Application of Jordan controlled form for nonlinear optimal control systems design

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Abstract: - The problem of optimal control system design for nonlinear control systems (plants) has been considered in many works. But the majority of the proposed design methods are focused on linear models which are received by the usual linearization. Therefore the real control systems are quasi optimal practically. Known methods of the optimal systems design on the basis of nonlinear models are very difficultly for practical use. Therefore new design methods of the nonlinear optimal control systems are claimed. This paper presents the application of a Jordan controlled form to analytical design of the optimal control system for nonlinear plants, equations of which are transformed to this form. For this reason the definition of Jordan controlled form and some features of the transformation to this form of the nonlinear systems equations are considered. The suggested method of the optimal control systems design includes two steps. At the first step a linearization control is designed on the base of the known stabilizing control and the nonlinear transformation of the plant state variables. In new variables the equations of the close system with the linearization control is linear. It gives possibility to apply the known method LQ to these linear equations on the second step. The resulting nonlinear control system is optimal in the sense of a minimum of nonlinear quadratic criteria. Coefficients values of the criteria can be chosen in accordance with the desired transients of the nonlinear optimal control system. The design problem has a solution if a plant is controllable. The optimal control is a feedback on the state variables of the plant. Efficiency of the stabilizing control and the suggested design method of nonlinear optimal control systems are shown on the examples of designing and simulation of nonlinear control systems.

Key-Words: - Nonlinear plant, equation, Jordan controlled form, nonlinear transformation, linearization control, quadratic criteria, optimal control

1 Introduction

The optimal control systems widely use in practice, including for control of the nonlinear plants. Application of the traditional linearization method allows receiving the linear plant equations of the first approximation [1–5]. In this case the control systems are designed as optimal in sense of the minimum of quadratic criteria [1, 3, 4]. However the equations of the first approximation are not exact, therefore the found control is not optimal actually. Control systems include usually the given plant and a projected controller [5, 6, p. 270]. The method of a nonlinear transformation of the plant equations to the some simple forms is more effective and it is widely applied at the solution of the control design problem for a nonlinear systems. This technique allows, first, to simplify the solution of this problem and, secondly, to make it analytical.

In nonlinear cases the plant equations are transformed to the normal canonical control form [2, 7], quasi-linear form [8, 9], triangular form [10, 11], Lukyanov-Utkin regular form [12], a Jordan controlled form [13 – 15] and others forms. If equations of the plant are represented in a triangular form, the backstepping method to design of an adaptive control system is applied very easily [11]. The Lukyanov-Utkin regular form of equations allows decomposing complex design problem on several tasks of the smaller dimension [12]. Jordan controlled form of the systems equations gives possibility to reject the external disturbances [15].

But in practice the optimal controls is used more often [1 – 5]. In a nonlinear case the optimal control is designed usually on the basis of the Pontryagin Minimum Principle or the Hamilton–Jacobi–Bellman equation [16]. But designing of the optimal control by these methods is very complex, since in nonlinear cases these methods demand the solution of the equations in partial derivatives.

The main difficulty of the transformation method is to find suitable transformation of the nonlinear systems equations. Constructing methods of such transformations are known in the theoretical plan, but often they are very complex. Therefore the finding of suitable transformation frequently is more difficult than the subsequent designing of the control system [2, 7, 12].

In this article the features of the design problem of the optimal control systems on the basis of the Jordan controlled form (JCF) is considered. The possibility of the optimal control system design by this approach is caused by existence of a special linearization control, which transforms the nonlinear equations in JCF to linear equations with constant parameters. The analysis shows: the equations of many real nonlinear plants have JCF or may be represented in this form by simple change of the state variables designations [8]. Representation of the plant equations in the JCF allows ensuring the stability of the system equilibrium [13, 14], the full compensation of the influence of the bounded external disturbances [15], the required transient time and also the desired character of transients.

This article is organized as follows. The Jordan controlled form of the equations plant is given in the section 2. The statement of the optimal control design problem with an uncertain criterion is presented in the section 3. The solution of this problem is received here at the assumption, that the equations of a controlled system are submitted in JCF. Therefore in section 4 the transformation of the nonlinear systems equations to JCF is considered.

The design method of a linearization control on base of the stabilizing control is considered in the section 5. For clearness the design procedure of the stabilizing control is shown here on example of the concrete nonlinear plant. Linearizing property of this control is shown here also. The problem of the nonlinear optimal control systems design on base of the Jordan controlled form of the systems equations is solved in the section 6. The final section includes the examples of the optimal control system design. Possibilities of the optimal and stabilizing control are compared. The proof of the theorem about conditions of transformation possibility to the JCF of the nonlinear differential equations of the second order is resulted in the appendix.

2 JCF of Systems Equations

Suppose some system (plant) with a single control is described by the equation

$$\dot{x} = f(x) + e_n u_0, \tag{1}$$

where $x \in R^n$ is the state vector; $f(x) = [f_1(\bar{x}_2)\ f_2(\bar{x}_3)\ \dots f_{n-1}(\bar{x}_n)\ f_n(\bar{x}_n)]^T$ is the vector-function; $f_i(\bar{x}_{i+1})$ is the scalar, continuous function which is differentiable $n-i$ time on all its arguments; $\bar{x}_i = [x_1 \dots x_i]^T$ is the sub vector including first i state variables $x_1, \dots, x_i$; evidently $\bar{x}_n = x$; e_n is n-th column of the identity $n \times n$-matrix; $u_0 = u_0(x)$ is the scalar control.

Let $x^\circ = x^\circ(t, u_0^\circ)$ is a vector that describes the unperturbed motion of the system (1); u_0° is the appropriate control. Enter the deviations $\tilde{x} = x - x^\circ$ and $u = u_0 - u_0^\circ$. For clearness the equation of the system (1) in deviations are recorded in a scalar form:

$$\dot{\tilde{x}}_i = \phi_i(\tilde{x}_1, \dots, \tilde{x}_{i+1}), \quad i = \overline{1, n-1}, \tag{2}$$

$$\dot{\tilde{x}}_n = \phi_n(\tilde{x}_1, \dots, \tilde{x}_n) + u, \tag{3}$$

where $\tilde{x}_1, \dots, \tilde{x}_n$ are deviations of the variables of the system (1): $\tilde{x}_i = x_i - x_i^\circ$, $i = \overline{1, n}$; $\phi_i(\tilde{x}_1, \dots, \tilde{x}_{i+1}) = f_i(\bar{x}_{i+1}) - f_i(\bar{x}_{i+1}^\circ) = \phi_i(\bar{\tilde{x}}_{i+1})$, $i = \overline{1, n-1}$ are the nonlinear, differentiable according on all their arguments functions; $\phi_n(\tilde{x}_1, \dots, \tilde{x}_n) = f_n(x) - f_n(x^\circ) = \phi_n(\tilde{x})$; $\bar{\tilde{x}}_i = [\tilde{x}_1 \dots \tilde{x}_i]^T$; $u = u(\tilde{x})$ is the search control. The variables $\tilde{x}_i$, $i=\overline{1,n}$ are measurable and $\phi_i(0)=0$, $i=\overline{1,n}$; $\bar{\tilde{x}}_n=\tilde{x}$ evidently.

Unlike the triangular or normal canonical control forms, the design control problem for nonlinear plant (2), (3) has a solution when the following conditions for all $\tilde{x}\in\Omega_{\tilde{x}}\in R^n$ are true:

$$\left|\frac{\partial\phi_i(\tilde{x}_1,\dots,\tilde{x}_{i+1})}{\partial\tilde{x}_{i+1}}\right|\ge\varepsilon\ne 0\,,\quad i=\overline{1,n-1}\,. \qquad (4)$$

Here ε is any positive constant and $\Omega_{\tilde{x}}$ is some domain of the space R^n. This domain should include the equilibrium $\tilde{x}=0$. Evidently, the inequalities (4) are controllability conditions of the plant (1) or (2), (3) [2, 8, 13].

Definition. If the equations (2), (3) satisfy conditions (4), they are called the Jordan controlled form (JCF) [13].

Evidently, the canonical Frobenius form is a special case of JCF, where $\phi_i(\tilde{x})=\tilde{x}_{i+1}$, $i=\overline{1,n-1}$ (for $n>1$) and $\phi_n(\tilde{x})=-\alpha_0\tilde{x}_1-\alpha_1\tilde{x}_2-\ldots-\alpha_{n-1}\tilde{x}_n$ [1, 8]. From expressions (2), (3) follows also, that JCF is a generalization of the known triangular form of the differential equations of a nonlinear controlled system [10].

3 Statement of Optimal Control System Design

The problem of a optimal control system design for the plant (2), (3) consists in the definition of a control u under which the uncertain nonlinear quadratic criteria satisfies to next condition

$$J=\int_0^\infty[\tilde{x}^TQ(\tilde{x})\tilde{x}+\rho l^2(u)]dt\to\min_u\,. \qquad (5)$$

Here $\rho>0$ is given number; $Q(\tilde{x})$ and $l(u)$ are uncertain $n\times n$-matrix and scalar function. They shell be determined later.

The problem of the optimal control systems design is solved here on base of the system equations in JCF, therefore the transformation process of the nonlinear differential equations to this form we shall consider more in detail.

