Virtual Power Control and Sinusoidal Current Operation for Grid-Connected Inverter under Unbalanced Grid Voltages Conditions

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Abstract:- Power oscillation and current quality are the important performance targets for the grid-connected inverter under unbalanced and distorted grid voltages. In this work, the inherent reason for the current harmonic and power oscillations of the inverter are discussed. Then away from the positive and negative sequences analysis, novel strategy to determine the harmonic components of the grid currents during constant power control and to obtain relations for the power oscillations during sinusoidal current control are introduced. Based on the introduced analysis, a power control strategy is suggested. The proposed power control strategy uses a virtual power as a controlled variable. The virtual power depends on virtual grid voltages which can be extracted from actual grid voltages by using second order generalized integrator. By controlling the virtual power, sinusoidal grid currents with minimum THD is obtained under unbalanced and distorted grid voltages. For constant power operations, the actual power is used as a controlled variable. Based on an adaptive parameter, flexible control technique is used to switch between sinusoidal current control and constant power control operations. Simulation studies are carried out to confirm the effectiveness of the proposed controller to obtain constant power/sinusoidal current control operation under unbalanced and distorted grid voltages and to confirm the correctness of the introduced equations to calculate the grid current harmonic components and the power oscillations under unbalanced grid voltages.

Key-words: - Grid-connected VSI, unbalanced grid voltages, Power oscillations, Current harmonics, Renewable energy.

1 Introduction

The main control solutions for grid-connected voltage source inverter (VSI) are the vector oriented control (VOC) and direct power control (DPC) [1]. VOC decomposes the grid currents into active and reactive power components in synchronous frame, which are regulated by inner current control loops. The obtained inverter voltage vector references are subsequently synthesized by space voltage modulation (SVM). During balanced grid voltages, good steady state performance and quick response can be obtained in VOC, but it requires fine tuning work and accurate system parameters [2]. DPC has many obvious advantages under balanced grid voltages operations, such as its simple structure and rapid dynamic response, but it has disadvantages of high and variable switching frequency operation [3]. However, the actual grid voltages are usually unbalanced due to various grid faults and/or disturbances, especially in the weak grid system, which is common in renewable energy systems [4]. These unbalanced grid voltages influence the performance of the VSIs. Under unbalanced voltage of the grid, many efforts have been done to improve the grid-connected-VSI performance [5–11]. Different improved VOC and DPC strategies were proposed. Most of them need positive and negative sequence decomposition of voltages and currents components which needs long and tedious computations. Various strategies based on VOC have been proposed to cope with the voltage unbalances. In [12], to achieve constant active power, the current reference is derived and regulated in positive sequence synchronous frame. However, the control of the negative sequence current is insufficient due to the absence of negative sequence current controller. To achieve adequate control of both positive and negative sequence currents, a dual current control scheme is proposed in [13] to regulate them in their respective synchronous frame. Four proportional integral (PI) controllers and two notch filters are required, which imposes much tuning work and increases the computational burden. To reduce the control complexity of dual current controller, some improved methods have been proposed by using PI plus resonant controller [14], but they still require positive sequence and negative sequence extraction of grid voltages. To achieve symmetrical and sinusoidal grid currents, a power compensation block is added to the reference power of conventional DPC, which keeps the philosophy of DPC [15]. However, it needs a positive sequence voltage angle and positive/negative sequence extraction of grid voltage and currents. The power compensation block is simplified in [1] by eliminating the extraction of negative sequence current and three selective targets are implemented. Unfortunately, it still requires the extraction of negative sequence grid voltages and positive sequence grid currents to obtain the power compensation [2], [16].

In this work, the inherent reason for the current harmonic and power oscillations of the inverter is discussed with a quantitative analysis. Second, a new control strategy is proposed to achieve the coordinate control of power and current quality. The power control strategy depends on virtual power and virtual grid voltages. The proposed power control uses the virtual power as a controlled variable instead of the actual power to obtain sinusoidal balanced grid currents. The introduced solution is very simple owing to the elimination of positive/negative sequence extraction of grid voltages/currents, current reference calculation or power compensation block, as required in prior VOC-based or DPC-based solutions.

2 Problem Analysis

A photovoltaic grid connected system which consists of three phase two-level inverter connected to the utility grid via 3-phase inductance is shown in Fig. 1. Under unbalanced grid voltages operations and away from the positive and negative sequences analysis, the next section introduces novel strategy to determine the harmonic components of the grid currents during constant power control and to obtain relations for the power oscillations during sinusoidal current control.

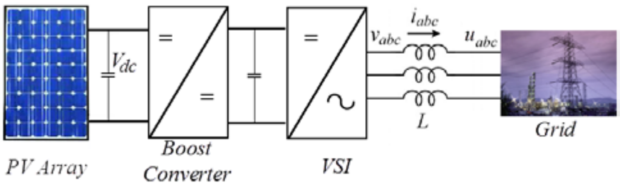

Fig.1 PV grid connected system.

