PARAMETER OPTIMIZATION OF AUTOMOTIVE SUSPENSION SYSTEM BASED ON PARTICLE SWARM ALGORITHM

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Abstract: – In this paper, the dynamic model of suspension system and road incentive model are established. The transfer functions of the parameters for the suspension system are derived, and they are quantized objectively. Then, taking the body vibration acceleration as the objective function, the suspension dynamic deflection and tire dynamic load as the main constraint condition, and the stiffness and damping of the front and rear suspension as optimization variables, the suspension is optimized with a particle swarm optimization algorithm, whose results are compared with the neighborhood transfer genetic algorithm. The optimization results show that the vehicle vibration acceleration is significantly reduced after optimization, and the riding comfort of vehicle and comfort of occupant are improved effectively.

Key-Words: - Vehicle, Suspension system, Parameter optimization, Particle swarm algorithm

1 Introduction

The suspension system is the general term of all power transmission device between the frame (or frameless body) and the axle (or wheel), its main function is to transmit the reaction force of vertical, longitudinal and lateral direction generated by the external factors, as well as the torque formed by these forces to the frame or frameless body. Thus, it can reduce impact and vibration of auto body, and improve the vehicle ride comfort. Meanwhile, the suspension system can adjust the body posture timely in the vehicle roll or pitch, to ensure the normal driving of vehicles, and improve the handling stability. Therefore, the design quality of suspension system has great influence on the comfort and safety of the vehicle.

In recent years, a lot of researches on the control method of the suspension system have been done by the scholars (Crews *et al.*, 2011; Unger *et al.*, 2013; Fei and Xin, 2012; Zheng *et al.*, 2010), involving the control strategies of the suspension spring stiffness and shock absorber damping adjustment, and so on (Riahi and Balochian, 2012; Kaldas and Kemal, 2011; Wang, *et al.*, 2009; Li and Zheng, 2005). Taking the vehicle ride comfort

as control objectives, many new control strategies are applied to the research of the suspension system, such as the skyhook damping control, optimal control, adaptive control, neural network control, sliding mode variable structure control, and fuzzy control (Li *et al.*,2014; Fang *et al.*, 2011; Soleymani, 2012). Although related research on the suspension system has become more and more fierce, but the parameter optimization of suspension system is still rarely reported.

However, the parameter optimization is not only the foundation of the system design, but also the key factors that influence the suspension performance. Therefore, taking the passive suspension as the research object, the parameters of the system are optimized in this paper. The dynamic model of suspension system and road incentive model are established. The performance function of three design indicators for the body acceleration, suspension dynamic deflection and tire dynamic load are proposed, and their performance is analyzed. Then, the suspension system is optimized based on the dynamic particle swarm optimization, which will provide a theoretical basis and technical support for the

design and development of vehicle suspension system.

2 Dynamic Model

2.1 Dynamic model of the suspension system It is generally considered that the car is symmetrical about its longitudinal axis, and the roughness functions of the two wheels are equal. In this case, the car only the vertical and pitching motion, and the complex suspension can be simplified as a 4 DOF plane model, as shown in figure 1.



Figure 1. The suspension model.

The suspention model can be depicted as:

$$\begin{cases} (m_{2f} + m_{2c}b^{2}/L^{2})\ddot{z}_{2f} + K_{2f}(z_{2f} - z_{1f}) + C_{2f}(\dot{z}_{2f} - \dot{z}_{1f}) + (m_{2c}ab/L^{2})\ddot{z}_{2r} = 0 \\ (m_{2r} + m_{2c}b^{2}/L^{2})\ddot{z}_{2r} + K_{2r}(z_{2r} - z_{1r}) + C_{2r}(\dot{z}_{2r} - \dot{z}_{1r}) + (m_{2c}ab/L^{2})\ddot{z}_{2r} = 0 \\ m_{1f}\ddot{z}_{1f} - K_{2f}(z_{2f} - z_{1f}) - C_{2f}(\dot{z}_{2f} - \dot{z}_{1f}) + K_{1f}(z_{1f} - q_{f}) = 0 \\ m_{rr}\ddot{z}_{1r} - K_{2r}(z_{2r} - z_{1r}) - C_{2r}(\dot{z}_{2r} - \dot{z}_{1r}) + K_{1r}(z_{1r} - q_{r}) = 0 \end{cases}$$
(1)

