Dynamics analysis of Cournot triopoly game model with heterogeneous players in exploitation of a renewable resource

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Abstract: Based on a dynamical multi-team Cournot game in the exploitation of a renewable resource, a new dynamic Cournot triopoly game model with one bounded rational player and two team players is established to find a renewable resource. By using the theory of bifurcations of dynamical systems, we studied the stability of the system, obtaining a point at which local stable regions reachthe Nash equilibrium. Besides, we analyzed the resulting adjustment speed parameters and the weight parameter of the system, which could otherwise affect the dynamics behaviors of the system. The complexity of the system is demonstrated by means of bifurcation diagram, the Lyapunov exponents, the phase portrait, the time history diagram and the fractal dimension. Furthermore, the linear feedback control method is used to control the chaos of the system. The derived results have very important theoretical and practical significance to the renewable resource firms and market.

Key–Words: Complexity; Chaos; Dynamic Cournot triopoly game; Bounded rationality; Team players

1 Introduction

Research on the complexity of nonlinear economic systems is a very hot topic in the economic and management area. In recent years, a series of dynamic game models on the output decision (Cournot model) and price decision (Bertrand model) have been studied in the related references. Agiza [1] and Kopel [2] considered bounded rationality and established duopoly Cournot model with linear cost functions. From then on, the model has been extended to multi-oligopolistic market. Bischi et al. [3] suppose that all of the two firms determine their output based on the reaction functions, that is, all of the two players take naive strategy. Agliari et al. [4] studied a dynamic triopoly Cournot game with isoelastic demand function, linear cost function and naive players. Agiza and Elsadany [5] investigated a dynamic game model with two-types of players: one is bounded rational player, and the other is adaptive expectation player. Elabbasy et al.[6] investigated the dynamics of a nonlinear triopoly game with three-types of players which are bounded rational player, adaptive player and naive player. Tramontana [7], and Tramontana and Elsadany [8] studied the period-doubling bifurcations and the Neimark-Sacker bifurcation in a duopoly and triopoly with isoelastic demand function and heterogeneous players, respectively. Tomasz [9] analyzed the complex dynamics of a Bertrand duopoly game with bounded rational and adaptive players. Elettreby et al.[10] introduced a dynamic multi-team Bertrand four oligarchs game model with bounded rational and naive players, and researched the dynamics and control of this model. Sun and Ma [11] introduced a triopoly Bertrand model with nonlinear demand functions in Chinese cold rolled steel market, and researched the complexity and the control of the model. Matsumoto and Nonaka [12] researched the complexity of Cournot duopoly game model with complementary goods and naive players. Tramontana et al. [13] studied a piecewise-smooth Cournot duopoly game on the global bifurcations. Agliari [14] analyzed the global bifurcations of basins of a dynamic triopoly game. Yassen and Agiza [15] considered a Cournot duopoly game model with bounded players and the model with delayed bounded rationality, and derived that the model with delay has more possible to stable at the Nash equilibrium state. In these literatures, the vast majority of models suppose that the oligarchs have non-cooperation factor. However, there are few models studied the cooperation factor between the oligarchs.

Based on a dynamical multi-team Cournot game in exploitation of a renewable resource[16], a new dynamic Cournot triopoly game model with team players in exploitation of a renewable resource is built up. Suppose the inverse demand function is linear form, but the cost functions are nonlinear form. In this model, the bounded rational players regulate output speed according to marginal profit (or team marginal profit), and decide their output. By theoretical analysis and numerical simulation, the stable regions about the output adjustment speed parameters are derived. It is shown that the adjustment of the output speed and the weight parameter can change the stability of the system and even lead chaos to occur. It has an important theoretical and applied significance to the complexity research of new nonlinear dynamic Cournot game model in exploitation of renewable resource.

The paper is organized as follows. In Section 2, the dynamic Cournot triopoly game model with team players in exploitation of a renewable resource is presented. In Section 3, the existence and the local stability the Nash equilibrium point are studied, and the stable regions of the Nash equilibrium point are obtained. In Section 4, we investigate the output adjustment speed parameters and the weight parameter which effect on the dynamics behaviors of the system. Numerical simulation method is used to show complex dynamics of the system by means of the bifurcation diagram, the Lyapunov exponents, the phase portrait, the time history diagram and the fractal dimension. In Section 5, control bifurcation and chaos of the model is considered with the linear feedback control method. Finally, some conclusions are made.

2 The model

Suppose that there are three mainly oligopoly enterprises X_1, X_2, X_3 in exploitation of a renewable resource. The enterprise $X_i \quad (i = 1,2,3)$ makes the optimal output decision, and suppose the t-output is $q_i(t) \quad (i = 1,2,3)$. At each period t, the price p is determined by the total output $Q_T(t) = q_1(t) + q_2(t) + q_3(t)$.

According to Ref. [16], we also propose the renewable resource market with the linear inverse demand function:

$$p(t) = a - bQ_T(t). \tag{1}$$

and the cost function of the enterprise $X_i \quad (i = 1,2,3)$ is as follow:

$$C_i(t) = c_i + \frac{d_i q_i^2(t)}{Q_T(t)}, \quad (i = 1,2,3). \tag{2}$$

where $c_i \quad (i = 1,2,3)$ is the fix cost, and $d_i \quad (i = 1,2,3)$ is positive parameter.

We can get the profit of the enterprise $X_i \quad (i = 1,2,3)$ as follow:

$$\pi_i(t) = [a - bQ_T(t)]q_i(t) - c_i - \frac{d_i q_i^2(t)}{Q_T(t)}, \quad (i = 1,2,3). \tag{3}$$

Here, it is assumed that the renewable resource firms X_1, X_2 establish a team, then the profit of the team is:

$$\begin{aligned} \pi^{team}(t) & \\ = \quad & \theta[(a - bQ_T(t))q_1(t) - c_1 - \tfrac{d_1 q_1^2(t)}{Q_T(t)}] \\ & +(1-\theta)[(a - bQ_T(t))q_2(t) - c_2 - \tfrac{d_2 q_2^2(t)}{Q_T(t)}], \end{aligned} \tag{4}$$

where the θ is the weight parameter of the profit of renewable resource firm X_1 in the team.

