Global Finite-Time Output Feedback Stabilization of Uncertain High-Order Time-Varying Nonlinear Systems

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Abstract: This paper investigates the problem of global finite-time stabilization by output feedback for a class of high-order time-varying nonlinear systems with uncertainties. By introducing sign function and necessarily modifying the homogeneous domination approach, a continuous output feedback controller is successfully constructed to guarantee the global uniform finite-time stability of the resulting closed-loop system. A simulation example is provided to illustrate the effectiveness of the proposed approach.

Key–Words: High-order time-varying nonlinear systems, Output feedback, Homogeneous domination approach, Sign function, Global finite-time stabilization

1 Introduction

In this paper, we consider the following high-order time-varying nonlinear systems:

$$\begin{aligned} &\dot{x}_i = x_{i+1}^{p_i} + f_i(t,x,u), \quad i = 1,\cdots,n-1 \\ &\dot{x}_n = u^{p_n} + f_n(t,x,u) \end{aligned} \tag{1}$$

where $x = (x_1,\cdots,x_n)^T \in R^n$, $u \in R$ are the system state and input, respectively; $p_i \in R^{\geq 1}_{odd} :=\{\frac{p}{q} \mid p$ and q are positive odd integers, and $p \geq q\}$ are said to be the high orders of the system; $f_i : R^+ \times R^n \times R \to R$, $i = 1,\cdots,n$ are unknown continuous functions of all the states and the control input.

The importance for studying such system is exemplified in the papers[1,2], where state feedback controllers were used to stabilize the underactuated, weakly coupled, unstable mechanical system. However, to globally stabilize of system (1) using only its measurable output has been widely recognized as a challenging problem due to the lack of nonlinear version of separation principle. Moreover, Jacobian linearization of system (1) at the origin being neither controllable nor feedback linearizable for the case of $p_i > 1$, leads to that the output feedback stabilization of system (1) becomes more complex. Mainly thanks to the homogeneous domination approach introduced in [3], the novelty of which is that no precise information about the nonlinearities is needed, the output feedback stabilization of system (1) has been well-studied and a number of interesting results have been achieved over the past years, for example, one can see [4-10] and the references therein. Nevertheless, it should be noted that most of the existing works only consider the feedback stabilizer that makes the trajectories of the systems converge to the equilibrium as the time goes to infinity.

Compared to the asymptotic stabilization via output feedback, the finite-time stabilization by output feedback is a relatively new problem. In fact, even in the case of global stabilization of system (1) using state feedback, there are very few results in the literature[11-16]. In the case when parts of the states are not measurable, to globally stabilize system (1) only using limited measurable states becomes challenging. Recently, some attempts have been made[17-19]. In particular, [19] solved the finite-time output feedback stabilization problem under the condition that f_i satisfies

$$|f_i(t,x,u)| \leq c\sum_{j=1}^{i}|x_j|^{\frac{r_i+\tau}{r_j}}$$

where τ is some ratios of odd integers in $(-\frac{2}{(2n+1)p_1\cdots p_{n-1}}, 0)$. Naturally, the following interesting problem is proposed: *Is it possible to relax the assumption on τ and r_i ? Under the weaker condition, can a finite-time output feedback stabilizing controller be designed?*

In this paper, by introducing a combined homogeneous domination and sign function approach, and overcoming some essential difficulties such as the weaker assumption on the system growth, the appearance of the sign function and the construction of a C^1, proper and positive definite Lyapunov function, we will focus on solving the above problem.

2 Mathematical Preliminaries

The following preliminaries are to be used throughout the paper.

Notations. Throughout this paper, the following notations are adopted. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n-dimensional space. For any vector $x = (x_1, \cdots, x_n)^T \in R^n$ denote $\bar{x}_i = (x_1, \cdots, x_i)^T \in R^i$, $i = 1, \cdots, n$, $|x| = (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$. K denotes the set of all functions: $R^+ \to R^+$, which are continuous, strictly increasing and vanishing at zero; K_∞ denotes the set of all functions which are of class K and unbounded. A sign function $sign(x)$ is defined as follows: $sign(x) = 1$, if $x > 0$; $sign(x) = 0$, if $x = 0$ and $sign(x) = -1$, if $x < 0$. Besides, the arguments of the functions will be omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or f.

Definition 1[20]**.** Weighted Homogeneity: For fixed coordinates $(x_1, \cdots, x_n) \in R^n$ and real numbers $r_i > 0, i = 1, \cdots, n$.

• the dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x) = (\varepsilon^{r_1} x_1, \cdots, \varepsilon^{r_n} x_n)$ for any $\varepsilon > 0$, where r_i is called the weights of the coordinates. For simplicity, we define dilation weight $\Delta = (r_1, \cdots, r_n)$.

• a function $V \in (R^n, R)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, \cdots, x_n)$ for any $x \in R^n \setminus \{0\}, \varepsilon > 0$.

• a vector field $f \in (R^n, R^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $f_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau + r_i} f_i(x)$, for any $x \in R^n \setminus \{0\}, \varepsilon > 0, i = 1, \cdots, n$.

• a homogeneous p-norm is defined as $\|x\|_{\triangle,p} = (\sum_{i=1}^n |x_i|^{p/r_i})^{1/p}$ for all $x \in R^n$, for a constant $p \geq 1$. For simplicity, in this paper, we choose $p = 2$ and write $\|x\|_\triangle$ for $\|x\|_{\triangle,2}$.

Lemma 1[20]**.** Given a dilation weight $\Delta = (r_1, \cdots, r_n)$, suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions of degree τ_1 and τ_2, respectively. Then $V_1(x)V_2(x)$ is also homogeneous with respect to the same dilation weight Δ. Moreover, the homogeneous degree of $V_1(x)V_2(x)$ is $\tau_1 + \tau_2$.

Lemma 2[20]**.** Suppose $V : R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following holds:

(i) $\partial V/\partial x_i$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.