The folowing analysis uses the fundamental component of phase-a as a reference such as:

$$u_a = U_a \sin\omega t \quad (1)$$

Assuming the unbalanced grid voltages have the following forms:

$$\begin{aligned} u_\alpha &= U_\alpha \sin(\omega t + \varphi_\alpha) \\ u_\beta &= -U_\beta \cos(\omega t + \varphi_\beta) \end{aligned} \quad (2)$$

Assuming the fundamental components of the grid currents are $i_{\alpha 1}$ and $i_{\beta 1}$, while the harmonics components are $i_{\alpha h}$ and $i_{\beta h}$ Then:

$$\begin{aligned} i_\alpha &= I_1 \sin(\omega t + \varphi) + i_{\alpha h} \\ i_\beta &= -I_1 \cos(\omega t + \varphi) + i_{\beta h} \end{aligned} \quad (3)$$

According to instantaneous power theory, the active and reactive power for three phase system can be expressed as follows:

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \frac{3}{2} \begin{bmatrix} u_\alpha & u_\beta \\ u_\beta & -u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (4)$$

To determine the voltage and current components in αβ stationary reference frame, the following equation is used:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad (5)$$

Where: x represents the voltage or current of the three phase system. From (4) the currents can be calculated as follows:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2P}{3(u_\alpha^2 + u_\beta^2)} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} + \frac{2Q}{3(u_\alpha^2 + u_\beta^2)} \begin{bmatrix} u_\beta \\ -u_\alpha \end{bmatrix} \quad (6)$$

2.1 Constant power control operations

In the power control loop, the actual power will follow the reference power. If the reference power is constant, the actual power will be constant as well; consequently, the current will contain harmonics if the grid voltages are unbalanced. The next section introduces novel strategy to obtain the harmonic components of the grid currents during constant power operations and unbalanced distorted grid voltages.

2.1.1 Analysis of current harmonics under unbalanced grid voltages

From (2) and (6) and assuming that ($\varphi_\alpha = \varphi_\beta = 0$) and $Q = 0$, the following equations can be obtained:

$$i_\alpha = \frac{\frac{2}{3} P U_\alpha \sin\omega t}{U_\alpha^2 \sin^2 \omega t + U_\beta^2 \cos^2 \omega t} \quad (7)$$

$$i_\alpha = \frac{\frac{2}{3} P U_\alpha \sin\omega t}{\frac{1}{2}(U_\alpha^2 + U_\beta^2) + \frac{1}{2}(U_\alpha^2 - U_\beta^2)\cos 2\omega t} \quad (8)$$

$$i_\alpha[(U_\alpha^2+U_\beta^2)+(U_\alpha^2-U_\beta^2)\cos 2\omega t]=\frac{4}{3}PU_\alpha\sin\omega t \quad (9)$$

Substituting from (3) in (9):

$$(U_\alpha^2+U_\beta^2)I_1\sin\omega t+I_1(U_\alpha^2-U_\beta^2)\cos 2\omega t\sin\omega t$$
$$+i_{\alpha h}(U_\alpha^2+U_\beta^2)+i_{\alpha h}(U_\alpha^2-U_\beta^2)\cos 2\omega t=\frac{4}{3}PU_\alpha\sin\omega t \quad (10)$$

By temporarily neglecting the fourth term of the left hand side in (10) and doing some trigonometric operations:

$$(U_\alpha^2+U_\beta^2)I_1\sin\omega t+\frac{I_1}{2}(U_\alpha^2-U_\beta^2)\sin 3\omega t$$
$$-\frac{I_1}{2}(U_\beta^2-U_\alpha^2)\sin\omega t+i_{\alpha h}(U_\alpha^2+U_\beta^2)=\frac{4}{3}PU_\alpha\sin\omega t \quad (11)$$

By equating the wave forms which have the same frequency in both equation sides:

$$i_{\alpha h}=\frac{I_1}{2}\frac{U_\beta^2-U_\alpha^2}{U_\alpha^2+U_\beta^2}\sin 3\omega t=i_{\alpha 3} \quad (12)$$

$$I_1=\frac{\frac{8}{3}PU_\alpha}{3U_\alpha^2+U_\beta^2} \quad (13)$$

$$i_{\alpha 3}=I_3\sin 3\omega t \quad (14)$$

$$I_3=\frac{I_1}{2}\frac{U_\alpha^2-U_\beta^2}{U_\alpha^2+U_\beta^2} \quad (15)$$

As it can be depicted from (12-15), the peak values of the fundamental and third harmonic components of the grid current can be precisely calculated based on the reference power and the actual grid voltages. By the same way, the next harmonic component can be calculated by considering the fourth term in (10) based on the following equation:

$$i_{\alpha n}=I_n\sin n\omega t \quad (16)$$

$$I_n=\frac{I_{n-2}}{2}\frac{U_\alpha^2-U_\beta^2}{U_\alpha^2+U_\beta^2} \quad (17)$$

Where: n=3, 5, 7…

The presented equations (12-17) can be used as a guide to determine the performance of any constant power controller. Also, these equations can be used to determine the grid current THD under defined unbalanced grid voltages and constant power operations.

The next analysis introduces another contribution to determine and calculate the grid current harmonic components under balanced and distorted grid voltages during constant power operations.

2.1.2 Analysis of current harmonics under distorted balanced grid voltages

Assuming distorted balanced grid voltages with a fifth harmonic component. The peak values of the fundamental and the fifth harmonic component are defined as U and U_5 in the following equation:

$$u_\alpha=U\sin\omega t+U_5\sin 5\omega t$$
$$u_\beta=-U\cos\omega t-U_5\cos 5\omega t \quad (18)$$

Based on (18) the following equation can be obtained:

$$u_\alpha^2+u_\beta^2=(U\sin\omega t+U_5\sin 5\omega t)^2$$
$$+(-U\cos\omega t-U_5\cos 5\omega t)^2 \quad (19)$$

The following equation can be obtained by neglecting the term of U_5^2:

$$u_\alpha^2+u_\beta^2=U^2+2UU_5\sin 5\omega t*\sin\omega t+2UU_5\cos 5\omega t*\cos\omega t$$
$$u_\alpha^2+u_\beta^2=U^2+2UU_5\cos 4\omega t \quad (20)$$

From (18), (20) and (6) and for unity power factor operation, the following equation can be deduced:

$$i_\alpha[U^2+2UU_5\cos 4\omega t]=\frac{2}{3}P(U\sin\omega t+U_5\sin 5\omega t) \quad (21)$$