4 Transformations of the Systems Equations to JCF

The equations in deviations of the nonlinear systems can be converted to JCF frequently by change of their variables designation. For example, the slightly changed system of the differential equations, considered in the book [17, p. 196], look like

$$\dot{x}_1=x_2-x_1^2\,;\;\;\dot{x}_2=x_1x_2+u\,;\;\;\dot{x}_3=x_2\,. \qquad (6)$$

The form of the equations (6), evidently, does not meet JCF, but they are converted to this form, if their variables to designate as follows: $x_1=\tilde{x}_1$, $x_2=\tilde{x}_3$, $x_3=\tilde{x}_2$. As a result the equations (6) take the form

$$\dot{\tilde{x}}_1=\tilde{x}_2-\tilde{x}_1^2=\phi_1(\tilde{x})\,;\quad\dot{\tilde{x}}_2=\tilde{x}_3=\phi_2(\tilde{x})\,;$$
$$\dot{\tilde{x}}_3=\tilde{x}_1\tilde{x}_3+u=\phi_3(\tilde{x}_1)+u\,. \qquad (7)$$

Here $n=3$ and $\partial\phi_1(\tilde{x})/\partial\tilde{x}_2=\partial\phi_2(\tilde{x})/\partial\tilde{x}_3=1\ne 0$, i.e. the conditions (4) carry out in relation to the equations (7), hence, there equations have JCF.

If a nonlinear system of the differential equations have more complex kind the conditions (criterion) of the transformation possibility of these equations to the JCF are necessary. If the equations have the order above the third such criterion is unknown. Conditions of a transformation possibility of the third order system of the nonlinear differential equations to JCF are given in [8]. Corresponding conditions for a case of the second order nonlinear systems are considered here.

Suppose, a nonlinear controlled system (plant) has the second order and equation of this system in deviations has view

$$\dot{x}=f(x)+b(x)u_1\,, \qquad (8)$$

where $x=[x_1\;\;x_2]^T$ is a state vector of this system; $f(x)=[f_1(x)\quad f_2(x)]^T$, $b(x)=[b_1(x)\quad b_2(x)]^T$ are the nonlinear differentiable vector-functions, and $f_i(0)=0$, $i=1,2$; control $u_1=u_1(x)$ is a nonlinear function of the vector x.

First of all, we shall note, if the functions $b_1(x)\equiv 0$ but $b_2(x)\ne 0$ and the partial derivative $\partial f_1(x)/\partial x_2\ne 0$ at everything $x\in\Omega_{x2}\in R^2$ the equation (8) has JCF with the control $u(x)=b_2(x)u_1(x)$. If the function $b_2(x)\equiv 0$ but $b_1(x)\ne 0$ and partial derivative $\partial f_2(x)/\partial x_1\ne 0$ at everything $x\in\Omega_{x2}$ this equation can be transformed to JCF by change of the variables and control designation as shown above.

Therefore we shall assume further that the conditions $b_1(x)\ne 0$ and $b_2(x)\ne 0$ are satisfied in domain Ω_{x2} and we shall enter an determinant

$$G(x)=\det[f(x)\quad b(x)]\,. \qquad (9)$$

Theorem 1. If the nonlinear vector-functions $f(x)$ and $b(x)$ from the equation (8) in some domain Ω_{x2} satisfy to the next condition

$$K(x) = \frac{\partial G}{\partial x_1} b_1(x) + \frac{\partial G}{\partial x_2} b_2(x) - \\ -\left(\frac{\partial b_1(x)}{\partial x_1} + \frac{\partial b_2(x)}{\partial x_2} \right) G(x) \neq 0\,, \qquad (10)$$

there is an invertible, continuous transformation $x = \psi(\tilde{x})$, $\psi(0) = 0$, which transforms the equation (8) to JCF.

The proof of this theorem is given in appendix. The inequality (10) is the condition (criterion) of the transformation possibility of the second order systems equations such as (8) to JFC. If for a given system this inequality is carried out to find the transformation of such systems to JCF it is enough to integrate of the partial derivatives which follow from expressions (68) – (71). The constants of integration and the integrating multiplier $\mu(\tilde{x})$ are chosen on the conditions $\phi_i(0) = 0$, $i = 1, 2$ and $\Delta(\tilde{x}) \neq 0$ which follow from the expressions (68) and (69).

Validity of the theorem statement we shall show on the example.

Example 1. The converter increasing a voltage of a direct current includes any source of the constant voltage, the controlled switchboard, the inductance and the capacity is connected with active load. Currents through inductance and capacity are switched with some period. As shown in [18], the average changes of the inductance current and the voltage on capacity are described by the equations

$$\dot{I}_L = -U_C L^{-1}(1-\tau) + U_{SR} L^{-1}\,,$$
$$\dot{U}_C = C^{-1}(I_L(1-\tau) - U_C R^{-1})\,. \qquad (11)$$

Here I_L is a current in the inductance *L*; U_C is a voltage on the capacity *C* and on the load resistance R; U_{SR} is the voltage of the source of a constant current with unlimited power; $\tau = T_{CH}/T$ is the relative duration of the capacity charge time, $\tau \in [0, 1]$; *T* is the period of currents switching; T_{CH} is the duration of the capacity charge time at the period. The equations (11) are necessary to transform to JCF.

To solve the task the converter equations in deviations are found first of all. The steady state values of the current I_L° and the voltage U_C° (at $\tau = \tau^\circ = Const$) are determined according to the equations (11) by the next expressions:

$$\tau^\circ = 1 - U_{SR}/U_C^\circ\,, \quad I_L^\circ = U_{SR}/(1-\tau^\circ)^2 R\,, \qquad (12)$$

where U_C° – the required output increased voltage on the converter load, i.e. $U_C^\circ > U_{SR}$.

To receive the equations of the converter in the deviations, the new state variables and control are entered so:

$$x_1 = I_L - I_L^\circ,\;\; x_2 = U_C - U_C^\circ,\;\; u_1 = \tau - \tau^\circ. \qquad (13)$$

The equations in the deviations are defined by differentiation on time of the variables x_1, x_2 (13) in view of the equations (11):

$$\dot{x}_1 = -\frac{(1-\tau^\circ)}{L} x_2 + \frac{(x_2 + U_C^\circ)}{L} u_1\,,$$
$$\dot{x}_2 = -\frac{(1-\tau^\circ)}{C} x_1 - \frac{x_2}{RL} - \frac{(x_1 + I_L^\circ)}{C} u_1\,. \qquad (14)$$

Control u_1 is contained into both equations (14), i.e. there equations do not meet JCF on the form. To solve the considered task in the beginning we shall estimate a possibility of transformation to JCF the equations (14).

With this purpose the determinant $G(x)$ is determined on (9) and the function $K(x)$ is determined on (10). As the result we shall receive: $G(x) = [2U_C^\circ x_2 - U_{SR} R x_1 + x_2^2]/RLC$ and $K(x) = -(x_2 + U_C^\circ)[U_{SR} RC + 2L(x_1 + I_L^\circ)]/R(LC)^2$ or in view of the designations (13) the function $K(x) = -U_C[U_{SR} RC + 2LI_L)]/R(LC)^2$. The voltage $U_C = x_2 + U_C^\circ$ and the current $I_L = x_1 + I_L^\circ$ do not change the signs in operating modes. Hence, the condition (10) is carried out and equations (14) can be transformed to JCF.

Let $\tilde{x} = \varphi(x)$ there is the inverse to transformation $x = \psi(\tilde{x})$, i.e. $\psi(\varphi(x)) = x$. In the equations (13) the functions $b_1(x) = -(x_2 + U_C^\circ)/L$, $b_2(x) = (x_1 + I_L^\circ)/C$ and $\partial b_2(x)/\partial x_2 = \partial b_1(x)/\partial x_1 = 0$, therefore the inverse transformation $\varphi(x)$ as: $\tilde{x}_1 = \varphi(x_1, x_2)$, $\tilde{x}_2 = x_2$ can be found easier.

As shown above the function $\varphi(x_1, x_2)$ can be determined by integrated of the partial derivatives $\partial\varphi(x)/\partial x_1 = -b_2(x)\mu(x)$ and $\partial\varphi(x)/\partial x_2 = b_1(x)\mu(x)$, where $\mu(x)$ is an integrating multiplier. These derivatives are integrated with $\mu(\tilde{x}) = \mu = 2LC$ for simplicity. Resulting transformation looks like

$$\tilde{x}_1 = \varphi(x) = (x_1 + I_L^\circ)^2 L + (x_2 + U_C^\circ)^2 C + \varphi_0\,,$$
$$\tilde{x}_2 = x_2\,. \qquad (15)$$

The constant

$$\varphi_0 = -(U_C^\circ)^2 C - (I_L^\circ)^2 L \qquad (16)$$

is found on the condition $\varphi(0) = 0$.

Direct transformation of the state vector $x = \psi(\tilde{x})$ is defined by the expressions:

$$x_1 = \psi_1(\tilde{x}) = \tilde{I}_L(\tilde{x}) - I_L^\circ, \quad x_2 = \psi_2(\tilde{x}) = \tilde{x}_2, \qquad (17)$$

where

$$\tilde{I}_L(\tilde{x}) = \sqrt{[\tilde{x}_1 - (\tilde{x}_2 + U_C^\circ)^2 C - \varphi_0] L^{-1}}. \qquad (18)$$

$\tilde{I}_L(\tilde{x})$ is the expression for the current in the inductance as the function of the new variables $\tilde{x}_1$ and $\tilde{x}_2$.

The direct transformation (17) exists at all values of the variables $\tilde{x}_1$ and $\tilde{x}_2$ since the expression of under the root equals $(x_1 + I_L^\circ)^2$ in the formula (18).