We propose that the enterprise $X_i \quad (i = 1,2,3)$ takes bounded rational strategy. Since the game between the enterprises is a continuous and long-term repeated dynamic process, the dynamic adjustment of the player in the triopoly game can be expressed as follow:

$$\begin{cases} q_i(t+1) = q_i(t) + \alpha_i q_i(t) \frac{\partial \pi^{team}(t)}{\partial q_i(t)}, \quad (i = 1,2), \\ q_3(t+1) = q_3(t) + \alpha_3 q_3(t) \frac{\partial \pi_3(t)}{\partial q_3(t)}, \end{cases} \tag{5}$$

where $\alpha_i \quad (i = 1,2,3)$ is the output adjustment speed parameter.

A new dynamic Cournot triopoly game with heterogeneous players in exploitation of a renewable resource is obtained. This map has the following form:

$$\begin{cases} q_1(t+1) = q_1(t) + \alpha_1 q_1(t) A, \\ q_2(t+1) = q_2(t) + \alpha_2 q_2(t) B, \\ q_3(t+1) = q_3(t) + \alpha_3 q_3(t) C. \end{cases} \tag{6}$$

where

$A = [\theta(a - bQ_T(t) - bq_1 - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)}) + (1 - \theta)(-bq_2(t) + \frac{d_2 q_2^2(t)}{Q_T^2(t)})]$

$B = [\theta(-bq_1(t) + \frac{d_1 q_1^2(t)}{Q_T^2(t)}) + (1 - \theta)(a - bQ_T(t) - bq_2 - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)})]$

$C = [a - bQ_T(t) - bq_3 - \frac{2d_3 q_3(t)}{Q_T(t)} + \frac{d_3 q_3^2(t)}{Q_T^2(t)}]$

3 The stability of the system

In system (6), $\alpha_i \quad (i = 1,2,3)$ is taken as bifurcation parameter, and the other parameters are as follows: $a = 6.29, b = 0.92, d_1 = 0.265, d_2 = 0.374, d_3 = 0.436, \theta = 0.5$.

Let the marginal profits equal to 0, we can get the Nash equilibrium point.The fixed points of system (6) satisfy the following equations:

$$\begin{cases} q_1[\theta(a-bQ_T-bq_1 \\ -\frac{2d_1q_1}{Q_T}+\frac{d_1q_1^2}{Q_T^2})+(1-\theta)(-bq_2+\frac{d_2q_2^2}{Q_T^2})]=0, \\ q_2[\theta(-bq_1+\frac{d_1q_1^2}{Q_T^2}) \\ +(1-\theta)(a-bQ_T-bq_2-\frac{2d_2q_2}{Q_T}+\frac{d_2q_2^2}{Q_T^2})]=0, \\ q_3(a-bQ_T-bq_3-\frac{2d_3q_3}{Q_T}+\frac{d_3q_3^2}{Q_T^2})=0. \end{cases} \tag{7}$$

The Eqs. (7) are solved and seven meaningful fixed points $p_1(1.3500, 0.9566, 2.0933), p_2(3.2745, 0, 0)$, $p_3(0, 3.2152, 0), p_4(0, 0, 3.1815), p_5(1.9515, 1.3827, 0)$, $p_6(2.2483, 0, 2.1201), p_7(0, 2.1919, 2.1461)$ are obtained. Here, we only consider the stability of the Nash equilibrium point $p_1(q_1^* = 1.35, q_2^* = 0.9566, q_3^* = 2.0933)$, and denote the total output at the Nash equilibrium point as $Q_T^* = q_1^* + q_2^* + q_3^*$.

We can calculate the Jacobian matrix of system (6) at the Nash equilibrium point p_1

$$J = \begin{pmatrix} 1+j_{11} & j_{12} & j_{13} \\ j_{21} & 1+j_{22} & j_{23} \\ j_{31} & j_{32} & 1+j_{33} \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned} j_{11} = {} & \alpha_1 q_1^*[\theta(-2b-\frac{2d_1}{Q_T^*}+\frac{4d_1q_1^*}{Q_T^{*2}}-\frac{2d_1q_1^{*2}}{Q_T^{*3}}) \\ & +(1-\theta)\frac{2d_2q_2^{*2}}{Q_T^{*3}}], \\ j_{12} = {} & \alpha_1 q_1^*[\theta(-b+\frac{2d_1q_1^*}{Q_T^{*2}}-\frac{2d_1q_1^{*2}}{Q_T^{*3}}) \\ & +(1-\theta)(-b+\frac{2d_2q_2^*}{Q_T^{*2}}-\frac{2d_2q_2^{*2}}{Q_T^{*3}})], \\ j_{13} = {} & \alpha_1 q_1^*[\theta(-b+\frac{2d_1q_1^*}{Q_T^{*2}}-\frac{2d_1q_1^{*2}}{Q_T^{*3}})-(1-\theta)\frac{2d_2q_2^{*2}}{Q_T^{*3}}], \\ j_{21} = {} & \alpha_2 q_2^*[\theta(-b+\frac{2d_1q_1^*}{Q_T^{*2}}-\frac{2d_1q_1^{*2}}{Q_T^{*3}} \\ & +(1-\theta)(-b+\frac{2d_2q_2^*}{Q_T^{*2}}-\frac{2d_2q_2^{*2}}{Q_T^{*3}})], \\ j_{22} = {} & \alpha_2 q_2^*[-\theta\frac{2d_1q_1^{*2}}{Q_T^{*3}} \\ & +(1-\theta)(-2b-\frac{2d_2}{Q_T^*}+\frac{4d_2q_2^*}{Q_T^{*2}}-\frac{2d_2q_2^{*2}}{Q_T^{*3}})], \\ j_{23} = {} & \alpha_2 q_2^*[-\theta\frac{2d_1q_1^{*2}}{Q_T^{*3}}+(1-\theta)(-b+\frac{2d_2q_2^*}{Q_T^{*2}}-\frac{2d_2q_2^{*2}}{Q_T^{*3}})], \\ j_{31} = {} & j_{32} = \alpha_3 q_3^*(-b+\frac{2d_3q_3^*}{Q_T^{*2}}-\frac{2d_3q_3^{*2}}{Q_T^{*3}}), \\ j_{33} = {} & \alpha_3 q_3^*(-2b-\frac{2d_3}{Q_T^*}+\frac{4d_3q_3^*}{Q_T^{*2}}-\frac{2d_3q_3^{*2}}{Q_T^{*3}}). \end{aligned}$$

