., N-[fN]. \end{cases}$$

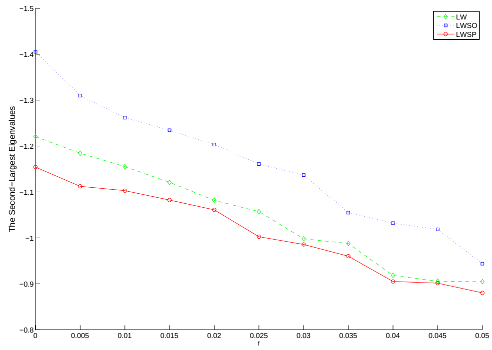

Figure 5: (color online)Synchronization robustness against random failures: changes of the second-largest eigenvalues of the local-world networks(solid line with circles), the local-world synchronization-optimal networks(dotted line with diamonds) and the local-world synchronization-preferential dynamical network(dash-dotted line with squares). Each curve in the figure is the average result of 5 groups of networks.

In the simulation, we take $N = 1000$. The original network with the local-world topology, the local-world synchronization-optimal topology or the local-world synchronization-preferential topology contains 1000 nodes and about 3000 connections. We have known from Refs.18 that the synchronizability of the original coupled network remains almost unchanged when a scale-free dynamical network has a random failure, for example, a very small fraction f of nodes are randomly removed; however, the synchronizability of the original coupled network is also decreased significantly or even destroyed when the network is attacked intentionally, for example, a small fraction f of "big" nodes are removed and then the original network changes significantly and even breaks into parts. These tell us: a scale-free network regarding its dynamical synchronization has the meanings of "robustness and yet fragility".

Table 2: A comparison of the decreased magnitudes of the second-largest eigenvalues when 5% nodes are respect to random and specific removed.

	Robustness	Fragility
LW	25.88%	92.79%
LWSO	32.83%	86.5%
LWSP	23.72%	81.4%

It has been shown as in Fig. 5 that the value of the second-largest eigenvalues of the 'LWSP' has de-

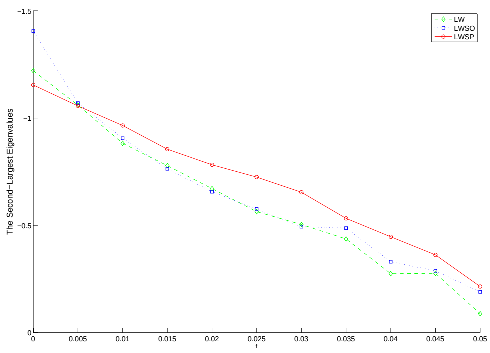

Figure 6: (color online)Synchronization fragility against specific attacks: changes of the second-largest eigenvalues of the local-world networks(solid line with circles), the local-world synchronization-optimal networks(dotted line with diamonds) and the local-world synchronization-preferential dynamical network(dash-dotted line with squares). Each curve in the figure is the average result of 5 groups of networks.

creased from -1.1541 to -0.8803 when as many as 5% of randomly chosen nodes are removed. Then the decreased magnitude of the second-largest eigenvalues of the 'LWSO' and the 'LW' is almost 32.83 percent and 25.88 percent, respectively. When more vertices are randomly removed, it is no significant that the reduction in the second-largest eigenvalue of the coupling matrix. This means that the 'LWSP' is most robust in the three models against random failures. Then we consider the fragility of the synchronizability with respect to deliberate attacks. A few isolated vertices or clusters may take place during the process of deliberate attacks. We removed the vertices with the highest degree and found that the second-largest eigenvalue of the 'LWSO' reduced from -1.4052 to -0.1895 when about 5% of the most connected vertices were removed at every time step. As shown in Fig. 6, the decreased magnitude of the second-largest eigenvalue of the 'LW' and the 'LWSP' is about 92.79 percent and 81.4 percent, respectively. This implies that the 'LWSP' is least vulnerable to specific removal of those most connected vertices in the three models. The details are as shown in Table II. Accordingly, the local-world synchronization-preferential dynamical network is particularly well-suited to tolerate random errors compared with the other two dynamical networks. Moreover the local-world synchronization-preferential dynamical network is particularly well-suited to tolerate intentional attacks in the three networks.

5 Conclusion

In this paper, we have proposed a local-world synchronization-preferential growth topology model. We also investigate the synchronizability of a class of continuous-time dynamical networks. Then the view have been validated that synchronizability is always improved as the maximum betweenness centrality B_{max} is reduced for heterogeneous networks. We have found some evidence indicating there may be some common features between synchronization and network traffic on a dynamical level. So this work may be helpful for understanding the intrinsic mechanism and the capability of network traffic. Then we also investigate the robustness of the synchronizability with respect to random failures and the fragility of the synchronizability with specific removal of nodes. Numerical simulations show that the local-world synchronization-preferential dynamical network is particularly well-suited to tolerate random errors compared with the other two dynamical networks. Moreover the proposed network is particularly well-suited to tolerate intentional attacks in the three networks.

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