The new equations of the converter are found by differentiation on time of the variables $\tilde{x}_1 = \varphi(x_1, x_2)$, $\tilde{x}_2 = x_2$ (15) in view of the expressions (12), (13), (16) – (18) and the equations (14). The resulted equations of the converter have view

$$\dot{\tilde{x}}_1 = \phi_1(\tilde{x}) = \phi_1(\tilde{x}_1, \tilde{x}_2), \qquad (19)$$

$$\dot{\tilde{x}}_2 = \phi_2(\tilde{x}_1, \tilde{x}_2) + u, \qquad (20)$$

where

$$\phi_1(\tilde{x}) = 2[U_\text{н} R(\tilde{I}_L(\tilde{x}) - I_L^\circ) - 2U_C^\circ \tilde{x}_2 - \tilde{x}_2^2]/R,$$

$$\phi_2(\tilde{x}) = [(1-\tau^\circ)(\tilde{I}_L(\tilde{x}) - I_L^\circ)R - \tilde{x}_2]/RC, \qquad (21)$$

$u = -\tilde{I}_L(\tilde{x}) C^{-1} u_1$ is the new control.

The left part of the condition (4) in relation to the equations (19) – (21) and again in view of the expressions (12), (13) and (15) – (18) has view

$$\frac{\partial \phi_1(\tilde{x})}{\partial \tilde{x}_2} = -\frac{2U_C}{R}\left[\frac{U_\text{н} RC}{LI_L} + 2\right].$$

As was marked above the current $I_L = x_1 + I_L^\circ$ of the inductance is positive and does not change the sign, therefore from this expression is follows, that the condition (4) with $n = 2$ is carried out, i.e. the equations (19), (20) have JCF similarly to the equations (2), (3) or (7).

On the basis of the controlled system equations in JCF a stabilizing and optimal controls can be found. The stabilizing control is found directly on the equations in JCF (2), (3), but the optimization problem (2) – (5) is solved here in two stages. For the solution of the last problem, the linearization control is designed for the nonlinear plant (2), (3) at the first stage. The optimal control is designed at the second stage.

5 Linearization Control Design

Used here a linearization control is a special case of the stabilizing control which was proposed in [13]. To design this control for the equations (2), (3) under the conditions (4) the transformation of the state vector $\tilde{x}$ to new state vector w is determined as follows:

$$w = w(\tilde{x}) = [w_1(\tilde{x}) \quad w_2(\tilde{x}) \quad \dots \quad w_n(\tilde{x})]^T, \qquad (22)$$

where

$$w_1 = \tilde{x}_1,$$

$$w_i(\bar{\tilde{x}}_i) = \sum_{\nu=1}^{i-1} \frac{\partial w_{i-1}}{\partial \tilde{x}_\nu} \phi_\nu(\bar{\tilde{x}}_{\nu+1}) + \lambda_{i-1} w_{i-1}(\bar{\tilde{x}}_{i-1}), \; i = \overline{2, n}, \qquad (23)$$

$\lambda_i \ge \varepsilon_1 > 0$ are some constants, $i = \overline{1, n-1}$ [8, 13].

The transformation $w(\tilde{x})$ (22), (23) is bounded and convertible by virtue of the conditions (4), i.e. in the domain $\Omega_{\tilde{x}} \in R^n$ there is a bounded inverse transformation $\tilde{x} = \tilde{x}(w)$ such that $\tilde{x}(w)\big|_{w(\tilde{x})} = \tilde{x}$.

The stabilizing control $u = u(\tilde{x})$ for the system (2), (3) is determined by the expression

$$u(\tilde{x}) = -\gamma_1^{-1}\left[\gamma_2(\tilde{x}) + \lambda_n w_n(\tilde{x})\right] - \phi_n(\tilde{x}), \qquad (24)$$

where

$$\gamma_1 = \gamma_1(\tilde{x}) = \frac{\partial w_n(\tilde{x})}{\partial \tilde{x}_n} = \prod_{i=1}^{n-1} \frac{\partial \phi_i(\bar{\tilde{x}}_{i+1})}{\partial \tilde{x}_{i+1}}, \qquad (25)$$

$$\gamma_2(\tilde{x}) = \sum_{\mu=1}^{n-1} \frac{\partial w_n(\tilde{x})}{\partial \tilde{x}_\mu} \tilde{\phi}_\mu(\bar{\tilde{x}}_{\mu+1}), \quad \tilde{x} \in \Omega_{\tilde{x}}, \qquad (26)$$

$\lambda_n \ge \varepsilon_1 > 0$. The variable $w_n(\tilde{x})$ is determined also by the expressions (23) [14, 15].

The important property of the control (24) – (26) is that the closed system (2), (3), (24) is described in the variables w_i, $i = \overline{1, n}$ by a system of the linear stationary differential equations which have the following kind:

$$\dot{w} = \Lambda_n w, \quad \Lambda_n = \begin{bmatrix} -\lambda_1 & 1 & \dots & 0 \\ 0 & -\lambda_1 & \ddots & 0 \\ \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \dots & -\lambda_n \end{bmatrix}, \qquad (27)$$

Note, the conditions (4) ensure the existence of the stabilizing control (24) – (26) in the domain $\tilde{x} \in \Omega_{\tilde{x}}$. The matrix Λ_n (27) coincides with the Jordan $n \times n$-cell [19, p. 142] with $\lambda_i = -\lambda$, $i = \overline{1, n}$. Therefore the system of the equations (2), (3) is called Jordan controlled form, if the conditions (4) carry out in some domain $\Omega_{\tilde{x}} \in R^n$ [13].

Evidently, the system (27) is asymptotically stable if $\lambda_i \geq \varepsilon_1 > 0$, $i = \overline{1, n}$. Since the transformation (22), (23) is convertible and bounded, then the equilibrium $\tilde{x} = 0$ of the system (2), (3), (24) – (26) with $\lambda_i \geq \varepsilon_1 > 0$, $i = \overline{1, n}$ also asymptotically stable in the domain $\Omega_{\tilde{x}} \in R^n$, i.e. the control (24) – (26) under shown conditions is the stabilizing control.

Let's show this property of the control (24) – (26) on an example.

Example 2. At some assumptions longitudinal movement of a aircraft is described by the equations

$$\dot{\theta} = T_V^{-1} \sin\alpha - gV^{-1}\cos\theta,$$

$$\ddot{\vartheta} = -T_\omega^{-1}\dot{\vartheta} - k_\alpha \sin\alpha + k_\delta \delta, \tag{28}$$

where θ is the corner of the trajectory inclination, α is the attack corner, δ is control input (the corner of the rudders deviation), $\vartheta = \theta + \alpha$; V is the flight speed, g is the gravity acceleration, T_V, T_ω, k_α, k_δ are the parameters of the aircraft [20].

The variables values: $\alpha^\circ = \arcsin(gT_V V^{-1}\cos\theta^\circ)$, $\vartheta^\circ = \theta^\circ + \alpha^\circ$ and the control $\delta^\circ = k_\alpha k_\delta^{-1} \sin\alpha^\circ$ correspond to the steady state movement of the aircraft. Let the designation of the deviations are: $\tilde{x}_1 = \theta - \theta^\circ$, $\tilde{x}_2 = \alpha - \alpha^\circ$, $\tilde{x}_3 = \dot{\alpha}$, $u = k_\delta(\delta - \delta^\circ)$. The equations (28) in these deviations will become

$$\dot{\tilde{x}}_1 = a_{12}\sin(\alpha^\circ + \tilde{x}_2) - a_{11}\cos(\tilde{x}_1 + \theta^\circ) = \phi_1(\tilde{x}_1, \tilde{x}_2),$$

$$\dot{\tilde{x}}_2 = \tilde{x}_3 = \phi_2(\tilde{x}),\ \ \dot{\tilde{x}}_3 = \phi_3(\tilde{x}) + u, \tag{29}$$

where

$$\phi_3(\tilde{x}) = -\sigma - (a_{31} + a_{12}\cos(\alpha^\circ + \tilde{x}_2))\tilde{x}_3 - k_\alpha \sin(\alpha^\circ + \tilde{x}_2),$$

$$\sigma = (a_{11}\sin(\tilde{x}_1 + \theta^\circ) + a_{31})\phi_1(\tilde{x}) - k_\delta \delta^\circ,\ \ a_{11} = gV^{-1},$$

$$a_{12} = T_V^{-1},\ \ a_{31} = T_\omega^{-1}.$$

If $\left|\alpha^\circ + \tilde{x}_2\right| < \pi/2$, the equations (29) have JCF, since $\partial\phi_1(\tilde{x}_1, \tilde{x}_2)/\partial\tilde{x}_2 = a_{12}\cos(\alpha^\circ + \tilde{x}_2) \neq 0$, and $\partial\phi_2(\tilde{x})/\partial\tilde{x}_3 = 1$. The variables w_i, the functions $\gamma_1(\tilde{x})$ and $\gamma_2(\tilde{x})$ are determined by expressions (22), (23), (25), (26):

$$w_1 = \tilde{x}_1,\ \ w_2 = \phi_1(\tilde{x}_1, \tilde{x}_2) + \lambda_1 \tilde{x}_1,$$

$$w_3(\tilde{x}) = \sigma_1(\tilde{x}_1)\phi_1(\tilde{x}) + \lambda_1\lambda_2\tilde{x}_1 + a_{12}\tilde{x}_3\cos(\alpha^\circ + \tilde{x}_2),$$

$$\gamma_1(\tilde{x}) = a_{12}\cos(\alpha^\circ + \tilde{x}_2),$$

$$\gamma_2(\tilde{x}) = [a_{11}a_{12}\cos\tilde{x}_1 \sin(\alpha^\circ + \tilde{x}_2) - a_{11}^2\cos 2(\tilde{x}_1 + \theta^\circ) +$$

$$+\lambda_1\lambda_2 + (\lambda_1 + \lambda_2)a_{11}\sin(\tilde{x}_1 + \theta^\circ)]\phi_1(\tilde{x}) + \sigma_2(\tilde{x}),$$

where

$$\sigma_1(\tilde{x}_1) = \lambda_1 + \lambda_2 + a_{11}\sin(\tilde{x}_1 + \theta^\circ),$$

$$\sigma_2(\tilde{x}) = [\sigma_1(\tilde{x}_1)\cos(\alpha^\circ + \tilde{x}_2) - \tilde{x}_3\sin(\alpha^\circ + \tilde{x}_2)]a_{12}\tilde{x}_3.$$