From (3) and (21):

$$(I_1\sin\omega t+i_{\alpha h})(U^2+2UU_5\cos 4\omega t)$$
$$=\frac{2}{3}P(U\sin\omega t+U_5\sin 5\omega t) \quad (22)$$

$$I_1U^2\sin\omega t+2I_1UU_5\cos 4\omega t*\sin\omega t+i_{\alpha h}U^2$$
$$+2i_{\alpha h}UU_5\cos 4\omega t=\frac{2}{3}P(U\sin\omega t+U_5\sin 5\omega t) \quad (23)$$

Temporarily, by neglecting the fourth term of the left hand side of (23), and by using trigonometric relations and equating the wave forms which have the same frequency in both sides of (23) the following equation can be deduced:

$$i_{\alpha 3}=I_1\frac{U_5}{U}\sin 3\omega t \quad (24)$$

By the same way and considering the neglected term of (23), the next harmonic can be obtained as follows:

$$i_{\alpha 7}=I_1\left(\frac{U_5}{U}\right)^2\sin 7\omega t \quad (25)$$

From (24) and (25), it can be depicted that the fifth harmonic superimposed on the grid voltages introduces considerable third and seventh harmonics in the grid current. So that, it can be stated that if harmonic of order (n) superimposed on the grid voltages will produce grid current harmonics based on the following equation:

$$i_{\alpha h} = I_1 \frac{U_n}{U} \sin(n-2)\omega t + I_1 (\frac{U_n}{U})^2 \sin(2n-3)\omega t \quad (26)$$

The obtained result in (26) can be used to determine the grid current THD under defined distorted grid voltages and constant power operations. Also, (26) can be used as a reference to determine the performance quality of any constant power controller. Super position theory can be used to determine the grid current THD under unbalanced distorted grid voltages during constant power operation using the preceding equations.

With the system parameters listed in table 1, as an example, the grid current harmonic components can be numerically calculated as follows.

- During unbalanced grid voltages using (13):

$$I_1 = \frac{\frac{8}{3} P U_\alpha}{3U_\alpha^2 + U_\beta^2} = \frac{\frac{8}{3} * 10000 * 283}{3 * 283^2 + 250^2} = 24.93A$$

From (17)

$$I_n = \frac{I_{n-2}}{2} \frac{U_\alpha^2 - U_\beta^2}{U_\alpha^2 + U_\beta^2} = \frac{I_{n-2}}{2} * 0.1234 = 0.0617 I_{n-2}$$

$$I_3 = 0.0617 I_1 = 1.54\ A$$

$$I_5 = 0.0617 I_3 = 0.095\ A$$

- During balanced but distorted grid voltages (U=311V, U_5=0.1U):

$$I_1 = \frac{\frac{8}{3} P U_\alpha}{3U_\alpha^2 + U_\beta^2} = \frac{\frac{8}{3} * 10000 * 311}{3 * 311^2 + 311^2} = 21.4A$$

$$I_3 = 21.4 * 0.1 = 2.14\ A$$

$$I_7 = 21.4 * (0.1)^2 = 0.214\ A$$

Table 1 Simulated System Rating

u_a	$300\angle 0^o$ V	u_α	$283\angle 0^o$ V
u_b	$250\angle -120^o$ V	u_β	$250\angle -90^o$ V
u_c	$250\angle 120^o$ V	P^*	10 kW
V_{dc}	560 V	Q^*	0 Var

2.2 Sinusoidal current control operations

Based on (6), to obtain sinusoidal grid current with constant power, u_α and u_β must be balanced sinusoidal wave forms. But for unbalanced grid voltages and sinusoidal current control, the power will contain oscillations. The next section introduces novel strategy to obtain relations for the power oscillations during sinusoidal current control based on virtual healthy grid voltages and virtual active and reactive power components. The virtual healthy grid voltages supposed to have the following form:

$$\begin{aligned} u_{\alpha 1} &= U_{max} \sin \omega t \\ u_{\beta 1} &= -U_{max} \cos \omega t \end{aligned} \quad (27)$$

Where: $u_{\alpha 1}$ and $u_{\beta 1}$ represent the virtual healthy grid voltages components in $\alpha - \beta$ stationary reference frame. The harmonics components of the unbalanced grid voltages related to the virtual healthy grid voltages can be obtained form (27) and (2) as:

$$\begin{aligned} u_{\alpha h} &= u_\alpha - u_{\alpha 1} = U_\alpha \sin(\omega t + \varphi_\alpha) - U_{max} \sin \omega t \\ u_{\beta h} &= u_\beta - u_{\beta 1} = -U_\beta \cos(\omega t + \varphi_\beta) + U_{max} \cos \omega t \end{aligned} \quad (28)$$

During balanced sinusoidal current control operations, the grid current harmonics components (low order) should equal to zero and the following equation is valid for unity power factor ($Q \neq 0$ due to power oscillations) :

$$\begin{aligned} i_\alpha &= I_1 \sin \omega t \\ i_\beta &= -I_1 \cos \omega t \end{aligned} \quad (29)$$

The active and reactive power components based on the virtual grid voltages can be calculated as follows:

$$\begin{aligned} P &= \frac{3}{2}[(u_{\alpha 1} + u_{\alpha h}) i_{\alpha 1} + (u_{\beta 1} + u_{\beta h}) i_{\beta 1}] \\ Q &= \frac{3}{2}[(u_{\beta 1} + u_{\beta h}) i_{\alpha 1} - (u_{\alpha 1} + u_{\alpha h}) i_{\beta 1}] \end{aligned} \quad (30)$$

$$\begin{aligned} P &= \frac{3}{2}(u_{\alpha 1} i_{\alpha 1} + u_{\beta 1} i_{\beta 1}) + \frac{3}{2}(u_{\alpha h} i_{\alpha 1} + u_{\beta h} i_{\beta 1}) \\ Q &= \frac{3}{2}(u_{\beta 1} i_{\alpha 1} - u_{\alpha 1} i_{\beta 1}) + \frac{3}{2}(u_{\beta h} i_{\alpha 1} - u_{\alpha h} i_{\beta 1}) \end{aligned} \quad (31)$$