(ii) There is a constant c such that $V(x) \leq c\|x\|_\triangle^\tau$. Moreover, if $V(x)$ is positive definite, then $\underline{c}\|x\|_\triangle^\tau \leq V(x)$, where $\underline{c}$ is a constant.

Lemma 3 [21]**.** Consider the nonlinear system

$$\dot{x} = f(t, x) \ \ with \ f(t, 0) = 0 \quad x \in R^n \tag{2}$$

where $f : R^+ \times U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. Suppose there is a C^1 function $V(t, x)$ defined on $\hat{U} \subseteq U_0 \times R$, where $\hat{U}$ is a neighborhood of the origin, class K functions π_1 and π_2, real numbers $c > 0$ and $0 < \alpha < 1$, for $t \in [t_0, T)$ and $x \in \hat{U}$ such that (i) $\pi_1(|x|) \leq V(t, x) \leq \pi_2(|x|)$, $\forall t \geq t_0$, $\forall x \in \hat{U}$; (ii)$\dot{V}(t, x) + cV^\alpha(t, x) \leq 0$, $\forall t \geq t_0, \forall x \in \hat{U}$. Then, the origin of (2) is uniformly finite-time stable with $T \leq \frac{V^{1-\alpha}(t_0, x(t_0))}{c(1-\alpha)}$ for initial condition $x(t_0)$ in some open neighborhood $\hat{U}$ of the origin at initial time t_0. If $\hat{U} = U_0 = R^n$ and π_1 and π_2 are class K_∞ functions, the origin of system (2) is globally uniformly finite-time stable.

Lemma 4[22]**.** For $x \in R$, $y \in R$, $p \geq 1$ and $c > 0$ are constants, the following inequalities hold: (i) $|x + y|^p \leq 2^{p-1}|x^p + y^p|$, (ii) $(|x| + |y|)^{1/p} \leq |x|^{1/p} + |y|^{1/p} \leq 2^{(p-1)/p}(|x| + |y|)^{1/p}$, (iii) $||x| - |y||^p \leq ||x|^p - |y|^p|$, (iv) $|x|^p + |y|^p \leq (|x| + |y|)^p$, (v) $|[x]^{1/p} - [y]^{1/p}| \leq 2^{1-1/p}|x - y|^{1/p}$, (vi) $|[x]^p - [y]^p| \leq c|x - y||x - y|^{p-1} + |y|^{p-1}|$.

Lemma 5[23]**.** If $p = \frac{a}{b} \in R_{odd}^{\geq 1}$ with $a \geq b \geq 1$ being some real numbers, then for any $x, y \in R$

$$|x^p - y^p| \leq 2^{1-\frac{1}{b}} \Big|[x]^a - [y]^a\Big|^{\frac{1}{b}}$$

Lemma 6[24]**.** Let x, y be real variables, then for any positive real numbers a, m and n, one has

$$a|x|^m|y|^n \leq b|x|^{m+n} + \frac{n}{m+n}\Big(\frac{m+n}{m}\Big)^{-\frac{m}{n}} a^{\frac{m+n}{n}} b^{-\frac{m}{n}} |y|^{m+n},$$

where $b > 0$ is any real number.

Lemma 7[23]**.** $f(x) = sgn(x)|x|^a$ is continuously differentiable, and $\dot{f}(x) = a|x|^{a-1}$, where $a \geq 1, \in R$. Moreover, if $x = x(t)$, $t \geq 0$, then $\frac{df(x(t))}{dt} = a|x|^{a-1}\dot{x}(t)$.

3 Output feedback controller design

3.1 Assumption

The following assumption is imposed on system (1) in this paper.

Assumption 1. For $i = 1, \cdots, n$, there are constants $c > 0$ and $\tau \in (-\frac{1}{\sum_{l=1}^{n} p_1 \cdots p_{l-1}}, 0)$ such that

$$|f_i(t, x, u)| \le c \sum_{j=1}^{i} |x_j|^{\frac{r_i+\tau}{r_j}} \tag{3}$$

where $r_1 = 1$, $r_{i+1} = \frac{r_i+\tau}{p_i}$, $i = 1, \cdots, n$ and $\sum_{l=1}^{n} p_1 \cdots p_{l-1}$ for the case of $l = 1$.

The objective of this paper is to design an output feedback controller for system (1) under Assumption 1 such that the closed-loop system is globally finite-time stable.

Remark 1. In the recent paper [19], it is assumed that $\tau = \frac{p}{q}$ with p being any even integer and q being any odd integer, then r_i is always a ratio of odd integers. Therefore, an interesting problem is how to design a finite-time output feedback controller for (1) under the weaker assumption of τ and r_i being arbitrary real numbers in some interval. In this paper, we will ingeniously combine homogeneous domination theory and sign function approach to solve this problem. Furthermore, it should be mentioned that the value range of τ in Assumption 1 is larger than that in [19].

We introduce an equivalent coordinates transformation:

$$z_1 = x_1, \;\; z_i = \frac{x_i}{L^{\kappa_i}}, \;\; i = 2, \cdots, n, \;\; v^{p_n} = \frac{u^{p_n}}{L^{\kappa_n+1}} \tag{4}$$

where $\kappa_1 = 0$, $\kappa_{i+1} = \frac{\kappa_i+1}{p_i}$, $i = 1, \cdots, n-1$ and $L > 1$ is a constant to be determined. Then, under (4), system (1) is transformed into:

$$\begin{aligned} \dot{z}_i &= L x_{i+1}^{p_i} + \frac{f_i}{L^{\kappa_i}}, \qquad i = 1, \cdots, n-1 \\ \dot{z}_n &= L v^{p_n} + \frac{f_n}{L^{\kappa_n}} \\ y &= z_1 \end{aligned} \tag{5}$$