The found here expressions determine the stabilizing control for the aircraft (28) or (29) by the expression (24). This control has view

$$u(\tilde{x}) = -(a_{12}\cos(\alpha^\circ + \tilde{x}_2))^{-1}[\gamma_2(\tilde{x}) + \lambda_3 w_3(\tilde{x})] - \phi_3(\tilde{x}),$$

$$\left|\alpha^\circ + \tilde{x}_2\right| < \pi/2. \tag{30}$$

On Fig. 1 the schedules of the deviations $\tilde{x}_i(t)$, $i = 1,\ 2,\ 3$ are shown. They are received as the simulation result of the closed system (29), (30) in MATLAB with $\alpha_{11} = 0.07$, $\alpha_{12} = 0.5$, $\alpha_{31} = 0.1$, $k_{\alpha 1} = 0.1$, $k_{d1} = 1$, $\lambda_1 = 3.5$, $\lambda_2 = 5.5$, $\lambda_3 = 8$ and $\tilde{x}_0 = [0.35\ \ -0.15\ \ 0]$. All the systems deviations aspire to zero.

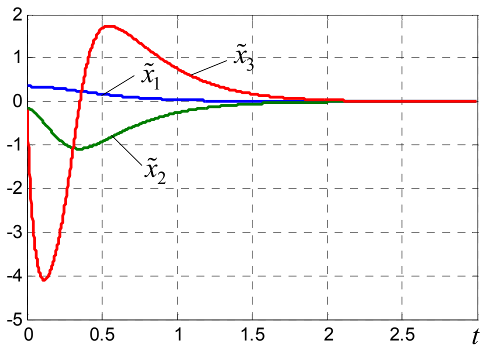


Fig. 1– Deviations of the aircraft variables

It is easy to check up, that the equations of the closed system (29), (30) in the variables w_i, $i = 1, 2, 3$ are linear and looks like:

$$\dot{w} = \begin{bmatrix} -3.5 & 1 & 0 \\ 0 & -5.5 & 1 \\ 0 & 0 & -8 \end{bmatrix} w.$$

This equation corresponds to expressions (27) with $n = 3$ completely. Hence, the control (24) – (26) is the stabilizing control, really.

The expressions (23) – (27) are fair at all values of the constants λ_i. The necessary in further linearization control follows from these expressions, if the conditions (4) are carried out and $\lambda_i = 0$, $i = \overline{1, n}$. This control is described by the next expression

$$u_{lin}(\tilde{x}) = -\gamma_{o1}^{-1}(\tilde{x})\gamma_{o2}(\tilde{x}) - \phi_n(\tilde{x}), \tag{31}$$

where the functions $\gamma_{o1}(\tilde{x})$ and $\gamma_{o2}(\tilde{x})$ are determined also by the expressions (25), (26). But in these expressions the variables w_i should be replaced by the variables

$$w_{o1} = \tilde{x}_1 ,$$

$$w_{oi}(\bar{\tilde{x}}_i) = \sum_{\nu=1}^{i-1} \frac{\partial w_{oi-1}}{\partial \tilde{x}_\nu} \phi_\nu(\bar{\tilde{x}}_{\nu+1}) , \quad i = \overline{1, n} . \tag{32}$$

Note, the control (31), (32) depends only from properties of the given nonlinear plant (2), (3). This linearization control is used at the problem solution of the optimal control system design.

6 Optimal Control System Design

After definition of the linearization control we can determine the uncertain matrix $Q(\tilde{x})$ and the function $l(x)$ in the nonlinear quadratic criteria from the optimality condition (5). Let

$$Q(\tilde{x}) = S^T(\tilde{x}) \bar{Q} S(\tilde{x}) , \quad l(u) = \gamma_{o1}(\tilde{x})[u - u_{lin}(\tilde{x})] ,$$

where $\bar{Q} \geq 0$ is a symmetrical numerical matrix; the matrix $S(\tilde{x})$ is a matrix from a quasilinear representation of the transformation $w_o(\tilde{x}) = S(\tilde{x})\tilde{x}$ (32) [8, 9]. This representation is determined [8] by the expressions

$$S(\tilde{x}) = \int_0^1 w'_o(\theta \tilde{x}) d\theta , \quad w'_o(\tilde{x}) = \partial w_o(\tilde{x}) / \partial \tilde{x} . \tag{33}$$

The matrix $S(\tilde{x})$ (33) is non-singular by virtue of the condition (4).

Thus the condition (5) finally looks like

$$J = \int_0^\infty [\tilde{x}^T S^T(\tilde{x}) \bar{Q} S(\tilde{x}) \tilde{x} + \rho \gamma_{o1}^2(\tilde{x})[u - u_{lin}(\tilde{x})]^2] dt \to \min_u . \tag{34}$$

where the function $u_{lin}(\tilde{x})$ is determined by the expression (31).

Values of the matrix $\bar{Q}$ coefficients and the number ρ are chosen in accordance with the desired transient of the nonlinear optimal control system.

For solution of the optimization problem (2), (3), (34) the control u at the equation (3) is taken in the form

$$u = u_{lin}(\tilde{x}) + \gamma_{o1}^{-1}(\tilde{x}) v . \tag{35}$$

Here v is a new control input of the close system (2), (3), (35). The equations (2), (3) are recorded in view of the control (35) as follows:

$$\dot{\tilde{x}} = \phi_{lin}(\tilde{x}) + e_n \gamma_{o1}^{-1}(\tilde{x}) v , \tag{36}$$

where $\phi_{lin}(\tilde{x}) = [\phi_1(\tilde{x}) \dots \phi_n(\tilde{x})]^T + e_n u_{lin}(\tilde{x})$.

Since the control $u_{lin}(\tilde{x})$ is determined by the expressions (25), (26) and (31), (32) therefore in accordance with the equations (27) the nonlinear system (36) is described at the state variables w_{oi}, $i = \overline{1, n}$ by the next expressions

$$\dot{w}_o = \bar{\Lambda}_n w_o + e_n v , \quad \bar{\Lambda}_n = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix} , \tag{37}$$

Diagonal elements of the matrix $\bar{\Lambda}_n$ in (37) equal to zero as the linearization control $u_{lin}(\tilde{x})$ is defined with $\lambda_i = 0$, $i = \overline{1, n}$.

At the variables w_{oi}, $i = \overline{1, n}$ in view of the equality $w_o(\tilde{x}) = S(\tilde{x})\tilde{x}$ and the expression (35) the criteria in the condition (34) looks like

$$J = \int_0^\infty [w_o^T \bar{Q} w_o + \rho v^2] dt . \tag{38}$$

As is well-known, the optimal control v_{OC} minimizing the quadratic criteria (38) on the trajectories of the system (37) is determined by the expression

$$v_{OC} = -\rho^{-1} e_n^T P w_o . \tag{39}$$

Here P is the symmetric, positive definite matrix, which is a solution of the Riccati equation

$$P \bar{\Lambda}_n + \bar{\Lambda}_n^T P - P e_n \rho^{-1} e_n^T P + \bar{Q} = 0 . \tag{40}$$

where $\bar{Q} \geq 0$, $\rho > 0$ are the matrix and the number from the quadratic criteria (34) and (38) [1, 3, 8].

Theorem 2. If the matrix $\bar{Q} \geq 0$ and the number $\rho > 0$ in the Riccati equation (40) are taken from the condition (34), the optimal control is defined in the equation (3) by expression

$$u_{OC} = u_{lin}(\tilde{x}) - \gamma_{o1}^{-1}(\tilde{x}) \rho^{-1} e_n^T P S(\tilde{x}) \tilde{x} . \tag{41}$$

The proof of this theorem is not given here, as its statement is evidently enough, in view of convertibility of the transformation $w_o(\tilde{x}) = S(\tilde{x})\tilde{x}$.

Note also, the theorem 2 can be proved with using the condition of a local minimum of integral [21, p. 322] and the dependences of the matrix $S(\tilde{x})$ and the functions $\gamma_{o1}(\tilde{x})$, $u_{lin}(\tilde{x})$ only from the nonlinear functions of the equations (2), (3).

The expressions (25), (26), (31), (32), (33), (40) and (41) are the mathematical base of the propose method of the optimal control system design for nonlinear controlled systems (plants). This method is much easier in comparison with other methods of the optimization problem solution, for example, by using the Pontryagin Minimum Principle or the Hamilton–Jacobi–Bellman equation [16]. If the equations of the given plant are transformed to JCF the design procedure of the optimal control system is completely analytical.

The Riccati equation (40) is solved by the known MATLAB function ARE as usual. Values of the matrix $\bar{Q}$ factors and number ρ are appointed by the iterations method for reception of the satisfactory transients. The properties of the nonlinear optimal system (2), (3), (41) can be changed by a choice of the criteria parameters as in the linear case.

When the equations of the given controlled system (plant) have a general view, design of the nonlinear optimal control systems by the proposed method is carried out in the following sequence:

- the equations of the plant are wrote in deviations if it is necessary;
- the equations in deviations are converted to JCF if it is necessary;
- the linearization control is designed on the basis of the equations in JCF;
- the nonlinear quadratic criteria is determined finally and the optimal control is found;
- the parameters of the quadratic criteria get out on desirable character of transient.

Efficiency of the optimal control systems design by the proposed method we shall show on examples.