Furthermore, we can get the characteristic polynomial of system (6) at p_1

$$f(\lambda) = \lambda^3 + B_2\lambda^2 + B_1\lambda + B_0, \tag{9}$$

where

$$\begin{aligned} & B_2 = -(3+j_{11}+j_{22}+j_{33}), \\ & B_1 = -j_{12}j_{21}-j_{13}j_{31}-j_{23}j_{32}+(1+j_{22})(1+j_{33}) \\ & +(1+j_{11})(2+j_{22}+j_{33}), \\ & B_0 = j_{13}(1+j_{22})j_{31}-j_{12}j_{23}j_{31}+j_{12}j_{21}(1+j_{33}) \\ & -j_{13}j_{21}j_{32} \\ & -(1+j_{11})[(1+j_{22})(1+j_{33})-j_{23}j_{32}]. \end{aligned}$$

According to the Jury test [17], the necessary and sufficient condition of the local stability of Nash equilibrium is the following four conditions satisfied.

i) $f(1) = B_2 + B_1 + B_0 + 1 > 0$,

ii) $-f(-1) = -B_2 + B_1 - B_0 + 1 > 0$,

iii) $|B_0| < 1$,

iv) $|B_0^2 - 1| > |B_1 - B_2 B_0|$.

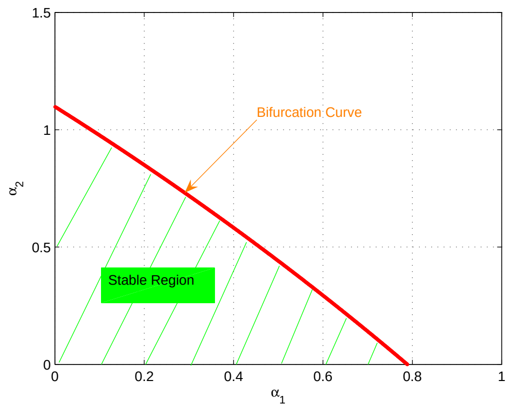

Figure 1: The stable region of Nash equilibrium point in the phase plane of (α_1, α_2), and $\alpha_3 = 0.12$

The local stable region of Nash equilibrium point can be obtained by solving the above equations. It is a bounded in the region of hyperbolic plane with positive $(\alpha_1, \alpha_2, \alpha_3)$. If α_3 held fixed, the stable region in the phase plane of (α_1, α_2) can be obtained, such as the stable region (α_1, α_2) is shown in Fig. 1 when $\alpha_3 = 0.412$. The Nash equilibrium is stable for the values of (α_1, α_2) inside the stable region. we can analogously obtain the stable region in the phase plane of (α_1, α_3) and (α_2, α_3) in Figs. 2 and 3 when α_2 and α_1 hold fixed, respectively. The meaning of the stable region is that all of the three renewable resource firms will eventually maintain at Nash equilibrium output after finite games whatever initial output are chosen in the local stable region. It is valuable to study that the enterprises increase the output adjustment speed

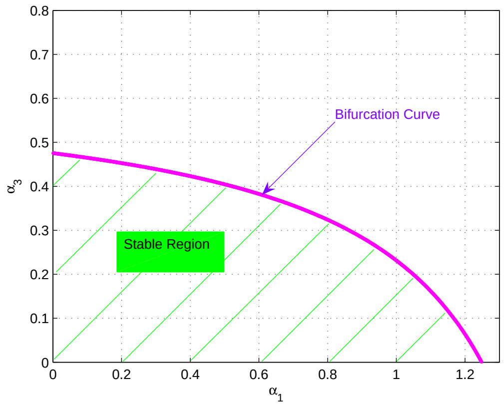

Figure 2: The stable region of Nash equilibrium point in the phase plane of (α_1, α_3), and $\alpha_2 = 0.1$

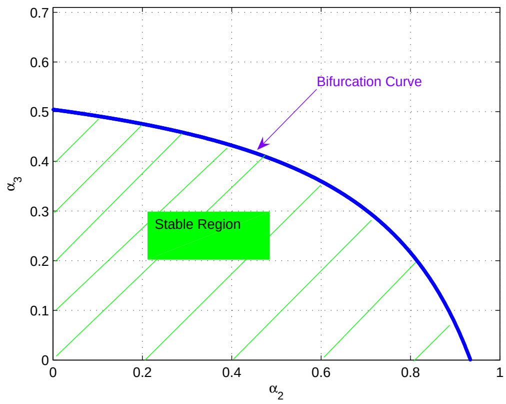

Figure 3: The stable region of Nash equilibrium point in the phase plane of (α_2, α_3), and $\alpha_1 = 0.46$

in order to get more profit. Though output adjustment speed parameters are unrelated to the Nash equilibrium point, the system will become unstable and even fall into chaos if one player adjusts output speed too fast and pushes $\alpha_i (i = 1, 2, 3)$ out of the stable region. Numerical simulation method is used to analyze the characteristics of system (6) with the change of $\alpha_i (i = 1, 2, 3)$. Numerical results such as bifurcation diagrams, strange attractors, the Lyapunov exponents, sensitive dependence on initial conditions and fractal structure will be discussed in the following section.

4 Dynamic characteristics of the system

In this game, the enterprises make the optimal output decision to get the maximum profit and adjust their

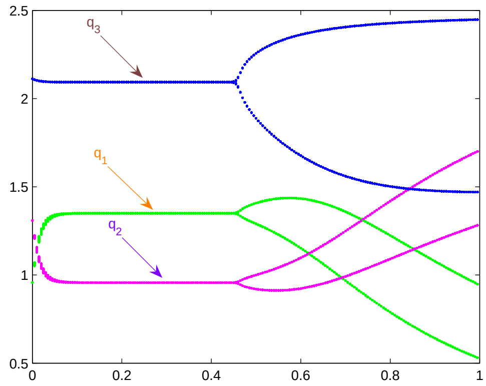

Figure 4: Bifurcation diagram of system (6) with $\alpha_1 \in (0, 1]$, and $(\alpha_2 = 0.476, \alpha_3 = 0.412)$

output based on the last period marginal profit. In the oligopoly market, the players have the driving force to increase their output in the hope of achieving more profits. The renewable resource firms can adjust their output speed to increase their output. So, the output adjustment speed parameter $\alpha_i (i = 1, 2, 3)$ affects the game results very much. The effect of output adjustment speed parameter $\alpha_i (i = 1, 2, 3)$ on system (6) will be analyzed in the following section.