The first term of (31) represents constant values P_1 and Q_1 which are called "virtual active and virtual reactive power". The following relation can be obtained using all preceeding equations and assuming that ($\varphi_\alpha = \varphi_\beta = 0$):

$$\begin{aligned} P &= P_1 + \frac{3I_1}{4}((U_{\alpha h} + U_{\beta h}) + (U_{\beta h} - U_{\alpha h}) \cos 2\omega t) \\ P &= P_1 + P_h \end{aligned} \quad (32)$$

$$\begin{aligned} Q &= Q_1 + \frac{3I_1}{4}(U_{\alpha h} - U_{\beta h}) \sin 2\omega t \\ Q &= Q_1 + Q_h \end{aligned} \quad (33)$$

Where:

$$\begin{aligned} P_h &= k_1 + k_2 \cos 2\omega t \\ k_1 &= \frac{3I_1}{4}(U_{\alpha h} + U_{\beta h}) \\ k_2 &= \frac{3I_1}{4}(U_{\beta h} - U_{\alpha h}) \\ U_{\alpha h} &= U_\alpha - U_{max} \\ U_{\beta h} &= U_\beta - U_{max} \end{aligned} \quad (34)$$

Power oscillations can be accurately determined using (32-33) which is one of the contributions of this work. Most litretures introduces the power oscillations in different ways based on positive and negative sequence analysis.

As can be depicted from (32), power oscillations mainly depend on the difference between the virtual healthy grid voltages and the actual unbalanced grid voltages. As the difference increases, power oscillations increase as well. Also, the middle term (k_1) of (32) must be taken into acount to obtain limited balanced grid currents. During balanced grid voltages (less than the virtual grid voltages), $U_{\alpha h}$ and $U_{\beta h}$ are equal, consequently, the power oscillations will disappear. In this case, the injected power to the grid must be less than the reference power by the term k_1 to obtain limited current. Injecting the same reference power to grid will increase the grid curent and affect the inverter switches and system protection.

With the system parameters listed in table 1, the power oscillations, as an example, can be numerically obtained as follows.

$$U_{\alpha h} = 283 - 311 = -28$$

$$U_{\beta h} = 250 - 311 = -61$$

$$P = 10000 - 1431 - 530.55\cos 2\omega t = 8569 - 530.55\cos 2\omega t$$

$$Q = 0 + 618.75\sin 2\omega t$$

3 Suggested Solution

In the power control loop, the actual power will follow the reference power. If the reference power is constant, the actual power will be constant as well; consequently, the current will be unbalanced with unbalanced grid voltage. On the other hand, by controlling the virtual power P_1 *and* Q_1, it will follow the constant reference power and limited balanced sinusoidal currents can be obtained. In this case, the actual power will be free to follow its value in (23-24). Then the proposed solution is to control the virtual power not the actual power to obtain sinusoidal current and to control the actual power to obtain constant power. Adaptable control of current harmonics and power oscillations can be achieved by mixing the controlled variables based on the following equation:

$$P_{controlled} = m\,P_1 + (1-m)\,P \qquad Q_{controlled} = m\,Q_1 + (1-m)\,Q \qquad (35)$$

Where: m is the adaptive coefficient. That is a very simple and attractive solution to obtain switched control between sinusoidal balanced grid current or constant power or mixing them to get the desired performance with unbalanced grid voltages while using constant reference power. There is no need for reference power compensation. Also, there is no need for any positive or negative sequence component for any variable. The block diagram of the proposed system is shown in Fig.2.

Then, the challenge is to find the virtual healthy grid voltages components $u_{\alpha 1}$ and $u_{\beta 1}$ at any condition disregarding the condition of the actual grid voltages to calculate and control the virtual power.

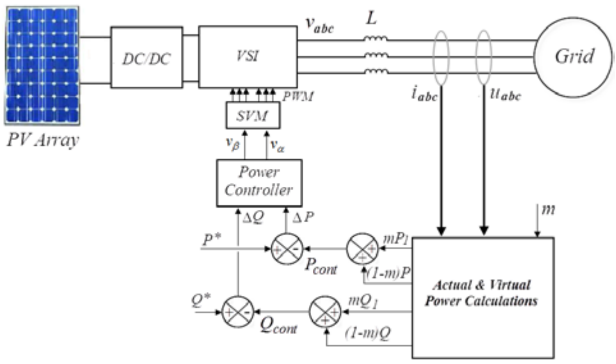

Fig. 2 Block diagram of the proposed system.