where $f_c \sim m_{2r} \sim m_{1r} \sim m_{1r} \sim m_{2c}$ are the body quality distribution at the front axle, rear axle, front wheel, rear wheel and center of mass, respectively; z_{1f} , z_{1r} , z_{2f} , z_{2r} are the displacement of the front wheel, rear wheel, the front axle of the body and the rear axle of the body, respectively; \dot{z}_{1f} , \dot{z}_{2f} , \dot{z}_{1r} , \dot{z}_{2r} are their corresponding velocity; $\ddot{\mathbf{z}}_{1f}$, $\ddot{\mathbf{z}}_{2f}$, $\ddot{\mathbf{z}}_{1r}$, $\ddot{\mathbf{z}}_{2r}$ are their corresponding acceleration; $\phi = \frac{z_{2f} - z_{2r}}{I}$ is the body pitch angle; $\ddot{\phi}$ is the body pitch acceleration; $K_{1f} \sim K_{1r} \sim K_{2f} \sim K_{2r}$ are the stiffness of the frontwheel, rear-wheel, front and rear suspension spring, respectively; L is the wheelbase; $a \, b$ are the distance from the front and rear axle to the center of mass, respectively; C_{2f} , C_{2f} are the damping coefficient of the front and rear suspension, respectively; $q_r \sim q_r$ are the displacement input of the front and rear road roughness, respectively.

 $X = \begin{bmatrix} \dot{z}_{2f} & z_{2f} - z_{1f} & \dot{z}_{1f} & z_{1f} - q_f & \dot{z}_{2r} & z_{2r} - z_{1r} & \dot{z}_{1r} & z_{1r} - q_r \end{bmatrix}^{T}$ is selected as the state variable vector, and $U = \begin{bmatrix} \dot{q}_{f} & \dot{q}_{r} \end{bmatrix}^{T}$ is defined as the input vector, where \dot{q}_{f} and \dot{q}_{r} are the velocity input of the front and rear road roughness. $Y = \begin{bmatrix} \ddot{z}_{2f} & z_{2f} - z_{1f} & K_{1f}(z_{1f} - q_{f}) & \ddot{z}_{2r} & z_{2r} - z_{1r} & K_{1r}(z_{1r} - q_{r}) & \ddot{\phi} \end{bmatrix}^{T}$ is defined as the output vector, where $z_{2f} - z_{1f}$ and $z_{2r} - z_{1r}$ are the dynamic deflection of the front and rear suspension, and $K_{1f}(z_{1f} - q_{f})$, $K_{1r}(z_{1r} - q_{r})$ are the dynamic load of the front and rear tire, respectively.

The dynamic differential equations are transformed into the state equation and output equation as follows:

$$\begin{array}{c} X = AX + BU \\ Y = CX \end{array}$$
(2)

where

2.2 Road incentive model

The road roughness is often used as the vehicle vibration input, whose statistical properties are mainly described by the road power spectral density.

The road input to wheel is filtered white noise, that is:

$$\dot{q} = -2\pi f_0 q + 2\pi \sqrt{G_0 u} w(t) \tag{3}$$

where f_0 is the lower cutoff frequency; G_0 is the road roughness; $\omega(t)$ is the Gauss white noise of zero mean.

3 System Performance Analysis

Considering the body acceleration, suspension dynamic deflection and tire dynamic load, the evaluation indicators of the suspension system are focused on the vehicle ride comfort, which involve the structural parameters of the suspension stiffness, the damping, and so on.

Assuming that the system adopts the road roughness \dot{q} as the input of the road model, the suspension dynamic model is transformed by Laplace method. Then the transfer function from the road input to the body acceleration, suspension dynamic deflection and wheel dynamic load can be obtained.

The transfer function of the vehicle vibration acceleration can be expressed as:

$$H_{\bar{z}_{2},\bar{\gamma}\bar{q}} = \frac{\ddot{Z}_{22}(s)}{\dot{q}(s)}$$

$$= \frac{c_{22}k_{13}s^{2} + k_{13}k_{22}s}{m_{12}m_{22}s^{4} + (m_{11} + m_{21})c_{21}s^{3} + (k_{11}m_{21} + k_{22}m_{11} + k_{22}m_{21})s^{2} + c_{22}k_{13}s + k_{13}k_{22}} \quad (4)$$

$$(i=f,r)$$

The transfer function of the suspension dynamic deflection can be expressed as:

$$H_{(z_{2}-z_{1})-\dot{q}} = \frac{z_{2}(s) - z_{1}(s)}{\dot{q}(s)} = \frac{-k_{1}m_{2}s}{m_{1}m_{2}s^{4} + (m_{1} + m_{2})c_{2}s^{3} + (k_{1}m_{2} + k_{2}m_{1} + k_{2}m_{2})s^{2} + c_{2}k_{1}s + k_{1}k_{2}}$$
(5)
(i=f,r)