4.1 The effect of output adjustment speed on the system

The stability of Nash equilibrium point will change if company X_1 accelerates output adjustment speed and pushes α_1 out of the stable region. Fig. 4 shows a one-parameter bifurcation diagram with respect to α_1 when $(\alpha_2 = 0.476, \alpha_3 = 0.412)$. For $\alpha_1 < 0.4615$, system (6) at the Nash equilibrium point, that is, the output of the three firms are in the equilibrium state. For $\alpha_1 = 0.4615$, system (6) undergoes 2 period doubling bifurcation.

Furthermore, when $(\alpha_2 = 0.476, \alpha_3 = 0.562)$, with output adjustment speed α_1 increasing, the output evolution of duopoly starts with 2 period orbits, through period doubling and ends with chaotic state. From the bifurcation diagram Fig. 5 and the corresponding Lyapunov exponents diagram Fig. 6, we can see $\alpha_1 \in (0, 0.3636]$ is the domain of 2 period orbits of system (6), $\alpha_1 \in (0.3636, 0.4825]$ is the domain of 4 period orbits of system (6), $\alpha_1 \in (0.4825, 0.5175]$ is the domain of 8 period orbits of system (6), $\alpha_1 \in (0.7343, 0.7552)$ is the domain of period orbits window of system (6), for $\alpha_1 \in (0.5175, 0.7343] \cup (0.7552, 1]$, system (6) is in a chaotic state. Calculation of the Lyapunov exponent is used to analyze the quantitative characteris-

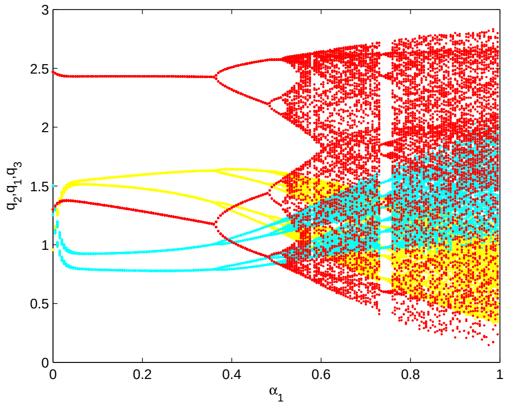

Figure 5: Bifurcation diagram of system (6) with $\alpha_1 \in (0,1]$, and $(\alpha_2 = 0.476, \alpha_3 = 0.562)$

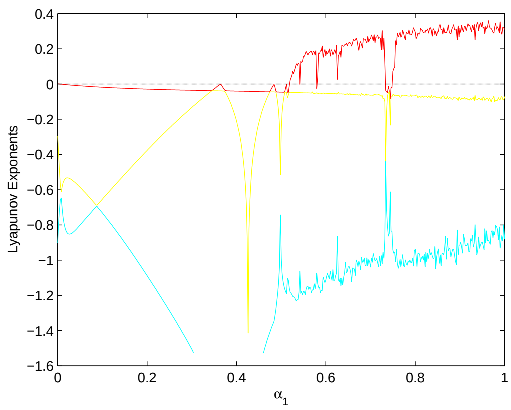

Figure 6: The corresponding Lyapunov exponents diagram with $\alpha_1 \in (0,1]$, and $(\alpha_2 = 0.476, \alpha_3 = 0.562)$

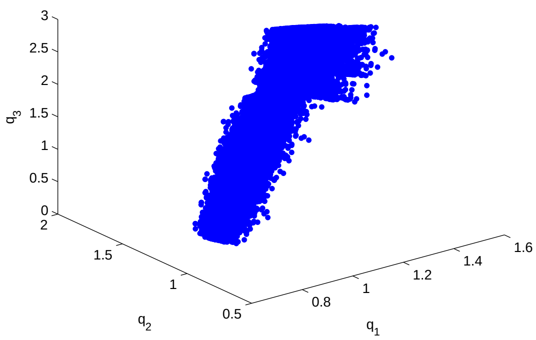

Figure 7: Chaos attractors of system (6) for $(\alpha_1 = 0.729, \alpha_2 = 0.476, \alpha_3 = 0.562)$

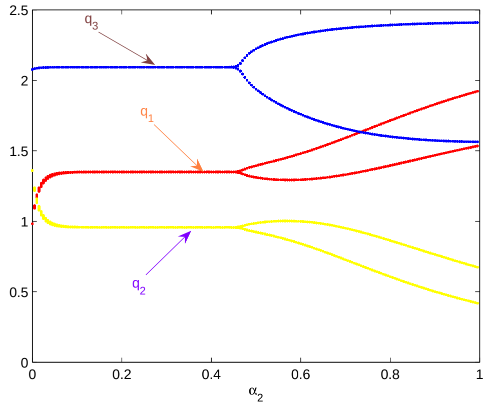

Figure 8: Bifurcation diagram of system (6) with $\alpha_2 \in (0,1]$, and $(\alpha_1 = 0.468, \alpha_3 = 0.412)$

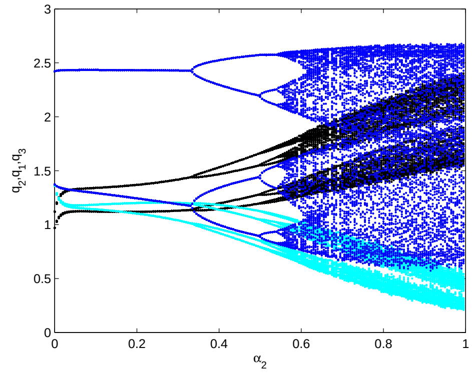

Figure 9: Bifurcation diagram of system (6) with $\alpha_2 \in (0,1]$, and $(\alpha_1 = 0.468, \alpha_3 = 0.562)$

tics of the dynamic system. The system is in chaotic state if the largest Lyapunov exponent is positive. In addition, the positive Lyapunov exponent is larger, the system is more obviously in chaotic state. Fig. 7 show the chaos attractors of system (6) at initial point $(q_{1_0} = 0.957, q_{2_0} = 1.36, q_{3_0} = 1.78)$ for $(\alpha_1 = 0.729, \alpha_2 = 0.476, \alpha_3 = 0.562)$.