3.2 State-feedback controller design for nominal nonlinear system

We first construct a state feedback controller for the nominal nonlinear system of (5)

$$\begin{aligned} \dot{z}_i &= L z_{i+1}^{p_i}, \qquad i = 1, \cdots, n-1 \\ \dot{z}_n &= L v^{p_n} \end{aligned} \tag{6}$$

Step 1. Let $\xi_1 = [z_1]^{1/r_1}$ and choose the Lyapunov function

$$V_1 = W_1 = \int_{z_1^*}^{z_1} \left[[s]^{1/r_1} - [z_1^*]^{1/r_1} \right]^{2-\tau-r_1} ds \tag{7}$$

with $z_1^* = 0$. From (6), it follows that

$$\dot{V}_1 \le -nL\xi_1^2 + L[\xi_1]^{2-\tau-r_1}(z_2^{p_1} - z_2^{*p_1}) \tag{8}$$

where the virtual controller is chosen as

$$z_2^* = -n^{1/p_1} z_1^{(r_1+\tau)/p_1} := -\beta_1^{r_2}[\xi_1]^{r_2} \tag{9}$$

Step i $(i = 2, \cdots, n)$**.** In this step, we can obtain the following property, whose similar proof can be found in [23] and hence is omitted here.

Proposition 1. Assume that at step $i-1$, there is a C^1, proper and positive definite Lyapunov function V_{i-1}, and a set of virtual controllers $z_1^*, \cdots, z_i^*$ defined by

$$\begin{array}{rclrcl} z_1^* &=& 0, & \xi_1 &=& [z_1]^{1/r_1} - [z_1^*]^{1/r_1} \\ z_2^* &=& -\beta_1^{r_2}[\xi_1]^{r_2}, & \xi_2 &=& [z_2]^{1/r_2} - [z_2^*]^{1/r_2} \\ \vdots & & & \vdots & & \\ z_i^* &=& -\beta_{i-1}^{r_i}[\xi_{i-1}]^{r_i}, & \xi_i &=& [z_i]^{1/r_i} - [z_i^*]^{1/r_i} \end{array} \tag{10}$$

with $\beta_1 > 0, \cdots, \beta_{i-1} > 0$ being constant, such that

$$\begin{aligned} \dot{V}_{i-1} \le& -(n-i+2)L \sum_{j=1}^{i-1} \xi_j^2 \\ &+ [\xi_{i-1}]^{(2\sigma-\tau-r_{i-1})}(z_i^{p_{i-1}} - z_i^{*p_{i-1}}) \end{aligned} \tag{11}$$

Then the ith Lyapunov function defined by

$$V_i = V_{i-1} + \int_{z_i^*}^{z_i} \left[[s]^{1/r_i} - [z_i^*]^{1/r_i} \right]^{2-\tau-r_i} ds \tag{12}$$

is C^1, proper and positive definite, and there is $z_{i+1}^* = -\beta_i^{r_{i+1}/\sigma}[\xi_i]^{r_{i+1}/\sigma}$ such that

$$\begin{aligned} \dot{V}_i \le& -(n-i+1)L \sum_{j=1}^{i} \xi_j^2 \\ &+ L[\xi_i]^{2-\tau-r_i}(z_{i+1}^{p_i} - z_{i+1}^{*p_i}) \end{aligned} \tag{13}$$

Hence at step n, choosing

$$\begin{aligned} V_n &= \sum_{i=1}^{n} \int_{z_i^*}^{z_i} \left[[s]^{1/r_i} - [z_i^*]^{1/r_i} \right]^{2-\tau-r_i} ds \\ z_{n+1}^* &= -\beta_n^{r_{n+1}}[\xi_n]^{r_{n+1}} = -\Big[\sum_{i=1}^{n} \bar{\beta}_i [z_i]^{1/r_i} \Big]^{r_{n+1}} \end{aligned} \tag{14}$$

with $\bar{\beta}_i = \beta_n \cdots \beta_i$, from Proposition 1, we arrive at

$$\dot{V}_n \le -L \sum_{j=1}^{n} \xi_j^2 + L[\xi_n]^{2-\tau-r_n}(v^{p_n} - z_{n+1}^{*p_n}) \tag{15}$$

3.3 Reduced-order observer and gain assignment

Since $z_2, \cdots, z_n$ are unmeasurable, we construct a homogeneous observer

$$\begin{aligned} \dot{\eta}_i &= -Ll_{i-1}\hat{z}_i^{p_{i-1}} \\ \hat{z}_i &= [\eta_i + l_{i-1}\hat{z}_{i-1}]^{r_i/r_{i-1}}, \quad i = 2, \cdots, n \end{aligned} \tag{16}$$

where $\hat{z}_1 = z_1$ and $l_s > 0$; $s = 1, \cdots, n-1$ are the gains to be determined. By the certainty equivalence principle, we can replace z_i with $\hat{z}_i$ in (14) and obtain an output feedback controller

$$v(\hat{z}) = -\Big[\sum_{i=1}^{n} \bar{\beta}_i [\hat{z}_i]^{1/r_i}\Big]^{r_{n+1}} \tag{17}$$

where $\hat{z} = (\hat{z}_1, \hat{z}_2, \cdots, \hat{z}_n)$ and $\hat{z}_1 = z_1$. Considering

$$U_i = \int_{[\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}}^{[z_i]^{(2-\tau-r_{i-1})/r_i}} ([s]^{r_{i-1}/(2-\tau-r_{i-1})} - \gamma_i) ds \tag{18}$$

where $\gamma_i = \eta_i + l_{i-1}z_{i-1}$ and setting the observation error $e_i = [z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}]^{1/(r_i p_{i-1})}$, for $i = 2, \cdots, n$, from (6), (16), (18) and Lemma 7, it follows that

$$\begin{aligned} \dot{U}_i &= L\frac{\partial U_i}{\partial z_i} z_{i+1}^{p_i} + L\frac{\partial U_i}{\partial z_{i-1}} z_i^{p_{i-1}} - L\frac{\partial U_i}{\partial \eta_i} l_{i-1}\hat{z}_i^{p_{i-1}} \\ &= \frac{2-\tau-r_{i-1}}{r_i} L|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i} \\ &\quad \times([z_i]^{r_{i-1}/r_i} - \gamma_i) z_{i+1}^{p_i} \\ &\quad -Ll_{i-1}(z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}) \\ &\quad \times\Big([z_i]^{(2-\tau-r_{i-1})/r_i} - [\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}\Big) \\ &\quad -Ll_{i-1}(z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}) \\ &\quad \times\Big([\hat{z}_i]^{(2-\tau-r_{i-1})/r_i} - [\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}\Big) \end{aligned} \tag{19}$$

where $z_{n+1} = v(\hat{z})$.