7 Examples

Example 3. For the nonlinear controlled system [17, p. 188], which is described in deviations by the equations

$$\dot{x}_1 = x_2,\ \dot{x}_2 = x_3^3 + u,\ \dot{x}_3 = x_1 + cx_3^3, \tag{42}$$

to find two variants of an optimal control $u = u_{OC}(\tilde{x})$ on the condition (34), where $\rho = 0.5$ and matrix $\bar{Q}$ is equals $\bar{Q}_1 = \mathrm{diag}\{2 \quad 1 \quad 5\}$ or $\bar{Q}_2 = \mathrm{diag}\{50 \quad 12 \quad 3\}$.

The equations (42) are in deviations, but their form does not meet JCF, evidently. To present these equations in JCF their variables we shall designate as follows: $x_1 = \tilde{x}_2$, $x_2 = \tilde{x}_3$, $x_3 = \tilde{x}_1$. The resulting equations of the given controlled system look like

$$\dot{\tilde{x}}_1 = \tilde{x}_2 + c\tilde{x}_1^3 = \phi_1(\tilde{x});\quad \dot{\tilde{x}}_2 = \tilde{x}_3 = \phi_2(\tilde{x});$$

$$\dot{\tilde{x}}_3 = \tilde{x}_1^3 + u = \phi_3(\tilde{x}) + u. \tag{43}$$

The equations (43) satisfy to the conditions (4) since $\partial\phi_1(\tilde{x})/\partial\tilde{x}_2 = 1$ and $\partial\phi_2(\tilde{x})/\partial\tilde{x}_3 = 1$ for all $\tilde{x} \in R^3$. Therefore these equations have JCF and the considered task has a solution.

Further, according to the proposed method a linearization control is designed. For this purpose the transformation $w_o = w_o(\tilde{x})$ is determined by the expressions (32) in view of the equations (43) and looks like:

$$w_{o1} = \tilde{x}_1,\quad w_{o2} = c\tilde{x}_1^3 + \tilde{x}_2,$$

$$w_{o3} = 3c^2\tilde{x}_1^5 + 3c\tilde{x}_1^2\tilde{x}_2 + \tilde{x}_3 \tag{44}$$

or in the quasilinear vector-matrix form: $w_o(\tilde{x}) = S(\tilde{x})\tilde{x}$, where

$$S(\tilde{x}) = \begin{bmatrix} 1 & 0 & 0 \\ c\tilde{x}_1^2 & 1 & 0 \\ 3c^2\tilde{x}_1^4 + 2c\tilde{x}_1\tilde{x}_2 & c\tilde{x}_1^2 & 1 \end{bmatrix}. \tag{45}$$

The transformation (44) is not singular, convertible and bounded for all $\tilde{x} \in R^3$, $\|\tilde{x}\| < \infty$. The functions $\gamma_{o1}(\tilde{x})$ and $\gamma_{o2}(\tilde{x})$ are determined by the expressions (25), (26) using the equations (43) and the variables w_{oi}, $i = \overline{1,3}$ (44) as:

$$\gamma_{o1}(\tilde{x}) = 1,$$

$$\gamma_{o2}(\tilde{x}) = 3c(7c\tilde{x}_1^4\tilde{x}_2 + 5c^2\tilde{x}_1^7 + 2\tilde{x}_1\tilde{x}_2^2 + \tilde{x}_1^2\tilde{x}_3). \tag{46}$$

Now the linearization control is written on the expression (31) with $n = 3$ as:

$$u_{lin}(\tilde{x}) = -\gamma_{o2}(\tilde{x}) - \tilde{x}_1^3. \tag{47}$$

The matrix P as a solution of the Riccati equation (40) with the given matrix $\bar{Q}_1 = \mathrm{diag}\{2 \quad 1 \quad 5\}$ and $\rho = 0.5$ is

$$P_1 = \begin{bmatrix} 4.398 & 4.335 & 1 \\ 4.335 & 8.533 & 2.199 \\ 1 & 2.199 & 2.168 \end{bmatrix}. \tag{48}$$

Therefore, first variant of the optimal control, determining by the expressions (41) in view of the linearization control (47), the functions $\gamma_{o1}(\tilde{x}) = 1$, $\gamma_{o2}(\tilde{x})$ (46), $\rho = 0.5$ and the matrices $S(\tilde{x})$ (45), and $P = P_1$ (48) there is

$$u_{OC1} = -\gamma_{o2}(\tilde{x}) - \tilde{x}_1^3 - (2\tilde{x}_1 + 4.398\tilde{x}_2 + 4.336\tilde{x}_3 +$$

$$+13.01c\tilde{x}_1^2(c\tilde{x}_1^3 + \tilde{x}_2) + 4.398c\tilde{x}_1^3). \tag{49}$$

Similarly, the solution of the Riccati equation (40) with the matrix $\bar{Q}_2 = \mathrm{diag}\{50 \quad 12 \quad 3\}$ and $\rho = 0.5$ is the matrix

$$P_2 = \begin{bmatrix} 57.358 & 26.9 & 5.0 \\ 26.9 & 25.858 & 5.736 \\ 5.0 & 5.736 & 2.69 \end{bmatrix}. \tag{50}$$

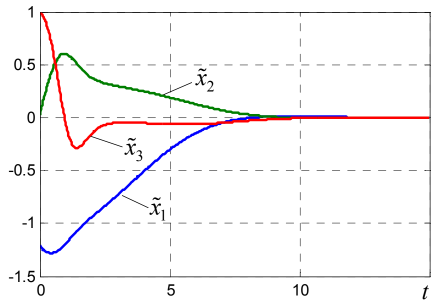
Hence, the second variant of the optimal control, determining by the expressions (41), (50) is:

$$u_{OC2} = -\gamma_{o2}(\tilde{x}) - \tilde{x}_1^3 - (10\tilde{x}_1 + 11.472\tilde{x}_2 + 5.38\tilde{x}_3 + \\ +16.14c\tilde{x}_1^2(c\tilde{x}_1^3 + \tilde{x}_2) + 5.38c\tilde{x}_1^3). \quad (51)$$

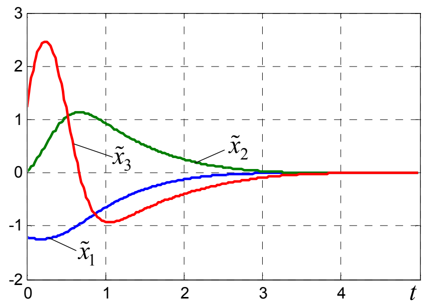
Transients of closed system (43), with the optimal controls (49) and (51) are submitted on Fig. 1,*a* and Fig. 1,*b* accordingly. These schedules are received by simulation of the optimal systems in MATLAB with $c = 0.2$ and $\tilde{x}_0 = [-1.2 \quad 0 \quad 1]$.

Reader can see, that transitive process on Fig. 1,*a* has big duration and the transitive process on Fig. 2,*b* has smaller duration.

Shown difference between the transients is caused by the various values of the matrixes $\bar{Q}_1$ and $\bar{Q}_2$ factors. Hence, the transient's character of the nonlinear optimal control systems really can be changed by a choice of the factors values of the nonlinear optimization criteria.



a)



b)

Fig. 2– Transients of the optimal control system

Example 4. For considered in example 2 converter a control system is designed here to compare the possibilities of the optimal and stabilizing controls for nonlinear systems. The optimal control is determined when in the condition (34) the number $\rho = 0.2$ and the matrixes:

$$\bar{Q}_1 = \begin{bmatrix} 5 & 0 \\ 0 & 400 \end{bmatrix}, \ \bar{Q}_2 = \begin{bmatrix} 6000 & 0 \\ 0 & 5 \end{bmatrix}, \ \bar{Q}_3 = \begin{bmatrix} 6000 & 170 \\ 170 & 5 \end{bmatrix}. \quad (52)$$

For the solution of the task the stabilizing and optimal controls are designed below on base of the converter equations in JCF.

The initial equations (11) of the given converter do not meet JCF; therefore they were transformed to this form in section 4. The equations (19), (20) are the resulting equations of the converter in JCF.

Design of the stabilizing control. As the converter equations have the order $n = 2$, for definition of this control the variables $w_1(\tilde{x})$, $w_2(\tilde{x})$ are found on the expressions (22), (23) and functions $\gamma_1(\tilde{x})$, $\gamma_2(\tilde{x})$ are found on the expressions (25), (26):

$$w_1 = \tilde{x}_1,$$

$$w_2 = 2[U_{SR}R(\tilde{I}_L(\tilde{x}) - I_L^\circ) - \tilde{x}_2^2 - 2U_C^\circ \tilde{x}_2]R^{-1} + \lambda_1\tilde{x}_1, \quad (53)$$

$$\gamma_1(\tilde{x}) = -\left(\frac{2U_{SR}C\sqrt{L^{-1}}}{\sqrt{(\tilde{x}_1 - \varphi_0) - (\tilde{x}_2 + U_C^\circ)^2 C}} + \frac{4}{R}\right)(\tilde{x}_2 + U_C^\circ), \quad (54)$$

$$\gamma_2(\tilde{x}) = \left(\frac{\partial w_2(\tilde{x})}{\partial \tilde{x}_1} + \lambda_1\right)\phi_1(\tilde{x}) = \\ = \left(\frac{U_{SR}}{\sqrt{L^{-1}\tilde{x}_1 + (I_L^\circ)^2}} + \lambda_1\right)\phi_1(\tilde{x}). \quad (55)$$

Here the constant φ_0 and the functions $\tilde{I}_L(\tilde{x})$, $\phi_1(\tilde{x})$ are determined by the expressions (16) and (18), (21).