Similarly, when $(\alpha_1 = 0.468, \alpha_3 = 412)$ and with the output α_2 output adjustment speed increasing, Fig. 8 shows the bifurcation diagram of system (6). We can see that for $\alpha_2 < 0.4615$, system (6) is stable at the Nash equilibrium point, for $\alpha_1 > 0.4615$, system (6) undergoes 2 period doubling bifurcation.

Furthermore, the bifurcation diagram Fig. 9 and the corresponding Lyapunov exponents diagram Fig. 10 show a one-parameter bifurcation diagram with respect to α_2 when $(\alpha_1 = 0.468, \alpha_3 = 0.562)$. We can see $\alpha_2 \in (0, 0.3357]$ is the domain of 2 period orbits of system (6), $\alpha_2 \in (0.3357, 0.50350]$ is the domain of 4 period orbits of system (6), $\alpha_2 \in (0.50350, 0.5455]$ is the domain of 8 period orbits of system

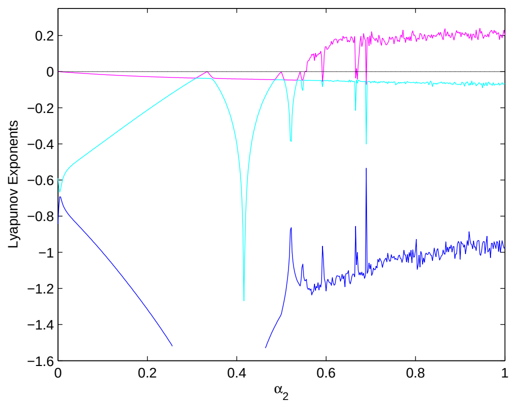

Figure 10: The corresponding Lyapunov exponents diagram with $\alpha_2 \in (0,1]$, and $(\alpha_1 = 0.468, \alpha_3 = 0.562)$

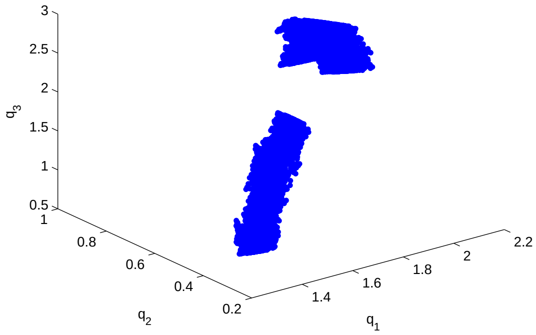

Figure 11: Chaos attractors of system (6) for $(\alpha_1 = 0.468, \alpha_2 = 0.792, \alpha_3 = 0.562)$

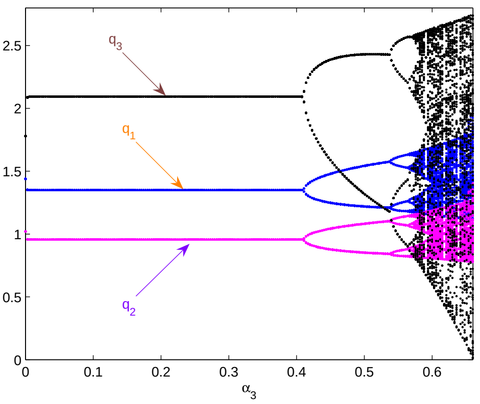

Figure 12: Bifurcation diagram of system (6) with $\alpha_3 \in (0, 0.6601]$, and $(\alpha_1 = 0.468, \alpha_2 = 0.476)$

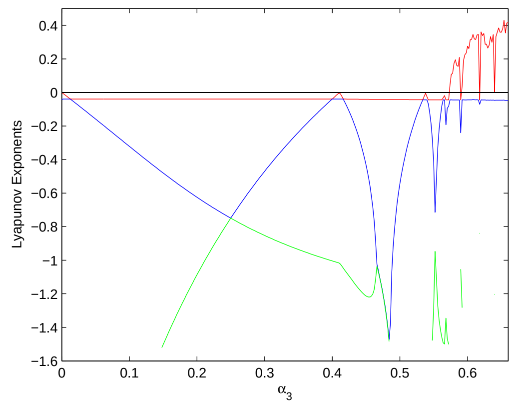

Figure 13: The corresponding Lyapunov exponents diagram with $\alpha_3 \in (0, 0.6601]$, and $(\alpha_1 = 0.468, \alpha_2 = 0.476)$

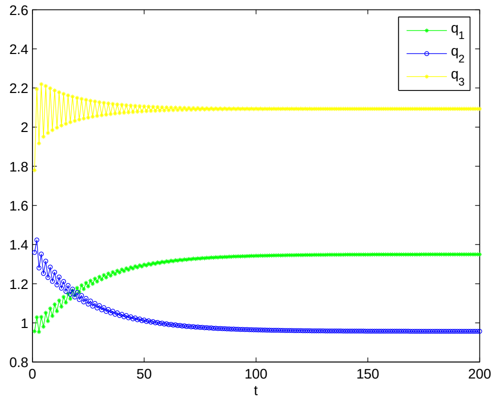

Figure 14: For $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.397)$, the time course diagram of system (6)

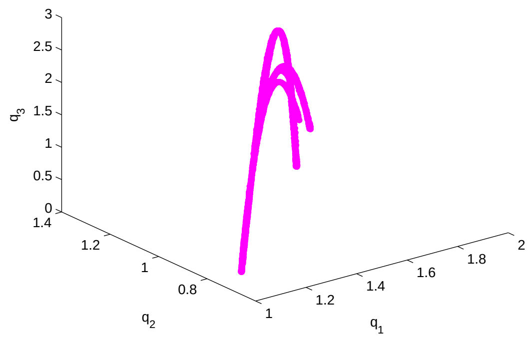

Figure 15: Chaos attractors of system (6) for $(\alpha_1 = 0.468, \alpha_2 = 476, \alpha_3 = 0.659)$

(6), $\alpha_2 \in (0.5455, 0.5594) \cup (0.6643, 0.6713]$ is the domain of period orbits window of system (6), for $\alpha_2 \in (0.5594, 0.6643] \cup (0.6713, 1]$, system (6) is in a chaotic state. Fig. 11 show the representative chaos attractors at initial point $(q_{1_0} = 0.957, q_{2_0} = 1.36, q_{3_0} = 1.78)$ for $(\alpha_1 = 0.468, \alpha_2 = 0.792, \alpha_3 = 0.562)$.