Each term on the right-hand side of (19) can be estimated by the following propositions whose proofs are given in Appendix.

Proposition 2. There exists a positive constant λ_i such that

$$\begin{aligned} & -l_{i-1}(z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}) \\ & \quad \times\Big([z_i]^{(2-\tau-r_{i-1})/r_i} - [\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}\Big) \\ & \leq -l_{i-1}\lambda_i e_i^2 \end{aligned} \tag{20}$$

Proposition 3. For $i = 2, \cdots, n-1$,

$$\begin{aligned} & \frac{2-\tau-r_{i-1}}{r_i}|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i}([z_i]^{r_{i-1}/r_i} - \gamma_i) z_{i+1}^{p_i} \\ & \leq \frac{1}{12}\sum_{j=i-1}^{i+1} \xi_j^2 + \alpha_i e_i^2 + g_i(l_{i-1}) e_{i-1}^2 \end{aligned} \tag{21}$$

where g_i is a continuous function of l_{i-1}, $\alpha_i > 0$ is a constant, and $g_2 = 0$.

Proposition 4. For the controller $v(\hat{z})$, we obtain

$$\begin{aligned} & \frac{2-\tau-r_{n-1}}{r_n}|z_n|^{(2-\tau-r_{n-1}-r_n)/r_n} \\ & \quad \times([z_n]^{r_{n-1}/r_n} - \gamma_n) v^{p_n} \\ & \leq \frac{1}{8}\sum_{j=1}^{n} \xi_j^2 + \bar{\alpha}\sum_{i=2}^{n} e_i^2 + g_n(l_{n-1}) e_{n-1}^2 \end{aligned} \tag{22}$$

where g_n is a continuous function of l_{n-1}, $\bar{\alpha} > 0$ is a constant.

Proposition 5. For $i = 3, \cdots, n$,

$$\begin{aligned} & -l_{i-1}(z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}) \\ & \quad \times\Big([\hat{z}_i]^{(2-\tau-r_{i-1})/r_i} - [\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}\Big) \\ & \leq \frac{1}{16}(\xi_{i-1}^2 + \xi_i^2) + e_i^2 + h_i(l_{i-1}) e_{i-1}^2 \end{aligned} \tag{23}$$

where h_i is a continuous function of l_{i-1}.

Choosing $U = \sum_{i=2}^{n} U_i$, by Propositions 2-5, we get

$$\begin{aligned} \dot{U} &= \frac{L}{2}\sum_{i=1}^{n} \xi_i^2 \\ &\quad +L\Big(-l_1\lambda_2 + \alpha_2 + \bar{\alpha} + g_3(l_2) + \theta_3(l_2)\Big) e_2^2 \\ &\quad +\sum_{i=3}^{n-1}\Big(-l_{i-1}\lambda_i + \alpha_i + 1 + \bar{\alpha} + g_{i+1}(l_i) \\ &\quad +\theta_{i+1}(l_i)\Big) e_i^2 + (-l_{n-1}\lambda_n + 1 + \bar{\alpha}) e_n^2 \end{aligned} \tag{24}$$

By (14), (17) and Assumption 1, we can estimate $[\xi_n]^{(2-\tau-r_n)/\sigma}(v^{p_n} - z_{n+1}^{*p_n})$ in (15) by the following proposition, whose proof is given in Appendix.

Proposition 6. There exists a positive constant $\tilde{\alpha}$ such that

$$[\xi_n]^{2-\tau-r_n}(v^{p_n} - z_{n+1}^{*p_n}) \leq \frac{1}{4}\sum_{i=1}^{n} \xi_i^2 + \tilde{\alpha}\sum_{i=2}^{n} e_i^2 \tag{25}$$

With the help of Proposition 6, defining $T = V_n + U$, combining (15) and (24), and recursively choosing

$$\begin{aligned} & l_{n-1} \geq \lambda_n^{-1}\Big(\frac{1}{4} + 1 + \bar{\alpha} + \tilde{\alpha}\Big) \\ & l_{i-1} \geq \lambda_i^{-1}\Big(\frac{1}{4} + \alpha_i \\ & \qquad +1 + \bar{\alpha} + \tilde{\alpha} + g_{i+1}(l_i) + \theta_{i+1}(l_i)\Big) \\ & i = n-1, \cdots, 3 \\ & l_1 \geq \lambda_2^{-1}\Big(\frac{1}{4} + \alpha_2 + \bar{\alpha} + \tilde{\alpha} + g_3(l_2) + \theta_3(l_2)\Big) \end{aligned} \tag{26}$$

we obtain

$$\dot{T} = -\frac{L}{4}\sum_{i=1}^{n}\xi_i^2 - \frac{L}{4}\sum_{i=2}^{n}e_i^2 \tag{27}$$

Note that from the construction of T, it can be verified that T is positive definite and proper with respect to $Z = (z_1, \cdots, z_n, \eta_2, \cdots, \eta_n)^T$. Denoting the dilation weight

$$\Delta = (\underbrace{r_1, \cdots, r_n}_{for\ z_1,\cdots,z_n}, \underbrace{r_1, \cdots, r_{n-1}}_{for\ \eta_2\cdots,\eta_n}) \tag{28}$$

the closed-loop system can be rewritten as

$$\dot{Z} = LE(Z) + F(Z) \tag{29}$$

where $E(Z) = (z_2^{p_1}, \cdots, z_n^{p_{n-1}}, \upsilon^{p_n}, \dot{\eta}_2 \cdots, \dot{\eta}_n)^T$ and $F(Z) = (f_1, \frac{f_2}{L^{\kappa_2}}, \cdots, \frac{f_n}{L^{\kappa_n}}, 0, \cdots, 0)^T$. Furthermore, from Definition 1, it can be shown that

$$T = V_n + \sum_{i=2}^{n} U_i \tag{30}$$

is homogeneous of degree $2 - \tau$ with respect to Δ.