The stabilizing control of the converter in variables $\tilde{x}_1$, $\tilde{x}_2$ (15) is determined by the expression (24) in view of the equations (19), (20), the expressions (53) – (55) and it looks like:

$$u_{SC}(\tilde{x}) = -\phi_2(\tilde{x}) - \gamma_1^{-1}(\tilde{x}) \times \\ \times \left[\left(\frac{U_{SR}}{\sqrt{L^{-1}\tilde{x}_1 + (I_L^\circ)^2}} + \lambda_1 + \lambda_2\right)\phi_1(\tilde{x}) + \lambda_1\lambda_2\tilde{x}_1\right]. \quad (56)$$

In the expressions (53), (55), (56) λ_1, λ_2 are varied parameters, which values determine character of the converter transients with using of the stabilizing control $u_{SC}(\tilde{x})$.

Design of the optimal control. For solution of this problem, according to the proposed method the variables $w_{o1}(\tilde{x})$, $w_{o2}(\tilde{x})$ and the function $\gamma_{o2}(\tilde{x})$ are found on the expressions (32) at $n = 2$, (21) and equation (19), (20) correspondently:

$$w_{o1} = \tilde{x}_1,$$

$$w_{o2}(\tilde{x}) = 2[U_н R(I_L(\tilde{x}) - I_L^\circ) - \tilde{x}_2^2 - 2U_C^\circ \tilde{x}_2]/R,$$

$$\gamma_{o2}(\tilde{x}) = \frac{\partial w_{o2}}{\partial \tilde{x}_1}\phi_1(\tilde{x}) = \frac{U_{SR}\phi_1(\tilde{x})}{\sqrt{L^{-1}\tilde{x}_1 + (I_L^\circ)^2}}. \quad (57)$$

The function $\gamma_{o1}(\tilde{x}) = \gamma_1(\tilde{x})$ is determined here by the expression (54), as well as in the previous case.

The linearization control is determined by the expressions (31), (21), (54), (57) and looks like

$$u_{lin}(\tilde{x}) = -\frac{U_{SR}\phi_1(\tilde{x})}{\gamma_{o1}(\tilde{x})\sqrt{L^{-1}\tilde{x}_1 + (I_L^\circ)^2}} - \phi_2(\tilde{x}). \quad (58)$$

Further, we determine the matrix $S(\tilde{x})$. In this case the system order $n = 2$, therefore the matrix $S(\tilde{x})$ has next view

$$S(\tilde{x}) = \begin{bmatrix} 1 & 0 \\ S_{21}(\tilde{x}) & S_{22}(\tilde{x}) \end{bmatrix}, \quad (59)$$

and the expressions (33) is possible to replace by the following formulas:

$$S_{21}(\tilde{x}) = \int_0^1 w'_{o21}(\tilde{x}_1\theta)d\theta,\ S_{22}(\tilde{x}) = \int_0^1 w'_{o22}(\tilde{x}_1, \tilde{x}_2\theta)d\theta, \quad (60)$$

where

$$w'_{o21} = \frac{\partial w_{o2}(\tilde{x})}{\partial \tilde{x}_1} = 2U_{SR}\tilde{I}'_{L1}(\tilde{x}) = \frac{U_{SR}}{\sqrt{L^{-1}\tilde{x}_1 + (I_L^\circ)^2}},$$

$$w'_{o22} = \frac{\partial w_{02}(\tilde{x})}{\partial \tilde{x}_2} = \frac{2}{R}(U_{SR}R\tilde{I}'_{L2}(\tilde{x}) - 2U_C^\circ - 2\tilde{x}_2) =$$

$$= \frac{-2U_{SR}(\tilde{x}_2 + U_C^\circ)CL^{-1}}{\sqrt{(\tilde{x}_1 - \varphi_0)L^{-1} - (\tilde{x}_2 + U_C^\circ)^2 CL^{-1}}} - \frac{4(U_C^\circ + \tilde{x}_2)}{R}.$$

Integration in the expression (60) is carried out with application of the formulas (191.01) and (321.01) [22]. In result we have received

$$S_{21}(\tilde{x}) = 2U_{SR}\left(\sqrt{L^{-1}\tilde{x}_1 + (I_L^\circ)^2} - I_L^\circ\right)/L^{-1}\tilde{x}_1, \quad (61)$$

$$S_{22}(\tilde{x}) = 2U_{SR}\tilde{x}_2^{-1}\sqrt{CL^{-1}}\left(\sqrt{(\tilde{x}_1 - \varphi_0)C^{-1} - (\tilde{x}_2 + U_C^\circ)^2} - \right.$$

$$\left. -\sqrt{(\tilde{x}_1 - \varphi_0)C^{-1} - (U_C^\circ)^2}\right) - \frac{2}{R}\left(2U_C^\circ + \tilde{x}_2\right). \quad (62)$$

The solution of the equation Riccati (40) with the matrix $\bar{Q}_1$ (52), vector $e_2^T = [0 \;\; 1]$ and factor $\rho = 0.2$ looks like

$$P = \begin{bmatrix} 2.3767 & 0.8944 \\ 0.8944 & 0.9787 \end{bmatrix}. \quad (63)$$

The product $e_n^T P S(\tilde{x})\tilde{x}$ on the basis of the expressions (59), (61) – (63) is given

$$e_n^T P\, S(\tilde{x})\tilde{x} = 0.894\tilde{x}_1 + 0.979[S_{21}(\tilde{x})\tilde{x}_1 + S_{22}(\tilde{x})\tilde{x}_2].$$

Hence, as $\gamma_{o1}(\tilde{x}) = \gamma_1(\tilde{x})$ (54), the optimal control, corresponding to the matrix $\bar{Q}_1$ on the expression (41) is determined by the next expression

$$u_{OC1}(\tilde{x}) = u_{lin}(\tilde{x}) - \gamma_1^{-1}(\tilde{x})\{4.472\tilde{x}_1 +$$
$$+ 4.8935[S_{21}(\tilde{x})\tilde{x}_1 + S_{22}(\tilde{x})\tilde{x}_2]\}. \quad (64)$$

The optimal controls $u_{OC}(\tilde{x})$, corresponding to the factor $\rho = 0.2$ and the matrixes $\bar{Q}_2$ and $\bar{Q}_3$ (52) are found similarly and have view:

$$u_{OC2}(\tilde{x}) = u_{lin}(\tilde{x}) - \gamma_1^{-1}(\tilde{x})\{20.9795\tilde{x}_1 +$$
$$+47.3495[S_{21}(\tilde{x})\tilde{x}_1 + S_{22}(\tilde{x})\tilde{x}_2]\}, \quad (65)$$

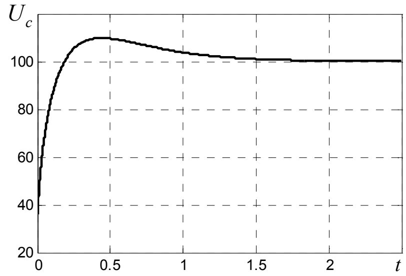
$$u_{OC3}(\tilde{x}) = u_{lin}(\tilde{x}) - \gamma_1^{-1}(\tilde{x})\{20.976\tilde{x}_1 +$$
$$+66.65[S_{21}(\tilde{x})\tilde{x}_1 + S_{22}(\tilde{x})\tilde{x}_2]\}. \quad (66)$$

Thus, the stabilizing control for the considered converter is described by the expressions (21), (54), (56) and it has two varied parameters. At the same time, the optimal control is described by the expressions (21), (54), (58), (41) and it has three varied parameters. In other words, the optimal control of the converter is more complex than the stabilizing control.

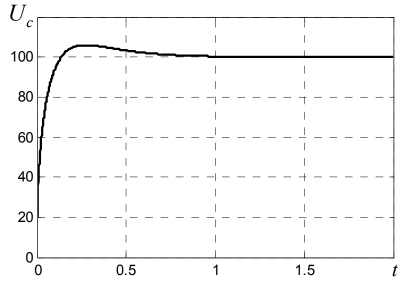
On the other hand, comparing the expressions (40), (41) with the expressions (23), (24) it is easy to conclude, that in generally case the optimal control has $n(n+1)/2$ varied parameters and the stabilizing control has only n such parameters. Hence, the optimal control designed by the proposed method has wide possibilities in comparison with the stabilizing control.

To investigate properties of the designed control system for the considered converter this system was simulated in MATLAB both with the stabilizing and with the optimal control. The simulation was spent with using of the converter equations in deviations (19), (20) with the various values of the initial conditions, values of the voltage of the power supply and the resistance of the load. But the basic attention was given to the dependence of the character voltage on the load from the controls parameters.

The transient's schedules of the load voltage with application of the stabilizing control (56) are shown on Fig. 3. These schedules are received with the voltage of the power supply $U_{SR} = 60$ V, the desired output voltage of the converter $U_C^\circ = 100$ and the load resistance $R = 0{,}2$ Ohm, $L = 0.55$ mGn, $C = 1.15$ μF. Schedules on Fig. 3,a correspond to values $\lambda_1 = 5$, $\lambda_2 = 3$ and schedules on Fig. 3,b would correspond to values $\lambda_1 = 5$, $\lambda_2 = 10$. In the first case the transient duration $t_{TR} = 1,28$ second and the overshot $\sigma = 6,9\,\%$. In the second case $t_{TR} = 0,38$ second and $\sigma = 6,0\,\%$.



a)



b)

Fig.3– Transients of the output voltage converter

Actual control of the converter (11) is the relative duration of time of the capacity charge: $\tau \in [0,1]$. This quantity is connected with the control $u = u(\tilde{x})$ from the equations (19), (20) by the expression $\tau = \tau^{\circ} - Cu(\tilde{x}) / \tilde{I}_L(\tilde{x})$. The example of the control transients τ is shown on Fig. 4.