The transfer function of the wheel dynamic load can be written as:

$$H_{F_{2}/G-q} = \frac{K_{u}(z_{u}(s) - q_{i}(s))}{Gq_{i}(s)} = \frac{-k_{u}[m_{u}m_{2}s^{4} + (m_{u} + m_{2})c_{2}s^{3} + (k_{2}m_{u} + k_{2}m_{2})s^{2}]}{(m_{i} + m_{i})g[m_{u}m_{2}s^{4} + (m_{u} + m_{2})c_{2}s^{3} + (k_{u}m_{u} + k_{2}m_{u})s^{2} + c_{2}k_{u}s + k_{u}k_{2}]}$$

$$(i=f,r)$$

4 Parameter Optimization of the Suspension System

4.1 Optimization model

4.1.1 Design variable

Because some parameters of the vehicle can't be chosen, the front suspension stiffness k_{2f} and rear suspension stiffness k_{2r} , the front suspension

damping coefficient c_{2f} , and rear suspension c_{2r} damping coefficient are taken as the optimization variables, that is, $X = (k_{2f}, k_{2r}, c_{2f}, c_{2r})$. Its upper and

lowerlimitsare $X_{max} = (55548.144, 54524.862, 3552.29, 3237.28),<math>X_{min} = (18516.048, 13374.954, 2527.35, 2312.34),$

initial value is X = (45480.0, 52290.0, 2546.5, 2840.6).

4.1.2 Objective function

Taking the frequency domain energy of the front and rear suspension vibration acceleration as the optimization objective function, the system is optimized with the frequency domain energy of the dynamic deflection and relative dynamic load as constraint condition. With all constraint conditions, the frequency domain energy of the front and rear suspension vibration acceleration can be selected as small as possible.

4.1.3. Constraints

1) Stiffness constraint

The static deflection of the suspension f_c is the ratio of the load F_w and the stiffness k_2 with the vehicle in full load and static conditions, that is, $f_c = F_w / k_2$. When the spring of the suspension is a linear spring, the static deflection of the front and rear suspension can be calculated by the following formula:

$$f_{cf} = m_{2f}g / k_{2f} \; ; \; f_{cr} = m_{2r}g / k_{2r}$$
(7)

where m_{2f} and m_{2f} are the body quality distribution at the front axle and rear axle, respectively; $k_{2f} \\ k_{2r}$ are the stiffness of the front and rear suspension spring, respectively; g is the acceleration of gravity.

The suspension static deflection f_c generally ranges from 10 to 30 cm, thus, the stiffness constrains of the front and rear suspension can be depicted as:

$$\begin{cases} 0.1 \le m_{2f}g/k_{2f} \le 0.3\\ 0.1 \le m_{2r}g/k_{2r} \le 0.3 \end{cases}$$
(8)

In the design of the front and rear suspension static deflection, f_{cf} and f_{cr} are always expected to be close to each other. Moreover, they can't be equal in order to prevent resonance. Usually, f_{cr} is set as $(0.6 \sim 0.8) f_{cf}$. Thus, it can be got as follows:

$$0.6 \le \frac{m_{2r}g / k_{2r}}{m_{2f}g / k_{2f}} \le 0.8$$
(9)

2) Damping constraints

The relative damping coefficient ξ is often used to evaluate the speed of the vibration attenuation. For the average elastic element, ξ is always chosen as 0.25~0.35. Thus, the range of damping can be given as:

$$\begin{cases} 0.25 \le \frac{c_{2f}}{2\sqrt{k_{2f}m_{2f}}} \le 0.35 \\ 0.25 \le \frac{c_{2r}}{2\sqrt{k_{2r}m_{2r}}} \le 0.35 \end{cases}$$
(10)

The limit stroke $[f_d]$ refers to the distance traveled from the equilibrium position to the maximum allowable deformation of the elastic element in the case of full load, and it is generally selected as 7~9cm. In this paper, $[f_d]$ is set as 7 cm. Thus, the front and rear suspension dynamic deflection should be met the following formulas:

$$\begin{cases} \sigma_{f_{d} \sim \dot{q}} \leq \left| \sigma_{f_{d} \sim \dot{q}} \right| = [f_{fd}]/3 \\ \sigma_{r_{f_{d}} \sim \dot{q}} \leq \left| \sigma_{f_{d} \sim \dot{q}} \right| = [f_{rd}]/3 \end{cases}$$
(11)

where $\sigma_{f_{d}\sim \dot{q}}$ and $\sigma_{r_{f_{d}\sim \dot{q}}}$ are the root mean square value of dynamic deflection of the front and rear suspension, respectively; $[f_{fd}]$ and $[f_{rd}]$ are the limit travel of the dynamic deflection of the front and rear suspension, respectively.

