A family of adaptive H_∞ controllers for switched dissipative Hamiltonian systems

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Abstract: This paper investigates adaptive H_∞ control problem for switched dissipative Hamiltonian systems (SDHSs). Firstly, using the dissipative Hamiltonian structural properties, such systems generated an augmented SDHS, with which some results on the controller design are then obtained. When there are external disturbances and parameter perturbations in such systems, a family of adaptive H_∞ controller is designed using the multiple Lyapunov functions (MLFs). Secondly, an algorithm for solving parameters of controllers is proposed with symbolic computation. The proposed method avoids solving Hamilton-Jacobi-Issacs inequalities (equations), and the obtained controllers are relatively simple in form and easy in operation. Finally, a numerical example is studied by using the new results proposed in this paper to SDHSs with external disturbances and parameter perturbations, and some numerical simulations are carried out to support our new results.

Key Words: adaptive H_∞ control; switched dissipative Hamiltonian systems (SDHSs); multiple Lyapunov functions (MLFs); symbolic computation.

1 Introduction

Switched systems are an important class of hybrid systems, which have developed rapidly in recent years. The motivation for studying switched systems is from the fact that many practical systems are inherently multi-models in the sense that several dynamical subsystems are required to describe their behaviour depending on various changing environmental factors. The widespread applications of switched systems are also motivated by increasing performance requirements in control, especially in the presence of large disturbances or uncertainties [1-4]. A common Lyapunov function for all subsystems is a necessary and sufficient condition for switched system to be asymptotically stable under arbitrary switching laws [5, 6]. It has been shown that the multiple Lyapunov functions (MLFs) approach proposed in [7] is an effective tool for choosing such switching laws. A hybrid nonlinear control methodology for a broad class of switched nonlinear systems with input constraints was proposed [8], which is the integrated synthesis via the MLFs. The sufficient conditions for exponential stability and weighted L_2 -gain developed for a class of switching signals with average dwell time [9]. The MLFs is derived a sufficient condition for the switched nonlinear system to be asymptotically stable with H_∞ -norm bound [10]. The switched dissipative Hamiltonian system (SDHS) is a kind of important nonlinear hybrid systems. Such system not only plays an important role in development of hybrid control theory, but also finds many applications in practical control designs for obtaining better control performance [11, 12]. The stability of SDHSs under arbitrary switching paths has been investigated via the dissipative Hamiltonian structural properties and the MLFs [13].

Adaptive control problems and the parameterization of all stabilizing controllers are two of the most researched and written about issues in modern control. Adaptive H_∞ control has proved to be a very useful method in nonlinear control systems which are sensitive to parameter uncertainty. With adaptive H_∞ control it is possible to estimate parameters errors and to compensate for those errors, and improve robustness. Some basic adaptive control methods for first-order, linear, nonlinear, single-input and multi-input systems have been described [14]. [15] described recursive methodology of back-stepping for nonlinear and adaptive control design. In [16], a novel approach for the solution of nonlinear adaptive control problems is proposed. The approach for the solution of nonlinear adaptive control problems is proposed. Furthermore, a family of controllers can solve the underlying H_∞ control problems for a controller satisfying the additional design objectives, which are internal stability and disturbance attention. A class of local nonlinear H_∞ controllers is parameterized as nonlinear fractional transformations on contractive, stable nonlinear parameters via output feedback have proposed [17,18]. In [19,20], the state-space formulas extended and a family of H_∞ state-feedback controller presented for n-dimensional nonlinear system. A family of reliable nonlinear H_∞ controller has been proposed via solving the HJ equations [21]. The generalized Hamiltonian system studied and a family of parameterized controller proposed in H_∞ control and adaptive control [22,23]. The controllers [17-23] are intended to solve the control problem for just one system. Particularly, there are few results on adaptive H_∞ control design of SDHSs so far.

Therefore, how to find a method for designing parameterized controller to solve adaptive H_∞ problem for SDHSs is a challenging issue. In this paper, we show that all the Hamiltonian function of the subsystems can be used as the MLFs for the SDHSs under a realistic assumption. We present a novel, straightforward and convenient method to design a family of controllers to insure that the SDHSs are adaptive H_∞ control.

The remainder of this paper is organized as follows. In Section 2, the problem of adaptive H_∞ control for SDHSs is formulated. The main contributions of this paper are then given in Section 3, in which a family of controllers and an algorithm for solving parameters of controller are provided, respectively. We present a numerical example for illustrating effectiveness and feasibility of controller in Section 4 and conclusions follow in Section 5.