3.1 Extraction of the virtual healthy grid voltages $u_{\alpha 1}$ and $u_{\beta 1}$

First, to obtain pure sinusoidal reference signal representing phase-a of the actual grid voltages, a Second Order Generalized Integrator (SOGI) based adaptive filter is used for filtering purposes and to generate $u_{\alpha 0}$ and $u_{\beta 0}$ at the same time. The SOGI block diagram is shown in Fig. 3, where more details are available in [17-18]. PLL is used to provide the operating frequency required to the adaptive filter. The generated orthogonal signals $u_{\alpha 0}$ and $u_{\beta 0}$ have the same peak value of the fundamental component of phase-a. To obtain $u_{\alpha 1}$ and $u_{\beta 1}$, which are required to calculate the virtual power P_1, the following equation is proposed:

$$\begin{bmatrix} u_{\alpha 1} \\ u_{\beta 1} \end{bmatrix} = \frac{U_{\max}}{\sqrt{u_{\alpha 0}^2 + u_{\beta 0}^2}} \begin{bmatrix} u_{\alpha 0} \\ u_{\beta 0} \end{bmatrix} \qquad (36)$$

Where $U_{\max}$ is the maximum value of the healthy grid voltage of phase-a. By using (36), the virtual grid voltages $u_{\alpha 1}$ and $u_{\beta 1}$ are available at any condition disregarding the condition of the actual grid voltages. Using these signals, the sinusoidal reference current can be obtained, where:

$$u_{\alpha 1}^2 + u_{\beta 1}^2 = \frac{U_{\max}^2 u_{\alpha 0}^2}{u_{\alpha 0}^2 + u_{\beta 0}^2} + \frac{U_{\max}^2 u_{\beta 0}^2}{u_{\alpha 0}^2 + u_{\beta 0}^2} = U_{\max}^2 \qquad (37)$$

So, by controlling the virtual power P_1 and Q_1 and substituting the virtual grid voltages instead of the actual grid voltages in (6):

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2P_1}{3U_{\max}^2}\begin{bmatrix} u_{\alpha 1} \\ u_{\beta 1} \end{bmatrix} + \frac{2Q_1}{3U_{\max}^2}\begin{bmatrix} u_{\beta 1} \\ -u_{\alpha 1} \end{bmatrix} \qquad (38)$$

Using this strategy, the grid currents are sinusoidal and balanced as it can be depicted from (38). The next step is to design the appropriate power control strategy.

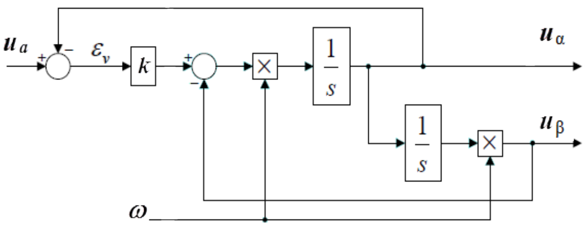

Fig.3 Adaptive filter based on the SOGI.

3.2 Power control strategy

. Fig. 4 shows the simplified equivalent circuit for ac side of the inverter in the α-β reference frame.
From Fig. 4, the relation between the inverter output voltage (v) and the grid voltage (u) in α-β reference frame during steady state operation can be given by:

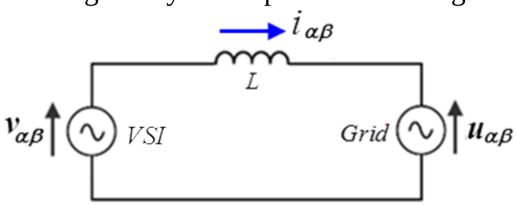

Fig. 4 System model in stationary reference frame.

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \begin{bmatrix} u_\alpha(t) \\ u_\beta(t) \end{bmatrix} + \begin{bmatrix} j\omega L i_\alpha(t) \\ j\omega L i_\beta(t) \end{bmatrix} \qquad (39)$$

Assuming sinusoidal current operation during steady state, hence:

$$\begin{bmatrix} j i_\alpha(t) \\ j i_\beta(t) \end{bmatrix} = \begin{bmatrix} -i_\beta(t) \\ i_\alpha(t) \end{bmatrix} \qquad (40)$$

Substituting from Eqn. (40) into Eqn. (39):

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \begin{bmatrix} u_\alpha(t) \\ u_\beta(t) \end{bmatrix} + \begin{bmatrix} -\omega L i_\beta(t) \\ \omega L i_\alpha(t) \end{bmatrix} \qquad (41)$$

The discrete form of (40) can be written as follows:

$$\begin{bmatrix} v_\alpha(k) \\ v_\beta(k) \end{bmatrix} = \begin{bmatrix} u_\alpha(k) \\ u_\beta(k) \end{bmatrix} + \begin{bmatrix} -\omega L i_\beta(k) \\ \omega L i_\alpha(k) \end{bmatrix} \qquad (42)$$

To force i_α and i_β to follow reference currents i_α^* and i_β^* a current controller is required. If there is an error between the reference current values and the actual current values, the current controller has to change the inverter voltage to compensate this error. To compensate this error during a time period T, the change in the inverter voltage can be obtained as:

$$\begin{bmatrix} \Delta v_\alpha(k) \\ \Delta v_\beta(k) \end{bmatrix} = \frac{L}{T}\begin{bmatrix} i^*_\alpha(k) - i_\alpha(k) \\ i^*_\beta(k) - i_\beta(k) \end{bmatrix} \qquad (43)$$

To force the actual current to follow the reference current by the end of a time period T, the adapted inverter voltage can be obtained using Eqns. (42) and (43) as follows:

$$\begin{bmatrix} v_\alpha(k) \\ v_\beta(k) \end{bmatrix} + \begin{bmatrix} \Delta v_\alpha(k) \\ \Delta v_\beta(k) \end{bmatrix} = \begin{bmatrix} u_\alpha(k) \\ u_\beta(k) \end{bmatrix} + \begin{bmatrix} -\omega L i_\beta(k) \\ \omega L i_\alpha(k) \end{bmatrix} + \frac{L}{T}\begin{bmatrix} i^*_\alpha(k) - i_\alpha(k) \\ i^*_\beta(k) - i_\beta(k) \end{bmatrix} \qquad (44)$$