Thus, the stabilizing control (56) provides the equilibrium stability of the examined converter. Changes of the control factors λ_1 and λ_2 cause the change of the transients' duration. The transients' character and the overshot change not essentially.

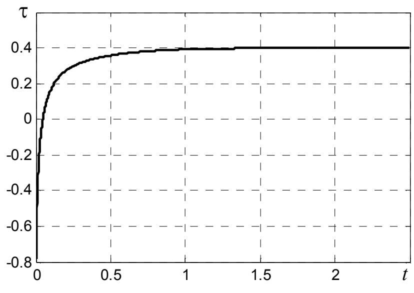
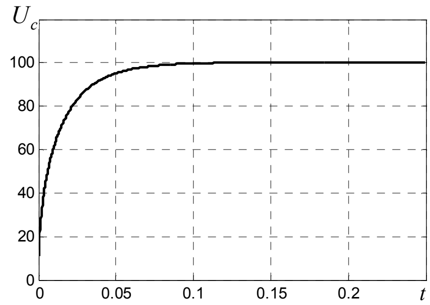


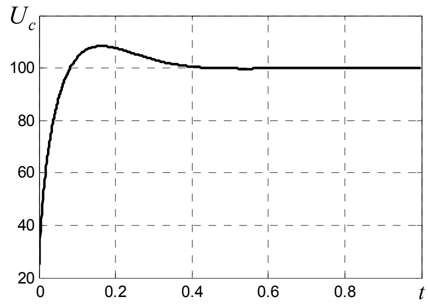
Fig. 4– Transients of the control converter

The control system of the converter with optimal control (64) – (66) was simulated with the same voltage of the power supply, a desirable output voltage and the load resistance with various values of the factor ρ and elements of the matrix $\bar{Q}$. Some results of the simulation are shown on fig. 5. They are received with $\tilde{x}_0 = [-150 \quad -95]$, $\rho = 0.2$ and matrixes (52):

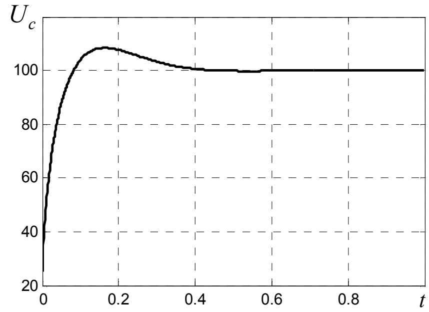
Schedules of the load voltage transients shown on Fig. 5,a are received with the optimal control (64); the schedules on Fig. 5,b are received with the optimal control (65) and schedules on Fig. 5,c are received with the optimal control (66). Control τ changes with the optimal controls similarly shown on Fig. 4.



a)



b)



c)

Fig. 5– Transients of the converter with optimal control

Apparently, with the optimal control the transient's duration is very small and depends from the matrix $\bar{Q}$ of the quadratic criteria. If $\bar{Q} = \bar{Q}_1$ then $t_{TR} = 0,05$ s, the overshot is absent. If $\bar{Q} = \bar{Q}_2$ or $\bar{Q} = \bar{Q}_3$ then $t_{TR} = 0,25$ s, the overshot $\sigma = 8,4\,\%$. Note, the not diagonal elements of the matrix $\bar{Q}$ do not influence practically on the character of the system's transients in this case (see Fig. 5,b and Fig. 5,c).

Thus, the optimal control (41) really provides the more wide possibilities to give projected system the desirable character of transients, in comparison with the stabilizing control (24).

The considered examples show also that the proposed method for the optimal control systems design is analytical, very simple and convenient for practical using at creation of the control systems for nonlinear plants.

8 Conclusion

Transformation of the nonlinear controlled systems (plants) equations to the Jordan controlled form allows using the proposed completely analytical method of the optimal control systems design. The resulted systems are optimal in the sense of a minimum of the nonlinear quadratic criteria. The optimal control is a nonlinear feedback on the state variables of the nonlinear plant.

If the plants equations converted to JCF, the optimal control system is designed by the proposed method in two stages. The linearization control is designed at the first stage. The linearization control is constructed on the basis of the nonlinear transformation of the nonlinear plant state variables to the new state variables. The plant equations under the linearization control become linear in the new variables. Simultaneously the nonlinear optimization criteria becomes as a usual quadratic criteria. This fact gives possibility to apply the known method LQ which uses usually to receive the optimal control for linear systems.

The equations of a many real nonlinear plants have the Jordan controlled form or can be transformed to this form without the big difficulties. Therefore the representation of the nonlinear controlled systems equations in Jordan controlled form is not hard restrictions. The proposed method of the optimal nonlinear control system design is shown on the examples.

From these examples follows, the nonlinear optimal control systems, designed by the proposed method, have more wide possibilities in comparison with the stabilization systems.

This research is supposed to be continued in a direction of development of the transformation methods to JCF of the nonlinear plants equations of the general view. Creation of the computer program for the automated design of the optimal control systems for different nonlinear plants is planned also.

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Appendix

Proof of the theorem 1. Jacobeans of the transformation $x = \psi(\tilde{x})$ is the matrix

$$J_{\psi\tilde{x}}(\tilde{x}) = \begin{bmatrix} \psi_{11}(\tilde{x}) & \psi_{12}(\tilde{x}) \\ \psi_{21}(\tilde{x}) & \psi_{22}(\tilde{x}) \end{bmatrix}, \qquad (67)$$

where $\psi_{ij}(\tilde{x}) = \partial\psi_i(\tilde{x})/\partial\tilde{x}_j$. Since $\dot{x} = J_{\psi x}(\tilde{x})\dot{\tilde{x}}$, then in view of the expressions (8), we have

$$\dot{\tilde{x}} = J_{\psi\tilde{x}}^{-1}(\tilde{x})[f(x) + b(x)u(x)]\big|_{x=\psi(\tilde{x})}. \qquad (68)$$

On the other hand, the inverse of the matrix $J_{\psi\tilde{x}}(\tilde{x})$ looks like

$$J_{\psi\tilde{x}}^{-1}(\tilde{x}) = \frac{1}{\Delta(\tilde{x})}\begin{bmatrix} \psi_{22}(\tilde{x}) & -\psi_{12}(\tilde{x}) \\ -\psi_{21}(\tilde{x}) & \psi_{11}(\tilde{x}) \end{bmatrix}, \qquad (69)$$

if $\Delta(\tilde{x}) = \psi_{11}(\tilde{x})\psi_{22}(\tilde{x}) - \psi_{21}(\tilde{x})\psi_{12}(\tilde{x}) \neq 0$.

Substituting this expression for $J_{\psi\tilde{x}}^{-1}(\tilde{x})$ in the equation (68), we conclude, at first, that according to the definition of JCF, the equality $b_1(\psi(\tilde{x}))\psi_{22}(\tilde{x}) - \psi_{12}(\tilde{x})b_2(\psi(\tilde{x})) \equiv 0$ should be carried out in domain $\Omega_{\tilde{x}2} \in R^2$. From here with taking in attention some multiplier $\mu(\tilde{x}) \neq 0$ follows, that possible to take

$$\psi_{22}(\tilde{x}) = b_2(\psi(\tilde{x}))\mu(\tilde{x}),$$
$$\psi_{12}(\tilde{x}) = b_1(\psi(\tilde{x}))\mu(\tilde{x}). \qquad (70)$$

If these conditions are carried out, the equality (68) coincides with the equations (2), (3) at $n = 2$ and $u(\tilde{x}) = u_1(\psi(\tilde{x}))/\mu(\tilde{x})$. In this case the function $\phi_1(\tilde{x})$ from the system of equations (68) is defined by expression

$$\phi_1(\tilde{x}) = [\psi_{22}(\tilde{x})f_1(\psi(\tilde{x})) - \psi_{12}(\tilde{x})f_2(\psi(\tilde{x}))]/\tilde{\Delta}(\tilde{x}), \qquad (71)$$

where $\tilde{x} \in \Omega_{\tilde{x}2}$, and

$$\tilde{\Delta}(\tilde{x}) = [\psi_{11}(\tilde{x})b_2(\psi(\tilde{x})) - \psi_{21}(\tilde{x})b_1(\psi(\tilde{x}))]\mu(\tilde{x}).$$

Second, the inequalities $\tilde{\Delta}(\tilde{x}) \neq 0$ and $\partial\phi_1(\tilde{x})/\partial\tilde{x}_2 \neq 0$ should be carried out at the domain $\Omega_{\tilde{x}2}$ too. The inequality $\tilde{\Delta}(\tilde{x}) \neq 0$ can be provided by the choice of the partial derivatives $\partial\psi_1(\tilde{x})/\partial\tilde{x}_1$ and $\partial\psi_2(\tilde{x})/\partial\tilde{x}_1$. Evidently, according to (70),

(71) the inequality $\partial\phi_1(\tilde{x})/\partial\tilde{x}_2 \not\equiv 0$ does not depend from the multiplier $\mu(\tilde{x})$.

Hence, the conditions of the transformation possibility of the equation (8) to JCF too do not depend from $\mu(\tilde{x})$. Therefore further we accept $\mu(\tilde{x}) = 1$. In this case the determinant $\tilde{\Delta}(\tilde{x})$ is defined by expression

$$\tilde{\Delta}(\tilde{x}) = [\psi_{11}(\tilde{x})b_2(\psi(\tilde{x})) - \psi_{21}(\tilde{x})b_1(\psi(\tilde{x}))] = \Delta(\tilde{x}).$$

Let's pass to finding-out of the conditions, at which the basic inequality $\partial\phi_1(\tilde{x})/\partial\tilde{x}_2 \not\equiv 0$ is carried out. For this purpose the function $\phi_1(\tilde{x})$ (71) is differentiated on $\tilde{x}_2$ in view of the known integralability condition of the two variables functions:

$$\frac{\partial^2\psi_i(\tilde{x})}{\partial\tilde{x}_1\partial\tilde{x}_2} = \frac{\partial^2\psi_i(\tilde{x})}{\partial\tilde{x}_2\partial\tilde{x}_1}, \quad i = 1, 2. \tag{72}$$

Argument of the nonlinear functions – the vector $\tilde{x}$ falls in the further expressions for their brevity.