Likewise, the bifurcation diagram Fig. 12 and the corresponding Lyapunov exponents diagram Fig. 13 show a one-parameter bifurcation diagram with respect to α_3 when $(\alpha_1 = 0.468, \alpha_2 = 476)$. One can observe that Nash equilibrium point is asymptotically stable for $0 < \alpha_3 < 0.4108$, for example, the time course diagram of system (6) shows in Fig. 12 for $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.397)$. For $\alpha_3 \in (0.4108, 0.5401]$ is a area of 2-cycle output fluctuation. $\alpha_3 \in (0.5401, 0.5678]$ is a area of 4-cycle area of output fluctuation. $\alpha_3 \in (0.5678, 0.5724]$ is a area of 8-cycle output fluctuation. For $\alpha_2 \in (0.5724, 0.6601]$, system (6) becomes into a chaotic state. Figs. 11 show the representative chaos attractors at initial point $(q_{1_0} = 0.957, q_{2_0} = 1.36, q_{3_0} = 1.78)$ for $(\alpha_1 = 0.468, \alpha_2 = 476, \alpha_3 = 0.659)$.

Form the above analysis, we can find that the stability of system (6) will be changed and even the complex dynamic behaviors occur with increasing of the output adjustment speed $\alpha_i\,(i = 1, 2, 3)$.

4.2 The weight parameter θ affects on the system

In this section, we study the weight parameter θ which affects the dynamic behaviors of system (6). Similarly, the bifurcation diagram Fig. 16 and the corresponding Lyapunov exponents diagram Fig. 17 show a one-parameter bifurcation diagram with respect to θ when $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.394)$. One can see that for $0.454545 < \theta < 0.559441$ system (6) is stable at different Nash equilibrium points, that is to say, the triopoly renewable resource firms are in different equilibrium states. The output q_3 changes only a little, but the output q_1, q_2 change very large. In both sides of the stable domain, system (6) undergoes period bifurcations to chaos with the weight parameter θ decreasing and increasing, which have only dynamics meaning, but they don't have economic meaning.

For $\theta \in (0, 0.041958]$, the output $q_1 = 0$, but output the q_2, q_3 are in chaotic state. For $\theta \in (0.041958, 0.059440]$, the output $q_1 = 0$, but output the q_2, q_3 have 8-cycle output orbits. For $\theta \in (0.059440, 0.115385]$, the output $q_1 = 0$, but output the q_2, q_3 have 4-cycle output orbits. For $\theta \in (0.115385, 0.454545]$, the output $q_1 = 0$, but output the q_2, q_3 have 2-cycle output orbits. For $\theta \in (0.559441, 0.884615]$, the output $q_2 = 0$, but out-

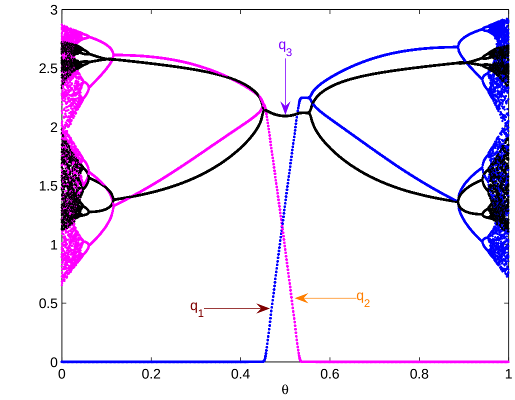

Figure 16: Bifurcation diagram of system (6) with $\theta \in (0, 1]$, and $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.394)$

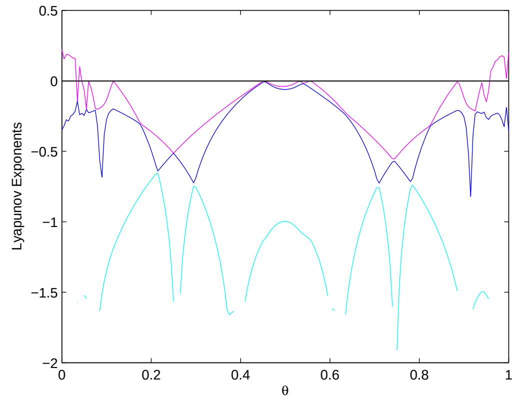

Figure 17: The corresponding Lyapunov exponents diagram with $\theta \in (0, 1]$, and $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.394)$