To find a direct relation between the inverter voltage and the active and reactive power, relations between currents and power can be used. As discussed before, it is proposed to calculate the reference current based on the following equations:

$$\begin{bmatrix} i^*_\alpha(k) \\ i^*_\beta(k) \end{bmatrix} = \frac{2}{3(u_{\alpha 1}^2(k) + u_{\beta 1}^2(k))}\begin{bmatrix} u_{\alpha 1}(k) & u_{\beta 1}(k) \\ u_{\beta 1}(k) & -u_{\alpha 1}(k) \end{bmatrix}\begin{bmatrix} P^*(k) \\ Q^*(k) \end{bmatrix} \qquad (45)$$

For the virtual power to be controlled, the actual current must be calculated based on the following equation:

$$\begin{bmatrix} i_\alpha(k) \\ i_\beta(k) \end{bmatrix} = \frac{2}{3(u_{\alpha 1}^2(k) + u_{\beta 1}^2(k))}\begin{bmatrix} u_{\alpha 1}(k) & u_{\beta 1}(k) \\ u_{\beta 1}(k) & -u_{\alpha 1}(k) \end{bmatrix}\begin{bmatrix} P_1(k) \\ Q_1(k) \end{bmatrix} \qquad (46)$$

Using Eqns. (44), (45) and (46), the direct relation between the inverter voltage and power can be obtained as,

$$\begin{bmatrix} v_\alpha(k) + \Delta v_\alpha(k) \\ v_\beta(k) + \Delta v_\beta(k) \end{bmatrix} = \begin{bmatrix} u_\alpha(k) \\ u_\beta(k) \end{bmatrix} + \frac{2L}{3T\left(u_{\alpha 1}^2(k) + u_{\beta 1}^2(k)\right)} \times \begin{bmatrix} u_{\alpha 1}(k)(\Delta P(k) + \omega T Q_1(k)) + u_{\beta 1}(k)(\Delta Q(k) - \omega T P_1(k)) \\ u_{\beta 1}(k)(\Delta P(k) + \omega T Q_1(k)) - u_{\alpha 1}(k)(\Delta Qk) - \omega T P_1(k)) \end{bmatrix} \qquad (47)$$

Where:

$$\Delta P(k) = P^*(k) - P_1(k)$$
$$\Delta Q(k) = Q^*(k) - Q_1(k) \qquad (48)$$

Using Eqn. (47), the required inverter voltage in α-β reference frame can be calculated directly to control the virtual power based on instantaneous errors of active and reactive power. This voltage is able to clear the power error by the end of the next sample. In this equation there are two terms named cross-coupling component terms ($\omega T\, Q_1(k)$, $\omega T P_1(k)$). The proposed controller with these cross-coupling components resembles the behavior of the decoupling branches present in synchronous frame controllers. Including these two terms, the proposed controller can achieve nearly zero steady-state tracking error in the controlled power signals. The actual power can be controlled by using P and Q instead of P_1 and Q_1 in the power control equations. From Eqn. (47), the controller simplicity is clear.

4 Simulation Results

A MATLAB/Simulink program is designed to simulate the proposed controlled system. Simulation studies are carried out to confirm:

- The effectiveness of the proposed controller to obtain constant power operation under unbalanced and/or distorted grid voltages.
- The correctness of the introduced equations to calculate the grid current harmonic components during constant power operation.
- The effectiveness of the proposed controller to get grid current with minimum THD under unbalanced and/or distorted grid voltages.
- The correctness of the introduced relations to calculate the power oscillations during sinusoidal current control under unbalanced grid voltages.

The obtained results are carried out in two categories. The first one is for the constant power operations and the second is for the sinusoidal current operations.

4.1 Constant power operations results

Fig. 5 shows the system performance with the listed parameters in table 1.

As shown in Fig. 5, the controlled variable is the actual power where it follows the reference power and constant power operation is obtained. Consequently, grid currents are distorted with third harmonic component as shown in Fig.6. The harmonic analysis of the MATLAB analyzer shown in Fig. 6 completely agree with the presented analysis in section 2. Where the calculated fundamental, third and seventh harmonic components are almost the same as obtained by the MATLAB/Simulink analyzer shown in Fig.6. ($I_1 = 24.93A, I_3 = 1.54A, I_5 = 0.095A$), which confirm the correctness of the presented analysis of section 2.

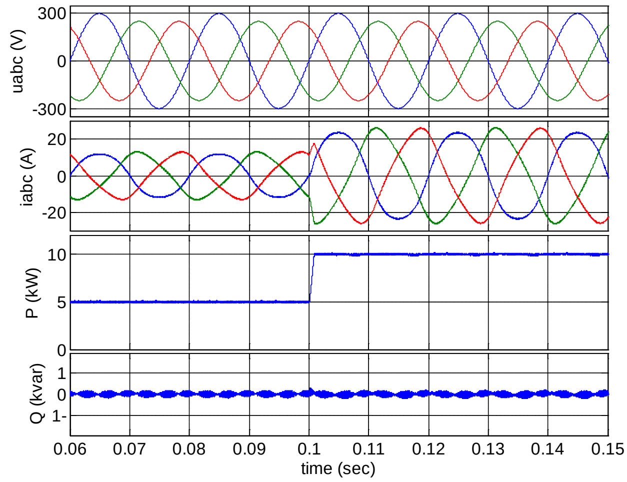

Fig.5 Constant power operation, from top to bottom: unbalanced grid voltages, grid currents, actual power (P), and actual reactive power (Q).

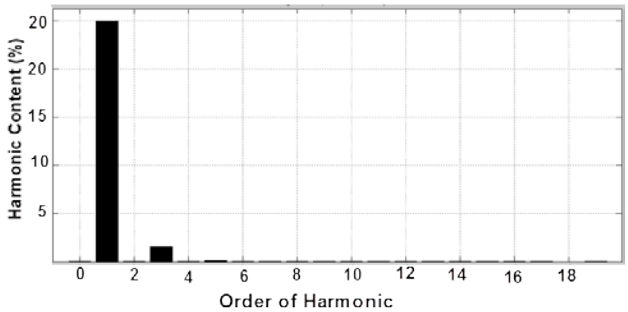

Fig. 6 Current harmonics under unbalanced grid voltages and constant power operation.