3.4 Stability analysis

The main results of the paper can be summarized into the following theorem:

Theorem 1. For the high-order nonlinear system (1) under Assumption 1, the output feedback controller $u^{p_n} = L^{\kappa_n+1}\upsilon^{p_n}$ in (4), (16) and (17), renders that the equilibrium at the origin of the closed-loop system is globally uniformly finite-time stable.

Proof. We prove Theorem 1 by three steps.

Step 1. We first prove that u^{p_n} preserves the equilibrium at the origin.

From (17) and $r_{n+1}p_n = r_n + \tau$, we have

$$\upsilon^{p_n}(\hat{z}) = -\Big[\sum_{i=1}^{n}\bar{\beta}_i[\hat{z}_i]^{1/r_i}\Big]^{r_n+\tau} \tag{31}$$

By which and the definitions of r_i's, we easily see that $u^{p_n} = L^{\kappa_n+1}\upsilon^{p_n}$ is a continuous function of $\hat{z}$ and $u^{p_n}(\hat{z}) = 0$ for $\hat{z} = 0$. This together with (16) and Assumption 1 implies that the solutions of Z-system is defined on a time interval $[0, t_M)$, where $t_M > 0$ may be a finite constant or $+\infty$, and u^{p_n} preserves the equilibrium at the origin.

Step 2. Because $T(Z)$ and $E(Z)$ are homogeneous of degree $2 - \tau$ and τ with respect to Δ, by Lemmas 1 and 2, there is constants c_1, c_2 and c_3, such that

$$c_1||Z(t)||_\Delta^{2-\tau} \le T(Z) \le c_2||Z(t)||_\Delta^{2-\tau} \tag{32}$$

$$\frac{\partial T(Z)}{\partial Z}LE(Z) \le -c_3L||Z(t)||_\Delta^2 \tag{33}$$

By (4), Assumption 1 and $L > 1$, we can find constants $\delta_i > 0$ and $0 < \nu_i \le 1$ such that

$$\begin{aligned}\Big|\frac{f_i(\cdot)}{L^{\kappa_i}}\Big| &\le \frac{c}{L^{\kappa_i}}\sum_{j=1}^{i}|x_j(t)|^{(r_i+\tau)/r_j}\\ &= c\sum_{j=1}^{i}L^{\kappa_j(r_i+\tau)/r_j-\kappa_i}|z_j(t)|^{(r_i+\tau)/r_j}\\ &\le \delta_i||Z(t)||_\Delta^{r_i+\tau}\end{aligned} \tag{34}$$

since it can be seen that by definition $r_j = \tau\kappa_j + 1/(p_1\cdots p_{j-1})$, so

$$\begin{aligned}&\frac{\kappa_j(r_i+\tau)}{r_j} - \kappa_i\\ &= \frac{\kappa_j(\tau\kappa_i+1/(p_1\cdots p_{i-1})+\tau)}{\tau\kappa_j+1/(p_1\cdots p_{j-1})} - \kappa_i\\ &= \frac{\tau\kappa_j - (\kappa_j-\kappa_i)/(p_1\cdots p_{i-1})}{\tau\kappa_j + 1/(p_1\cdots p_{j-1})}\\ &= \frac{\tau\kappa_j - (\kappa_j-\kappa_i)/(p_1\cdots p_{i-1})}{(\tau\sum_{l=1}^{j}p_1\cdots p_{l-2}+1)/(p_1\cdots p_{j-1})}\\ &\le 0\end{aligned} \tag{35}$$

Noting that for $i = 1, \cdots, n$, $\partial T(Z)/\partial Z_i$ is homogeneous of degree $2 - \tau - r_i$, from Lemma 5, we obtain

$$\begin{aligned}&\Big|\frac{\partial T(Z)}{\partial Z}F(Z)\Big|\\ &\le \sum_{i=1}^{n}\Big|\frac{\partial T(Z)}{\partial Z_i}\Big|\Big|\frac{f_i(\cdot)}{L^{\kappa_i}}\Big| \le \rho_1||Z(t)||_\Delta^2\end{aligned} \tag{36}$$

where ρ_1 is a positive constant.

According to (30), (32), (33) and (36), we get

$$\begin{aligned}\dot{T} &= \frac{\partial T(Z)}{\partial Z}LE(Z) + \frac{\partial T(Z)}{\partial Z}F(Z)\\ &\le -(c_3L-\rho_1)||Z(t)||_\Delta^2\\ &\le -\frac{(c_3L-\rho_1)}{c_1^{2/(2-\tau)}}T^{2/(2-\tau)}\end{aligned} \tag{37}$$

Hence, by choosing $L > max\{\rho_1/c_3, 1\}$ there exists a constant c^* such that

$$\dot{T} \le -c^*T^{2/(2-\tau)} \tag{38}$$

From (38) and Lemma 3, we obtain that the equilibrium $z = 0$ of the closed-loop ξ-systems (5), (16) and (17) is globally uniformly finite-time stable.