The dynamic load F_d between the wheels and ground is constantly alternating. When F_d is equal and opposite to the static load G, the wheels will jump off the ground, causing the vehicle to lose longitudinal and lateral force, and the safety of the vehicle will get serious deterioration. Therefore, the relative dynamic load of the front and rear tires should be satisfied with the following formulas:

$$\begin{cases} \sigma_{jF_d/G} \le 1/3 \\ \sigma_{rF_d/G} \le 1/3 \end{cases}$$
(12)

where $\sigma_{fF_d/G}$ and $\sigma_{rF_d/G}$ are the relative dynamic load of the front and rear tires, respectively.

4.2. Optimization algorithm design

Particle Swarm Optimization (PSO) is based on the group composed of particles, and the solution for each optimization problem is to find a particle in the feasible space. In order to allow the particles searched in the global scope and maintain the diversity, Dynamic Particle Swarm Optimization (DPSO) is used to optimize the suspension system.

If the D-dimensional space position vector for PSO is $x_i = (x_{i1}, x_{i2}, ..., x_{iD})$, and x_i represents a potentially feasible solution in the solution space. Then, it can be estimated whether it is the optimal solution according to the adaptive value calculated by the objective function. The D-dimensional space velocity vector of the *i* particle is $v_i = (v_{i1}, v_{i2}, ..., v_{iD})$, and its optimum position is $P_i = (P_{i1}, P_{i2}, ..., P_{iD})$. The optimal position of the particle swarm groups is $L_i = (L_{i1}, L_{i2}, ..., L_{iD})$, and their global optimal position is $G = (G_1, G_2, ..., G_D)$. The iterative formula can be expressed as follows:

$$v_{i}(t+1) = \omega v_{i}t + b_{1}r_{1}(p_{i}(t) - x_{i}(t)) + b_{2}r_{2}(L_{i}(t) - x_{i}(t)) + b_{2}r_{3}(G(t) - x_{i}(t))$$
(13)

where $b_1 \,\,,\,\, b_2 \,\,,\,\, b_3$ are the positive numbers; $r_1 \,\,,\,\, r_2 \,\,,\,\, r_3$ are the random numbers in [0,1]; The parameter ω is the inertia factor.

Suppose that ω decreases from ω_s to ω_e according to the number of cycles, and the maximum number of cycles is I_{max} , and the current number of cycles is I_c . Then, ω can be given as follows:

$$\omega = \omega_s - (\omega_s - \omega_e) \frac{I_c}{I_{\max}}$$
(14)

where ω_s and ω_e are the inertia factor of the initial and eventual optimization, respectively.

The position of the particles at the time t+1 can be got as follows:

$$x_{i}(t+1) = x_{i}(t) + v_{i}(t+1)$$
(15)

If particle swarm exceed the domain boundaries after being updated, the position of the particles should be re-adjusted so that it falls within the decision space. The new location can be calculated according to the following formula:

$$x_{i}(t+1) = x_{i}(t) + \lambda v_{i}(t+1)$$
(16)

$$\lambda = 2/(\gamma^2 + 2) \tag{17}$$

where λ is the speed adjustment coefficient in (0 1). γ is the number of adjustments; when γ 3, the particle velocity becomes reverse.

The distance $||x_i - x_k||$ between the particles i and k can be calculated by the following formula

$$\|x_{i} - x_{k}\| = \sqrt{\left(\sum_{i=1}^{D} \left(x_{iL} - x_{kL}\right)^{2}\right)/d}$$
(18)

where d is the dimension of the decision variable

The generation of the dynamic particle swarm M is the generated particle swarm, and partiswarm a is the nearest to particle swarm b, and distance between them is more than D_{max} , then particle swarm x_{M+1} needs to be generated. The dimensional component x_{M+1}^{ik} of the number particle can be got as follows:

$$x_{M+1}^{ik} = \left(x_a^{ik} + x_b^{ik}\right) / 2 + c_1 \left(-1\right)^{\text{round}(0.5+c_2)} \left|x_a^{ik} - x_b^{ik}\right| / 2 \quad (19)$$

where c_1 , c_2 are the random numbers in [0,1]; round (•) is the integral function, thus round $(0.5+c_2)$ is 0 or 1.