Using balanced grid voltages superimposed by a fifth order harmonic component of 10% peak value, Fig. 7 shows the system performance under constant power operation. Fig. 7 shows the effectiveness of the proposed controller to obtain constant power operation under balanced distorted grid voltages. As shown in Fig. 8, the grid current contains third and seventh order harmonic components as captured by the Matlab/Simulink simulator analyzer. The same values are calculated based on (13), (24) and (25), ($I_1 = 21.4A, I_3 = 2.14A, I_7 = 0.214A$) which confirm the correctness of the presented analysis of section.2.

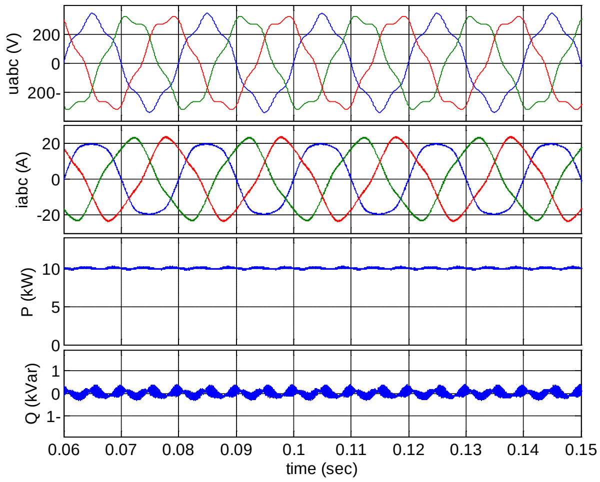

Fig. 7 System performance under balanced grid voltages superimposed by a fifth order harmonic component.

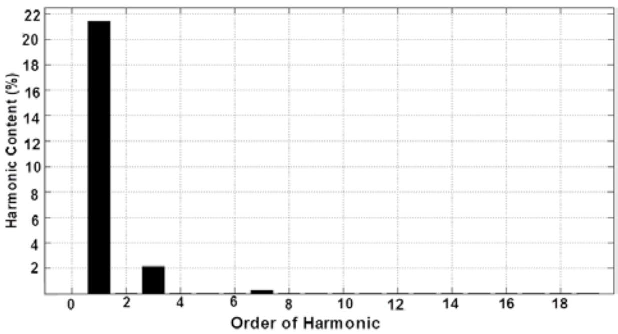

Fig. 8 Current harmonics under distorted balanced grid voltages during constant power operation.

To improve the current THD, the virtual power is used as a controlled variable as will be presented next.

4.2 Sinusoidal current operation

By controlling the virtual power to obtain sinusoidal current operations, Figs. 9-11 show system performance under unbalanced grid voltages with the same parameters of table 1. Using voltage signal of phase-a, the virtual grid voltages are extracted and presented in Fig. 9 with the actual grid voltages. As shown in Fig. 10, the virtual power follows the constant reference power of 10 kW. The actual power is running free to maintain balanced limited sinusoidal current. To limit the grid current to its rated values, the average value of the injected actual power is less than the reference power by the term k_1 which can be calculated from (34). The actual power contains second order harmonic oscillations. The peak value of the power oscillations can be calculated based on (32). The grid current has no low order harmonic components as shown in Fig.11, which confirms the superiority of the proposed controller to get grid current with a minimum THD. Fig. 12 shows the effectiveness of the proposed controller to obtain sinusoidal grid current under unbalanced distorted grid voltages. Fig. 13 shows the system performance under unbalanced and distorted grid voltages for sinusoidal current operation mode. The controlled variables are the active and reactive virtual power. Also the figure illustrates the tracking performance for step change in active power.

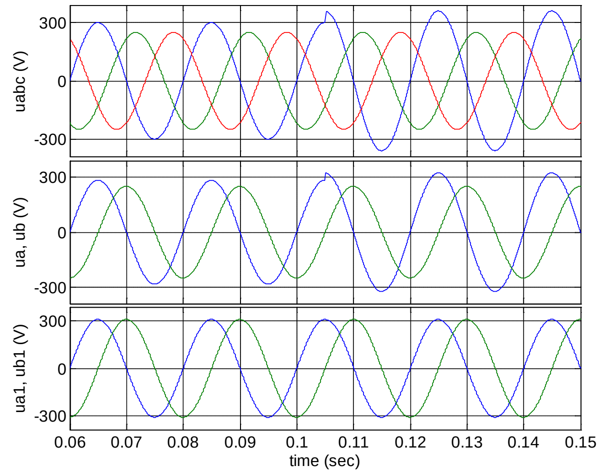

Fig. 9 Generating virtual grid voltage in αβ frame under unbalanced grid voltages during step change in phase-a voltage, from top to bottom, 3-phase grid voltages, actual u_α, u_β and virtual voltages $u_{\alpha 1}, u_{\beta 1}$.