Step 3. Since (4) is an equivalent transformation, the closed-loop system consisting of (1), $u^{p_n} = L^{\kappa_n+1}\upsilon^{p_n}$ in (4), (16) and (17), has the same properties as the system (5), (16) and (17). Thus, the proof is completed.

4 Extension

In this section, we show that the proposed design method can be extended to handle the high-order nonlinear system (1) in upper-triangular form.

Assumption 2. For $i = 1, \cdots, n$, there are constants $c > 0$ and $\tau \in (-\frac{1}{\sum_{l=1}^{n} p_1 \cdots p_{l-1}}, 0)$ such that

$$|f_i(t,x,u)| \le c \sum_{j=i+2}^{n} |x_j|^{\frac{r_i+\tau}{r_j}} \tag{39}$$

where $r_1 = 1$, $r_{i+1} = \frac{r_i+\tau}{p_i}$, $i = 1, \cdots, n-1$ and $\sum_{l=1}^{n} p_1 \cdots p_{l-1}$ for the case of $l = 1$.

By taking the same design procedure in Section 3, except for $L > 1$ being replaced by $0 < L < 1$, we can construct a continuous output feedback controller applicable to system (1), and thus obtain the following concluding theorem.

Theorem 2. For the high-order nonlinear system (1) under Assumption 2, the output feedback controller $u^{p_n} = L^{\kappa_n+1} \upsilon^{p_n}$ in (4), (16) and (17), renders that the equilibrium at the origin of the closed-loop system is globally uniformly finite-time stable.

Proof. The proof can be divided into four steps. Since the steps 1 and 2 of the proof are the same to those of Theorem 1 and hence are omitted here.

Step 3. Because $T(Z)$ and $E(Z)$ are homogeneous of degree $2-\tau$ and τ with respect to Δ, by Lemmas 1 and 2, there is a constant c_1, such that

$$\frac{\partial T(Z)}{\partial Z} LE(Z) \le -c_1 L ||Z(t)||_{\Delta}^2 \tag{40}$$

By (4), Assumption 2 and $0 < L < 1$, we can find constants $\bar{\delta}_i > 0$ and $\bar{\nu}_i > 0$ such that

$$\begin{aligned} &\Big|\frac{f_i(\cdot)}{L^{\kappa_i}}\Big| \\ &\le a \sum_{j=i+2}^{n} L^{\kappa_j(r_i+\tau)/r_j - \kappa_i}\Big(|z_j(t)|^{(r_i+\tau)/r_j} \\ &\quad + |z_j(t-d_j(t))|^{(r_i+\tau)/r_j}\Big) \\ &\le \bar{\delta}_i L^{1+\bar{\nu}_i}\Big(||Z(t)||_{\Delta}^{r_i+\tau} + \sum_{j=1}^{i} ||Z(t-d_j(t))||_{\Delta}^{r_i+\tau}\Big) \end{aligned} \tag{41}$$

since it can be seen that by definition $r_j = \tau\kappa_j + 1/(p_1 \cdots p_{j-1})$, so

$$\begin{aligned} &\frac{\kappa_j(r_i+\tau)}{r_j} - \kappa_i \\ &= \frac{\kappa_j(\tau\kappa_i + 1/(p_1\cdots p_{i-1}) + \tau)}{\tau\kappa_j + 1/(p_1\cdots p_{j-1})} - \kappa_i \\ &= \frac{\tau\kappa_j + (\kappa_i - \kappa_j)/(p_1\cdots p_{i-1})}{\tau\kappa_j + 1/(p_1\cdots p_{j-1})} > 1 \end{aligned} \tag{42}$$

Noting that for $i = 1, \cdots, n$, $\partial T(Z)/\partial Z_i$ is homogeneous of degree $2 - \tau - r_i$, from Lemma 5, one obtains

$$\begin{aligned} &\Big|\frac{\partial T(Z)}{\partial Z} F(Z)\Big| \\ &\le \sum_{i=1}^{n} \Big|\frac{\partial T(Z)}{\partial Z_i}\Big|\Big|\frac{f_i(\cdot)}{L^{\kappa_i}}\Big| \\ &\le \sum_{i=1}^{n} \bar{\rho}_{i1} L^{1+\bar{\nu}_0} ||Z(t)||_{\Delta}^2 \\ &\quad + \sum_{i=1}^{n} \bar{\rho}_{i2} L^{1+\bar{\nu}_0} ||Z(t-d_i(t))||_{\Delta}^2 \\ &\le \bar{\rho}_1 L^{1+\bar{\nu}_0} ||Z(t)||_{\Delta}^2 \\ &\quad + \bar{\rho}_2 L^{1+\bar{\nu}_0} \sum_{i=1}^{n} ||Z(t-d_i(t))||_{\Delta}^2 \end{aligned} \tag{43}$$

where $\bar{\rho}_{i1}$, $\bar{\rho}_{i2}$, $i = 1, \cdots, n$, $\bar{\rho}_1 = \sum_{i=1}^{n} \bar{\rho}_{i1}$, $\bar{\rho}_2 = max_{1\le i\le n}\{\bar{\rho}_{i2}\}$ and $\bar{\nu}_0 = max_{1\le i\le n}\{\bar{\nu}_i\} > 0$ are positive constants.