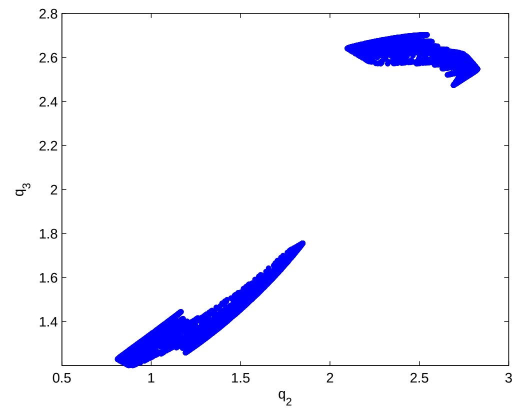

Figure 18: Chaos attractors of system (6) for $\theta = 0.0298$

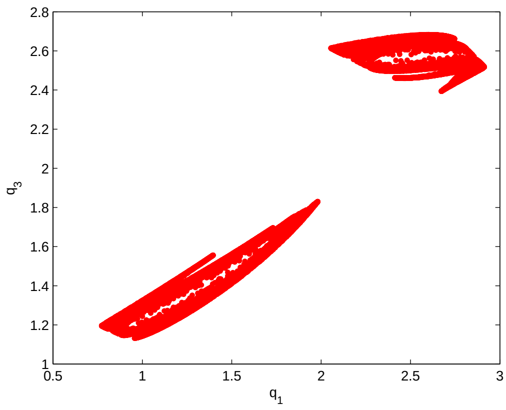

Figure 19: Chaos attractors of system (6) for $\theta = 0.985$

put the q_1, q_3 have 2-cycle output orbits. For $\theta \in (0.884615, 0.937063]$, the output $q_2 = 0$, but output the q_1, q_3 have 4-cycle output orbits. For $\theta \in (0.937063, 0.958042]$, the output $q_2 = 0$, but output the q_1, q_3 have 8-cycle output orbits. For $\theta \in (0.958042, 1]$, the output $q_2 = 0$, but output the q_1, q_3 are in chaotic state.

Figs. 18 and 19 show the representative chaos attractors at initial point $(q_{1_0} = 0.957, q_{2_0} = 1.36, q_{3_0} = 1.78)$ when $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.394)$, $\theta = 0.0298$ and $\theta = 0.985$, respectively.

Through the above analysis, we can see that the weight parameter θ not only affects on the stability of system (6) but also change the Nash equilibrium point. In both sides of the stable domain, system (6) undergoes period bifurcations to chaos with the weight parameter θ decreasing and increasing, which have no economic meaning.

4.3 Sensitive dependence on initial conditions

One of the most important characteristics of the chaos is extremely sensitive dependence on initial conditions. Figs. 20, 21, 22 respectively show the difference of the output q_1, q_2, q_3 with the change of time when system (6) has different initial conditions. We can see that there is almost no distinction between them in the beginning, but the difference between them is huge with the number of game increasing. It implies that a slight difference between initial values can lead to a great effect on the game results. For $(\alpha_1 = 0.625, \alpha_2 = 0.476, \alpha_3 = 0.562, \theta = 0.5)$, $(\alpha_1 = 0.468, \alpha_2 = 0.792, \alpha_3 = 0.562, \theta = 0.5)$ and $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.615, \theta = 0.5)$, it further proves that system (6) is in a chaotic state. When the system in a chaotic state, the market will be de-

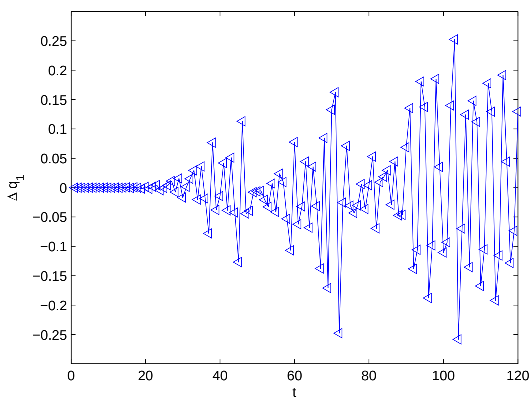

Figure 20: For $(\alpha_1 = 0.625, \alpha_2 = 0.476, \alpha_3 = 0.562, \theta = 0.5)$, and the initial conditions are (0.957, 1.36, 1.78) and (0.9571, 1.36, 1.78), difference of the output q_1 with the change of time

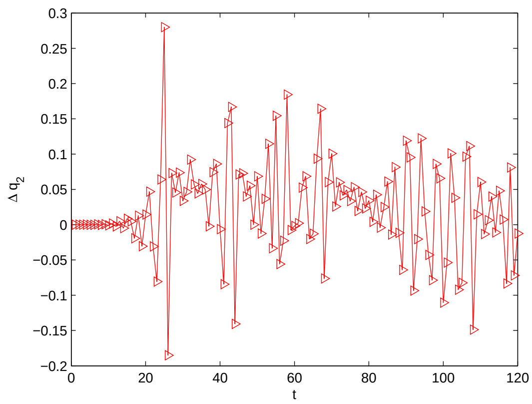

Figure 21: For $(\alpha_1 = 0.468, \alpha_2 = 0.792, \alpha_3 = 0.562, \theta = 0.5)$, and the initial conditions are (0.957, 1.36, 1.78) and (0.957, 1.3601, 1.78), difference of the output q_2 with the change of time

stroyed and it is difficult for the renewable resource companies to make long-term plan. So, every action from the renewable resource companies can cause great loss.

4.4 Fractal dimension

Fractal dimension can be used as another criterion to judge whether the system is in a chaotic state or not. There are many specific definitions of fractal dimension, but none of them can be taken as the universal one. According to Ref. [18], the following definition of fractal dimension is adopted in this paper.

$$d = j - \frac{\sum_1^j \lambda_i}{\lambda_{j+1}}, \tag{10}$$

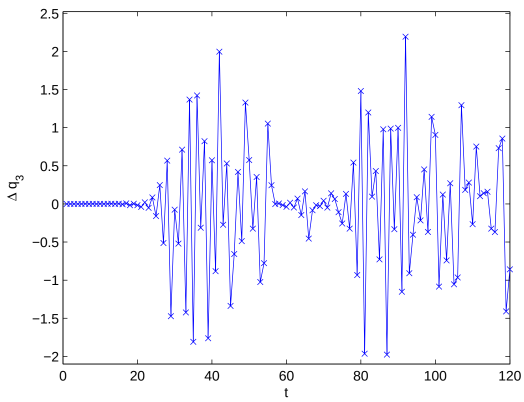

Figure 22: For $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.615, \theta = 0.5)$, and the initial conditions are (0.957, 1.36, 1.78) and (0.957, 1.36, 1.7801), the difference of the output q_3 with the change of time

where $\lambda_1 > \lambda_2 >, ..., \lambda_n$ are the Lyapunov exponents, and j is the maximum integer for which satisfies $\sum_1^j \lambda_i > 0$ and $\sum_1^{j+1} \lambda_i < 0$. If $\lambda_i \geq 0, i = 1, 2, ..., n$, the $d = n$. If $\lambda_i < 0, i = 1, 2, ..., n$, then $d = 0$.