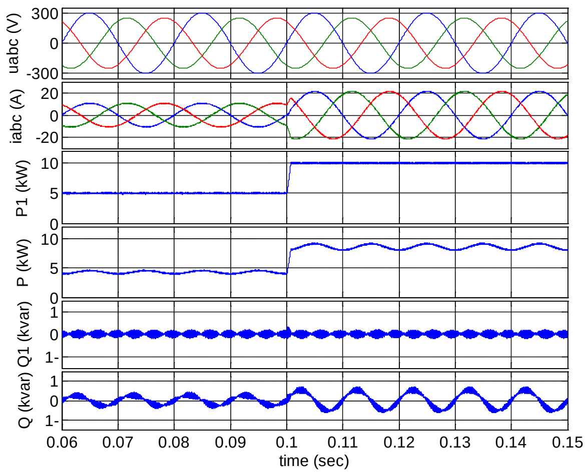

Fig. 10 Sinusoidal current operation, from top to bottom: Unbalanced grid voltages, grid currents, virtual P_1, actual P, virtual Q_1, and actual Q.

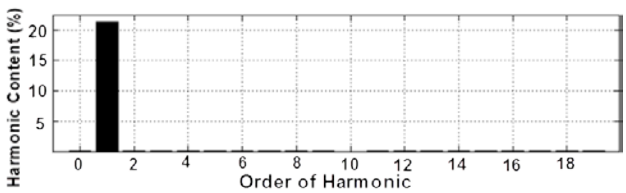

Fig11 Current harmonics under unbalanced grid voltages during sinusoidal current operation.

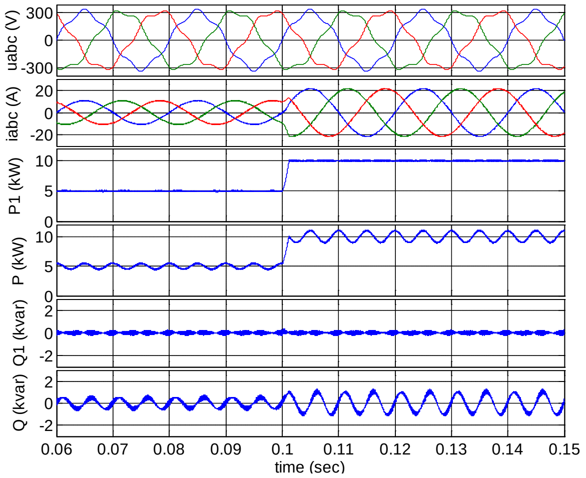

Fig. 12 System performance under balanced grid voltages superimposed by a fifth order harmonic component (sinusoidal current operations).

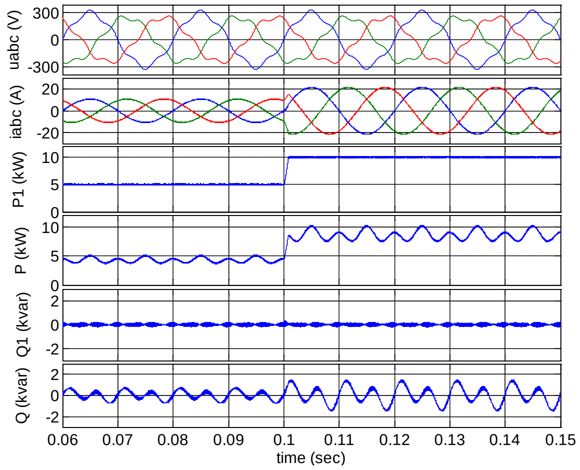

Fig. 13 System performance under unbalanced grid voltages superimposed by a fifth order harmonic component (sinusoidal current operations).

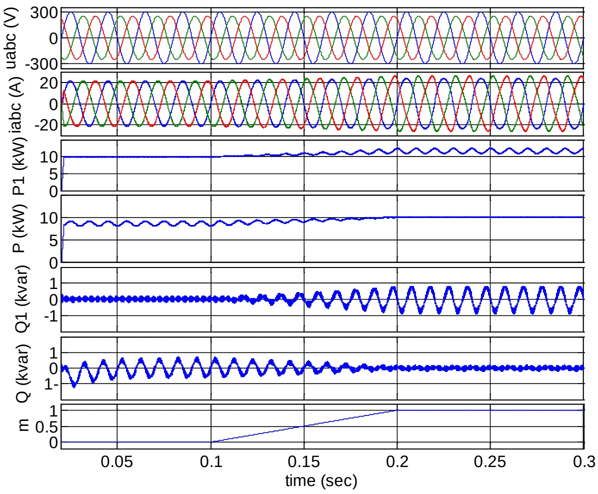

Fig. 14 Switching between constant power control and sinusoidal current control. Plots from top to bottom: unbalanced grid voltages, grid currents, virtual P_1, actual P, virtual Q_1, actual Q and adaptive factor m.

Fig. 14 shows the flexibility of the proposed controller to smoothly switch between constant power control and sinusoidal current control based on the value of the adaptive parameter m.

5 Conclusions

This paper introduces analysis and simulation of PV grid connected voltage source inverter under unbalanced and/or distorted grid voltages. The constant power operation mode and sinusoidal current operation mode are discussed and analyzed. The paper introduces novel strategy to determine the harmonic components of the grid currents during constant power control. The presented analysis can be used as a guide to determine the performance of any constant power controller for VSI and used to determine the grid current THD under defined unbalanced and/or distorted grid voltages. During sinusoidal current control mode, relations to determine power oscillations are derived based on virtual healthy grid voltages. Second order generalized integrator based filter is used to extract the virtual healthy grid voltages in α-β reference frame. The new strategy to inject sinusoidal current to the grid with minimum THD is based on virtual power control. The virtual power is calculated based on the extracted virtual healthy grid voltages. Controlling virtual power leads to sinusoidal current operation while controlling actual power leads to constant power operation. Matlab/simulink program is built to demonstrate the effectiveness of the proposed control strategy. Simulation results confirm the superiority of the proposed controller to get grid currents with a minimum THD under unbalanced and/or distorted grid voltages. Based on adaptive parameter, the proposed controller can smoothly switch between sinusoidal current control and constant power control which confirm the flexibility of the proposed controller.

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