5 Simulation Analysis

The corresponding particle swarm algorithm for the variables, objective function and constraints is designed, and the C ++ program is written. Then it is solved with ISIGHT software. The optimization results based on particle swarm optimization are compared with the Neighborhood Cultivation Genetic Algorithm (NCGA). Table 1 shows the comparison result of the parameter variables and performance indicators before and after optimization.

Table 1. Comparison result before and after optimization.

Parameter	Symbol	Before optimization	NCGA optimization	DPSO optimization
Front suspension stiffness	k_{2f}	45480.0	28761.0639	18516.0480

), ne	Front suspension damping coefficient	C_{2f}	2546.5	2546.6509	2795.1340
	Rear suspension damping coefficient	C _{2r}	2840.6	2323.3335	2481.6911
1.2)	Frequency- domain energy of front axle vibration acceleration	$\sigma^{\!2}(\ddot{Z}_{2f})$	0.280000	0.1607	0.1233
18)	Frequency- domain energy of rear axle vibration acceleration	$\sigma^{\!2}(\ddot{Z}_{2r})$	0.500276	0.2843	0.1702
: if cle the	Frequency- domain energy of front suspension dynamic deflection	$\sigma^2(\delta_{ m df})$	3.26E-4	2.93E-4	2.53E-4
na k- ri	Frequency- domain energy of rear suspension dynamic deflection	$\sigma^2(\delta_{ m dr})$	2.52E-4	2.66E-4	2.2E-4
19)	Frequency- domain energy of relative dynamic load of front wheel	$\sigma^2(F_{\rm df}/{ m G})$	0.004129	0.0023	0.0017

Rear

suspension stiffness

Frequency domain

energy of

relative

dvnamic load of rear

wheel

 $\sigma^2(F_{\rm dr}/{\rm G})$

33698.0818

17741.7304

0.0024

52290.0

k_{2r}

The suspension system is analyzed with Matlab software, and the simulation results are shown in figure 2 to figure 7. Figures 2 and figure 3 are the step response and Bode diagram of the front and rear axle vibration acceleration to the road input. It can be seen from the two graphs that the front and rear axle vibration acceleration is decreased by 56.0% and 66% with DPSO, and 42.6% and 43.2% with NCGA, than without optimization. Furthermore, compared to NCGA, the response speed of the system with DPSO is faster, and its amplitude is smaller. Thus the system can tend to a steady state faster. In addition, the amplitude and phase frequency characteristics of the system are improved as well.

0.006787

0.0039





(b) Bode diagram Figure 2. Response of front axle vibration acceleration to road input.





Figure 3. Response of rear axle vibration acceleration to road input.

Figures 4 and figure 5 are the step response and Bode diagram of the front and rear suspension dynamic deflection to the road input. It can be seen that the front and rear suspension dynamic deflection is decreased by 10.1% and 22.4% respectively, based on DPSO and NCGA. In comparison with NCGA, the system with DPSO has a smaller overshoot, and a faster response speed. Thus the optimized system can quickly reach a stable state, and the phase delay is smaller as well.





Figure 4. Response of front suspension dynamic deflection to road input.



Figure 6 and figure 7 are the step response and Bode diagram of the relative dynamic load of the front and rear wheel to the road input. As can be seen from the two graphs that the relative dynamic load of the front and rear wheel is reduced by 58.8% and 64.6% with DPSO, and 44.3% and 42.5% with NCGA, than without optimization.

converge to a steady state. In summary, the performance of the optimized suspension system is significantly better than that without optimization. Compared with NCGA, the suspension system based on DPSO can better improve the ride comfort of the vehicle, and further enhance the comfort of the occupant.

Compared with NCGA, the amplitude with DPSO

is decreased more significantly, and it can quickly



Figure 6. Response of front wheel relative dynamic load to road input.

Figure 5. Response of rear suspension dynamic deflection to road input.



C. Y. Wang, Z. J. Xu, W. Z. Zhao

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Figure 7. Response of rear wheel relative dynamic load to road input.

6 Conclusion

The dynamic model of the suspension system and road incentive model are established. The transfer functions of the parameters for the suspension system are derived, and they are quantized objectively. Then, Then, taking the body vibration acceleration as the objective function, the suspension dynamic deflection and tire dynamic load as the main constraint condition, and the stiffness and damping of the front and rear optimization suspension as variables. the suspension is optimized with DPSO, whose results are compared with NCGA. The optimization results show that the vehicle vibration acceleration is significantly reduced after optimization, and the riding comfort of vehicle and comfort of occupant are improved effectively.