According to (29), (32), (40) and (43), one gets

$$\begin{aligned} \dot{V} &= \frac{\partial T(Z)}{\partial Z} LE(Z) + \frac{\partial T(Z)}{\partial Z} F(Z) \\ &\quad + \sum_{i=1}^{n} \frac{\lambda}{1-\vartheta_i} ||Z(t)||_{\Delta}^2 - \sum_{i=1}^{n} \lambda ||Z(t-d_i(t))||_{\Delta}^2 \\ &\le -\Big(c_1 L - \bar{\rho}_1 L^{1+\bar{\nu}_0} - \sum_{i=1}^{n} \frac{\lambda}{1-\vartheta_i}\Big) ||Z(t)||_{\Delta}^2 \\ &\quad - \Big(\lambda - \bar{\rho}_2 L^{1+\bar{\nu}_0}\Big) \sum_{i=1}^{n} ||Z(t-d_i(t))||_{\Delta}^2 \end{aligned} \tag{44}$$

Therefore, by choosing

$$\lambda = \bar{\rho}_2 L^{1+\bar{\nu}_0} \tag{45}$$

and

$$0 < L < min\Big\{\Big(\frac{c_1}{\bar{\rho}_1 + \bar{\rho}_2 \sum_{i=1}^{n} \frac{1}{1-\vartheta_i}}\Big)^{1/\bar{\nu}_0}, 1\Big\} \tag{46}$$

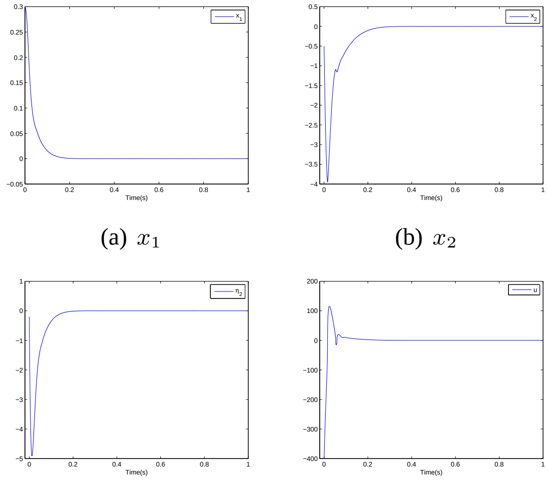
there exists a constant $\bar{c}^*$ such that

$$\dot{V} \le -\bar{c}^* ||Z(t)||_{\Delta}^2 \tag{47}$$

The rest of the proof is similar to that of Theorem 1 and hence is omitted here.

5 Simulation example

To illustrate the effectiveness of the proposed approach, we consider the following low-dimensional system



(a) x_1 (b) x_2

(c) η_2 (d) u

Figure 1: The responses of the closed-loop system (48) and (49).

$$\begin{array}{l} \dot{x}_1 = x_2^{5/3} + \frac{1}{10}x_1^{10/11} \\ \dot{x}_2 = u + \frac{1}{10}x_1^{5/11} + \frac{1}{8}x_2^{1/2} sin x_2 \\ y = x_1 \end{array} \tag{48}$$

where $p_1 = \frac{5}{3}$ and $p_2 = 1$. Choose $\tau = -\frac{1}{11} \in (-\frac{3}{8}, +\infty)$, then $r_1 = 1$, $r_2 = \frac{r_1+\tau}{p_1} = \frac{6}{11}$ and $r_3 = \frac{r_2+\tau}{p_2} = \frac{5}{11}$. By Lemma 6, it can be verified that $|f_1| \le \frac{1}{10}|x_1|^{10/11}$ and $|f_2| \le \frac{1}{10}(|x_1|^{5/11} + |x_2|^{5/6})$ satisfy Assumption 1 with $a = \frac{1}{10}$. Hence the controller proposed in this paper is applicable. Following the design procedure given in Section 3, we can get

$$\begin{array}{l} \dot{\eta}_2 = -Ll_1[\eta_2 + l_1 y]^{10/11} \\ u = -L^{8/5}\Big[\beta_2[\eta_2 + l_1 y] + \beta_2\beta_1[y]\Big]^{5/11} \end{array} \tag{49}$$

In the simulation, the gains in (49) are chosen as $\beta_1 = 2.2$, $\beta_2 = 24$, $l_1 = 20$ and $L = 9$. With the initial values $x_1(0) = 0.3$, $x_2(0) = -0.5$ and $\eta_2(\theta) = -0.2$, Figure 1 is obtained to demonstrates the effectiveness of the proposed controller.

6 Conclusion

In this paper, a continuous output feedback stabilizing controller is presented for a class of high-order nonlinear systems under a weaker condition. The controller designed preserves the equilibrium at the origin, and guarantees the global uniform finite-time stability of the systems. Some interesting problems are still remained, e.g., if the growth rate c in Assumption 1 is an unknown constant, how can design an adaptive finite-time output feedback controller for system (1)? In recent years, many results on control of stochastic nonlinear systems have been achieved [25-29], but these papers only consider the systems in time-invariant case. Hence an important issue is whether the results can be extended to the time varying counterpart.

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Appendix

Proof of Proposition 2. Noting that $(2-\tau-r_{i-1})/r_i p_{i-1} \ge 1$, by using Lemma 5 with $p=1$, $a = b = (2-\tau-r_{i-1})/r_i p_{i-1}$ and $e_i = [z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}]^{1/r_i p_{i-1}}$, one leads to

$$\begin{array}{l} -l_{i-1}(z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}) \\ \quad \times \Big([z_i]^{(2-\tau-r_{i-1})/r_i} - [\hat{z}_i]^{(2-\tau-r_{i-1})/r_i} \Big) \\ \le -l_{i-1}\lambda_i |e_i|^{1/r_i p_{i-1}} |e_i|^{(2-\tau-r_{i-1})/r_i p_{i-1}} \\ = -l_{i-1}\lambda_i e_i^2 \end{array} \tag{A1}$$

where $\lambda_i = 2^{(2r_i p_{i-1}-2)/(2-\tau-r_{i-1})} > 0$ is a constant.