As result, we shall receive, that the condition (4) in relation to the equations system (68) is equivalent to the following inequality

$$\Delta^{-2}\left\{\Delta\left[\begin{vmatrix} f_{11}\psi_{12} & b_1 \\ f_{21}\psi_{12} & b_2 \end{vmatrix} + \begin{vmatrix} f_1 & b_{11}\psi_{12} \\ f_2 & b_{21}\psi_{12} \end{vmatrix} + \begin{vmatrix} f_{12}\psi_{22} & b_1 \\ f_{22}\psi_{22} & b_2 \end{vmatrix} + \right.\right.$$
$$\left. + \begin{vmatrix} f_1 & b_{12}\psi_{22} \\ f_2 & b_{22}\psi_{22} \end{vmatrix}\right] - \begin{vmatrix} f_1(\psi) & b_1(\psi) \\ f_2(\psi) & b_2(\psi) \end{vmatrix}\left[\begin{vmatrix} \psi_{112} & b_1 \\ \psi_{212} & b_2 \end{vmatrix} + \right.$$
$$\left.\left. + \begin{vmatrix} \psi_{11} & b_{11}\psi_{12} \\ \psi_{21} & b_{21}\psi_{12} \end{vmatrix} + \begin{vmatrix} \psi_{11} & b_{12}\psi_{22} \\ \psi_{21} & b_{22}\psi_{22} \end{vmatrix}\right]\right\} \not\equiv 0. \tag{73}$$

Here and further $|\ |$ is designation of a determinant.

If in (73) the determinants are opened and similar members are resulted, this expression takes a kind

$$\begin{vmatrix} \psi_{11} & b_1 \\ \psi_{21} & b_1 \end{vmatrix}\left[\begin{vmatrix} f_{11} & b_1 \\ f_{21} & b_2 \end{vmatrix}\psi_{12} + \begin{vmatrix} f_1 & b_{11} \\ f_2 & b_{21} \end{vmatrix} + \begin{vmatrix} f_{12} & b_1 \\ f_{22} & b_2 \end{vmatrix}\psi_{22} + \right.$$
$$\left. + \begin{vmatrix} f_1 & b_{12} \\ f_2 & b_{22} \end{vmatrix}\psi_{22}\right] - \begin{vmatrix} f_1(\psi) & b_1(\psi) \\ f_2(\psi) & b_2(\psi) \end{vmatrix}\left[\begin{vmatrix} \psi_{112} & b_1 \\ \psi_{212} & b_2 \end{vmatrix} + \right.$$
$$\left. + \begin{vmatrix} \psi_{11} & b_{11} \\ \psi_{21} & b_{21} \end{vmatrix}\psi_{12} + \begin{vmatrix} \psi_{11} & b_{12} \\ \psi_{21} & b_{22} \end{vmatrix}\psi_{22}\right] \not\equiv 0. \tag{74}$$

In view of the equalities (70) and $\mu(\tilde{x}) = 1$ the inequality (74) becomes:

$$\Delta\left\{\left[\begin{vmatrix} f_{11} & b_1 \\ f_{21} & b_2 \end{vmatrix} + \begin{vmatrix} f_1 & b_{11} \\ f_2 & b_{21} \end{vmatrix}\right]b_1 + \left[\begin{vmatrix} f_{12} & b_1 \\ f_{22} & b_2 \end{vmatrix} + \right.\right.$$
$$\left.\left. + \begin{vmatrix} f_1 & b_{12} \\ f_2 & b_{22} \end{vmatrix}\right]b_2\right\} - \begin{vmatrix} f_1(\psi) & b_1(\psi) \\ f_2(\psi) & b_2(\psi) \end{vmatrix} D \not\equiv 0, \tag{75}$$

where

$$D = \left[\begin{vmatrix} \psi_{112} & b_1 \\ \psi_{212} & b_2 \end{vmatrix} + \begin{vmatrix} \psi_{11} & b_{11} \\ \psi_{21} & b_{21} \end{vmatrix}b_1 + \begin{vmatrix} \psi_{11} & b_{12} \\ \psi_{21} & b_{22} \end{vmatrix}b_2\right], \tag{76}$$

$$\psi_{112} = \frac{\partial\psi_{11}(\tilde{x})}{\partial\tilde{x}_2}, \quad \psi_{212} = \frac{\partial\psi_{21}(\tilde{x})}{\partial\tilde{x}_2}. \tag{77}$$

For definition of the partial derivatives $\psi_{112}(\tilde{x})$ and $\psi_{212}(\tilde{x})$ we shall take in attention the integralability condition (72) again. On these conditions

$$\psi_{112} = \psi_{112}(\tilde{x}) = \frac{\partial\psi_{11}(\tilde{x})}{\partial\tilde{x}_2} = \frac{\partial^2\psi_1(\tilde{x})}{\partial\tilde{x}_2\partial\tilde{x}_1} =$$
$$= \frac{\partial^2\psi_1(\tilde{x})}{\partial\tilde{x}_1\partial\tilde{x}_2} = \frac{\partial\psi_{12}(\tilde{x})}{\partial\tilde{x}_1}.$$

From here, in view of equality (70) and (77), we deduce

$$\psi_{112} == \frac{\partial b_1(\psi(\tilde{x}))}{\partial\psi_1}\frac{\partial\psi_1(\tilde{x})}{\partial\tilde{x}_1} + \frac{\partial b_1(\psi(\tilde{x}))}{\partial\psi_2}\frac{\partial\psi_2(\tilde{x})}{\partial\tilde{x}_1},$$

or

$$\psi_{112} = b_{11}\psi_{11} + b_{12}\psi_{21}. \tag{78}$$

Similarly

$$\psi_{212} = \frac{\partial\psi_{21}(\tilde{x})}{\partial\tilde{x}_2} = \frac{\partial\psi_2(\tilde{x})}{\partial\tilde{x}_2\partial\tilde{x}_1} = \frac{\partial\psi_2(\tilde{x})}{\partial\tilde{x}_1\partial\tilde{x}_2} = \frac{\partial\psi_{22}(\tilde{x})}{\partial\tilde{x}_1}$$

or

$$\psi_{212} = \frac{\partial b_2(\psi(\tilde{x}))}{\partial\psi_1}\frac{\partial\psi_1(\tilde{x})}{\partial\tilde{x}_1} + \frac{\partial b_2(\psi(\tilde{x}))}{\partial\psi_2}\frac{\partial\psi_2(\tilde{x})}{\partial\tilde{x}_1},$$

$$\psi_{212} = b_{21}\psi_{11} + b_{22}\psi_{21}. \tag{79}$$

The first determinant in the expression (76) in view of the received equality (78) and (79) can be presented to as follows:

$$\begin{vmatrix} \psi_{112} & b_1 \\ \psi_{212} & b_2 \end{vmatrix} = \begin{vmatrix} b_{11}\psi_{11} + b_{12}\psi_{21} & b_1 \\ b_{21}\psi_{11} + b_{22}\psi_{21} & b_2 \end{vmatrix} =$$
$$= \begin{vmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} \psi_{11} \\ \psi_{22} \end{bmatrix} & \begin{matrix} b_1 \\ b_2 \end{matrix} \end{vmatrix}.$$

Further, this expression is substituted in (76) and corresponding simplifications are carried out. As the result the expression (76) takes a kind

$$D = b_2(b_{11}\psi_{11} + b_{12}\psi_{21}) - b_1(b_{21}\psi_{11} + b_{22}\psi_{21}) +$$
$$+ b_1b_{21}\psi_{11} - b_1b_{11}\psi_{21} + b_{22}\psi_{11}b_2 - b_{12}\psi_{21}b_2 =$$
$$= b_2(b_{11} + b_{22})\psi_{11} - b_1(b_{11} + b_{22})\psi_{21} =$$

$$= (b_{11} + b_{22})\begin{vmatrix} \psi_{11} & b_1 \\ \psi_{21} & b_2 \end{vmatrix} = (b_{11} + b_{22})\Delta . \quad (80)$$

The expressions (78), (79) are substituted in (74) and the common multiplier Δ is taken out. It gives the inequality

$$\Delta\left\{\left[\left(\begin{vmatrix} f_{11} & b_1 \\ f_{21} & b_2 \end{vmatrix} + \begin{vmatrix} f_1 & b_{11} \\ f_2 & b_{21} \end{vmatrix}\right) b_1 + \left(\begin{vmatrix} f_{12} & b_1 \\ f_{22} & b_2 \end{vmatrix} + \right.\right.\right.$$

$$\left.\left.\left. + \begin{vmatrix} f_1 & b_{12} \\ f_2 & b_{22} \end{vmatrix}\right) b_2 \right] - (b_{11} + b_{22})\begin{vmatrix} f_1 & b_1 \\ f_2 & b_2 \end{vmatrix}\right\} \not\equiv 0 . \quad (81)$$

As $\Delta = \Delta(x) \neq 0$ by definition, the inequality (81) is equivalent to the condition (10) in view of the designation (9). Necessity of the theorem 1 conditions is proved. Sufficiency of its condition follows from the resulted expressions also. The theorem 1 is proved.

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