The Lyapunov exponents of system (6) are $\lambda_1 = 0.2127, \lambda_2 = -0.0607, \lambda_3 = -1.0576$ for $(\alpha_1 = 0.468, \alpha_2 = 0.792, \alpha_3 = 0.562)$. System (6) is in a chaotic state because the largest Lyapunov exponent λ_1 is positive. Fractal dimension demonstrates that the chaotic motion has self-similar structure, which is an important difference between the chaotic motion and the stochastic motion. The fractal dimension of system (6) is $d = 2 - \frac{\lambda_1 + \lambda_2}{\lambda_3} \approx 2.1437$. The fractal dimension also reflects the space density of the chaotic attractor. The larger the dimension of the chaotic attractor is, the larger the occupied space is. Thus, the structure of the chaotic attractor is more compact, and the system is more complexity and vice versa. The fractal dimension of the 3D discrete system(6) is more than 2, so the occupied space is big and the structure is tight, which can be seen in Fig. 11.

5 Chaos control

One can see that system (6) becomes unstable and eventually falls into chaos if the output adjustment speed parameter exceeds a critical value. All of the three renewable resource firms will be harmed and the market will become irregular when chaos occurs. Therefor, no one is able to make good strategies and decide reasonable output. To avert risk, it is good ideal for the triopoly renewable resource firms to stay at Nash equilibrium output.

In this section, the linear feedback control method

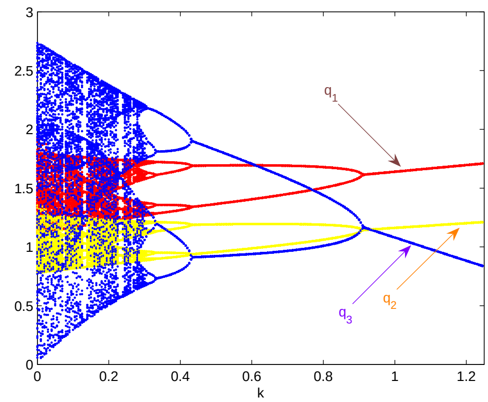

Figure 23: Bifurcation diagram of system (11) with $k \in (0, 1.25]$, $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.659)$

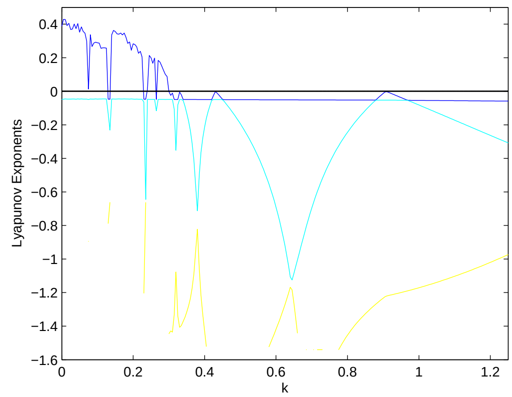

Figure 24: The corresponding Lyapunov exponents with $k \in (0, 1.25]$, $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.659)$

is used to control the effect of parameters α_3 on system (6). The system under controlled is as follows:

$$\begin{cases} q_1(t+1) = & q_1(t) + \alpha_1 q_1(t) A, \\ q_2(t+1) = & q_2(t) + \alpha_2 q_2(t) B, \\ q_3(t+1) = & q_3(t) + \alpha_3 q_3(t) C - k q_3(t). \end{cases} \tag{11}$$

where

$A = [\theta(a - bQ_T(t) - bq_1 - \frac{2d_1 q_1(t)}{Q_T(t)} + \frac{d_1 q_1^2(t)}{Q_T^2(t)}) + (1 - \theta)(-bq_2(t) + \frac{d_2 q_2^2(t)}{Q_T^2(t)})]$

$B = [\theta(-bq_1(t) + \frac{d_1 q_1^2(t)}{Q_T^2(t)}) + (1 - \theta)(a - bQ_T(t) - bq_2 - \frac{2d_2 q_2(t)}{Q_T(t)} + \frac{d_2 q_2^2(t)}{Q_T^2(t)})]$

$C = [a - bQ_T(t) - bq_3 - \frac{2d_3 q_3(t)}{Q_T(t)} + \frac{d_3 q_3^2(t)}{Q_T^2(t)}]$

where $k > 0$ is an adjustment parameter and other parameters are the same as above.

It can be seen from the bifurcation diagram Fig.

23 and the corresponding Lyapunov exponents path Fig. 24, for $(\alpha_1 = 0.468, \alpha_2 = 0.476, \alpha_3 = 0.659)$, controlled system (11) stabilized at Nash equilibrium point for $k > 0.9090$. It reveals that chaos control of the system (6) can realize through adding a negative linear feedback.

The purpose of system control is to take effective measures to regulate market behaviors for avoiding the occurrence of chaos. The linear feedback control method can make the system to regain equilibrium while the system (6) in a chaotic state, and effectively control the bifurcation and chaos behaviors of the system. This method can ensure that the renewable resource market orderly develop, and the renewable resource firms rationally and healthily compete.

6 Conclusions

A new dynamics of a nonlinear triopoly game with one bounded rational player and two team players in exploitation of renewable resource is built up in this paper. The stability of the Nash equilibrium point, the bifurcation and chaotic behaviors of the dynamic system are investigated. We found that bifurcation and chaos occur as the output adjustment speed parameter $\alpha_i (i = 1, 2, 3)$ increases and the weight parameter θ changes. The weight parameter θ not only affects the stability of system (6) but also changes the Nash equilibrium point. The oligopoly market may become unstable and even fall into chaos when the output adjustment speed parameters leave the stable region. The linear feedback control method is used to control the complexity of the system, which makes the system to regain stable states. It has a very theoretical and practical significance to research the complexity of new nonlinear dynamical system. As the traditional energy is non-renewable, it will be depleted in a few couple of years. In this way, the development of renewable resource is an inevitable choice. This paper also shows a guidance for the renewable resource companies to make strategies of their output and exploit the renewable resource, and it is helpful for the government to formulate relevant policies to manage the renewable resource market.

Acknowledgements: The research was supported by the National Natural Science Foundation of China (No. 61273231), Doctoral Fund of Ministry of Education of China (Grant No. 20130032110073)

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