Proof of Proposition 3. Using $\gamma_i = \eta_i + l_{i-1}z_{i-1}$, (10), (16) and Lemmas 4-6, it follows that

$$\begin{array}{l} \frac{2-\tau-r_{i-1}}{r_i}|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i}([z_i]^{r_{i-1}/r_i} - \gamma_i) z_{i+1}^{p_i} \\ = \frac{2-\tau-r_{i-1}}{r_i}|z_i|^{(2-\tau-r_{i-1}-r_i)/r_i} \\ \quad \times \Big([z_i]^{r_{i-1}/r_i} - [\hat{z}_i]^{r_{i-1}/r_i} + [\hat{z}_i]^{r_{i-1}/r_i} - \gamma_i \Big) z_{i+1}^{p_i} \\ \le \frac{2-\tau-r_{i-1}}{r_i}|\xi_{i+1} - \beta_i\xi_i|^{r_{i+1}p_i} \\ \quad \times |\xi_i - \beta_{i-1}\xi_{i-1}|^{(2-\tau-r_{i-1}-r_i)} \\ \quad \times \Big(|z_i^{p_{i-1}} - \hat{z}_i^{p_{i-1}}|^{r_{i-1}/p_{i-1}r_i} + l_{i-1}|z_{i-1} - \hat{z}_{i-1}| \Big) \Big) \\ \le k_{i3}\Big(|\xi_{i+1}|^{r_{i+1}p_i} + |\xi_i|^{r_{i+1}p_i} \Big) \\ \quad \times \Big(|\xi_i|^{2-\tau-r_{i-1}-r_i} + |\xi_{i-1}|^{2-\tau-r_{i-1}-r_i} \Big) \\ \quad \times (|e_i|^{r_{i-1}} + l_{i-1}|e_{i-1}|^{r_{i-1}}) \\ \le \frac{1}{12}\sum_{j=i-1}^{i+1} \xi_j^2 + \alpha_i e_i^2 + g_i(l_{i-1})e_{i-1}^2 \end{array} \tag{A2}$$

where $k_{i3} > 0$, $\alpha_i > 0$ are constants and g_i is a continuous function of l_{i-1}.

Proof of Proposition 4. By (10), (17) and the definition of e_i, one gets

$$\begin{aligned}|v^{p_n}(\hat{z})| &= \Big|\sum_{i=1}^{n}\bar{\beta}_i[\hat{z}_i]^{\sigma/r_i}\Big|^{p_n r_{n+1}/\sigma}\\ &\leq k_{i4}\Big(\sum_{i=1}^{n}|\xi_i|^{(r_n+\tau)/\sigma}+\sum_{i=1}^{n}|e_i|^{(r_n+\tau)/\sigma}\Big)\end{aligned} \quad (A3)$$

where k_{i4} is a positive constant.

Similar to (A2), with the use of Assumption 1, Lemmas 4-6 and (A3), (22) holds immediately.

Proof of Proposition 5. According to $\gamma_i = \eta_i + l_{i-1}z_{i-1}$, (10), Lemmas 3-5 and the definition of e_i, one obtains

$$\begin{aligned}& l_{i-1}(z_i^{p_{i-1}}-\hat{z}_i^{p_{i-1}})\\ &\quad\times\Big([\hat{z}_i]^{(2-\tau-r_{i-1})/r_i}-[\gamma_i]^{(2-\tau-r_{i-1})/r_{i-1}}\Big)\\ &\leq -l_{i-1}|e_i|^{r_i p_{i-1}}\Big|[\eta_i+l_{i-1}\hat{z}_{i-1}]^{(2-\tau-r_{i-1})/r_i}\\ &\quad-[\eta_i+l_{i-1}z_{i-1}]^{(2-\tau-r_{i-1})/r_{i-1}}\Big|\\ &\leq k_{n5}|e_i|^{r_i p_{i-1}}|e_{i-1}|^{r_{i-1}}\Big(|e_{i-1}|^{2-\tau-2r_{i-1}}\\ &\quad+|\xi_{i-1}|^{2-\tau-2r_{i-1}}+|\xi_i|^{2-\tau-2r_{i-1}}+|e_i|^{2-\tau-2r_{i-1}}\Big)\\ &\leq \frac{1}{16}(\xi_{i-1}^2+\xi_i^2)+e_i^2+h_i(l_{i-1})e_{i-1}^2\end{aligned} \quad (A4)$$

where k_{n5} is a positive constant and h_i is a continuous function of l_{i-1}.

Proof of Proposition 6. By (10), (22) and Lemmas 4-6, it follows that

$$\begin{aligned}&[\xi_n]^{(2\sigma-\tau-r_n)/\sigma}(v^{p_n}-z_{n+1}^{*p_n})\\ &=-[\xi_n]^{2-\tau-r_n}\Big(\Big[\sum_{i=1}^{n}\bar{\beta}_i[z_i]^{1/r_i}\Big]^{r_n+\tau}\\ &\quad-\Big[\sum_{i=1}^{n}\bar{\beta}_i[\hat{z}_i]^{1/r_i}\Big]^{r_n+\tau}\Big)\\ &\leq|\xi_n|^{2-\tau-r_n}\Big|\sum_{i=2}^{n}\bar{\beta}_i([z_i]^{1/r_i}-[\hat{z}_i]^{1/r_i})\Big|^{r_n+\tau}\\ &\leq k_{n6}|\xi_n|^{2-\tau-r_n}\Big(\sum_{i=2}^{n}|z_i-\hat{z}_i|\\ &\quad\times(|z_i-\hat{z}_i|^{(1-r_i)/r_i}+|z_i|^{(1-r_i)/r_i})\Big)^{r_n+\tau}\\ &\leq\bar{k}_{n6}|\xi_n|^{2-\tau-r_n}\Big(\sum_{i=2}^{n}|e_i|^{r_i}\\ &\quad\times(|e_i|^{1-r_i}+|\xi_{i-1}|^{1-r_i}+|\xi_i|^{1-r_i})\Big)^{r_n+\tau}\\ &\leq\frac{1}{4}\sum_{i=1}^{n}\xi_i^2+\tilde{\alpha}\sum_{i=2}^{n}e_i^2\end{aligned} \quad (A5)$$

where k_{n6}, $\bar{k}_{n6}$ and $\tilde{\alpha}$ are positive constants.

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