2 Problem Formulation

Consider the following SDHSs with disturbances and parameter perturbations

$$\begin{cases} \dot{x} = [J_{\lambda(t)}(x,p) - R_{\lambda(t)}(x,p)]\nabla H_{\lambda(t)}(x,p) \\ \quad + g_{\lambda(t)}(x)u + \bar{g}_{\lambda(t)}(x)\omega \\ z = h_{\lambda(t)}(x) g_{\lambda(t)}{}^T(x)\nabla H_{\lambda(t)}(x) \\ x(t_0) = x_0 \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector; $u \in \mathbb{R}^m$ is the controller; $\omega \in \mathbb{R}^s$ is the disturbances; $p \in \mathbb{R}^r$ is a constant unknown vector representing parameter perturbations; $J^T_{\lambda(t)}(x,p) = -J_{\lambda(t)}(x,p) \in \mathbb{R}^{n\times n}$, $J^T_{\lambda(t)}(x,0) = J_{\lambda(t)}(x)$; $0 \le R_{\lambda(t)}(x,p) \in \mathbb{R}^{n\times n}$, $R_{\lambda(t)}(x,0) = R_{\lambda(t)}(x)$; $g_{\lambda(t)}(x) \in \mathbb{R}^{n\times m}$ and $\bar{g}_{\lambda(t)}(x) \in \mathbb{R}^{n\times s}$ are sufficiently smooth functions; $h_{\lambda(t)}(x)$ is a weighting matrix; $z \in \mathbb{R}^q$ is the penalty; $H_{\lambda(t)}(x,p)$ is the subsystem's Hamiltonian function (the total energy) satisfying $H_{\lambda(t)}(x,p) > 0$, $H_{\lambda(t)}(x,0) = H_{\lambda(t)}(x)$ and $\nabla H_{\lambda(t)}(x,p) = \frac{\partial H}{\partial x}(x,p)$; the map $\lambda(t): [t_0, +\infty) \to \Lambda = \{1,2,\cdots N\}$ is a piecewise constant one, called the switching law or switching path, and $\lambda(t) = i(i = 1,2,\cdots,N)$ denotes that the $i\,th$ subsystem is realized. For an arbitrary switching law $\lambda(t)$, $\{t_m\}_{m=0}^{+\infty}$ is called the switching time sequence, which is assumed to satisfy $t_0 < t_1 < t_2 < \cdots < t_m < \cdots + \infty$. If $t_{m+1} = \infty$, the $i\,th$ subsystem of system (1) is always realized in $\lfloor t_m, +\infty)$ and the whole system is naturally stable.

To use the dissipative Hamiltonian structural properties and the MLFs, we propose two assumptions as follows.

Assumption 1. For $\forall i \in \Lambda$, the Hamiltonian function $H_i(x)$ satisfies $H_i(x) \in C^2$ and the Hessian matrix $\mathrm{Hess}(H_i(x_0)) > 0$.

Remark 1. Note that $H_i(x)$ has a local minimum at the equilibrium x_0 of system (1). It is straightforward that in Assumption 1, $H_i(x) \in C^2$ guarantees the existence of $\mathrm{Hess}(H_i(x))$ and $\mathrm{Hess}(H_i(x_0)) > 0$ guarantees that $H_i(x)$ is strict convex on some neighborhood of equilibrium x_0.

Assumption 2. For $\forall i \in \Lambda$ and $\forall x, y \in \mathbb{R}^n$, the Hamiltonian function $H_i(x)$ satisfies $H_i(x) > H_i(y) \Leftrightarrow \|x\|_P > \|y\|_P$, where $\|x\|_P \triangleq \sup_{\|x\|=1} x^T P x$ and $P > 0$ is a positive definite matrix.

Remark 2. Assumption 2 implies that the Hamiltonian function (candidate Lyapunov function)

$H_i(x)$ increases with the increase of $\|x\|_P$. Obviously, this assumption can be satisfied for many Hamiltonian systems. Thus, Assumption 2 is a realistic one.

Lemma 1. [13] If Assumption 2 holds, $u \equiv 0$, $\omega \equiv 0$ and $R_i(x) > 0$, $\forall i \in \Lambda$, then system (1) is globally asymptotically stable under the arbitrary switching laws $\lambda(t)$ with its dwell time $\tau > 0$.

Definition 1. The problem considered in this paper is to propose a family of adaptive H_∞ controller for systems (1), which can be described as: given a disturbance attenuation level $\gamma > 0$ and an arbitrary switching law $\lambda(t)$, we can obtain a family of adaptive H_∞ controller of the form

$$\begin{cases} u = u_{\lambda(t)} = \alpha_{\lambda(t)}\left(x, \hat{\theta}\right) \\ \dot{\hat{\theta}} = \beta_{\lambda(t)}\left(x, \hat{\theta}\right) \end{cases} \tag{2}$$

such that

R1: For $\forall i \in \Lambda$ $\forall \omega$, $t \in T_m$ and $m \in Z_+$, the inequality $\dot{H}_i(x) + \Psi_i(x) \le \frac{1}{2}\left\{\gamma^2 \|\omega\|^2 - \|z\|^2\right\}$ holds along the trajectories of the closed-loop system, which consisted of system (1) and the controller (2), where $\Psi_i(x) \ge 0$ is a scalar function.

R2: The closed-loop system (1) is asymptotically stable when $\omega \equiv 0$.

3 Main Results

In this section, we propose a family of adaptive H_∞ controller for system (1) by using the dissipative Hamiltonian structural properties and the MLFs and an algorithm for solving parameters with symbolic computation. The parameterization method suggests a framework to solve adaptive H_∞ control problem of SDHSs.

Notation 1. $\nabla H_i = \nabla H_i(x)$, $\Phi_i = \Phi_i(x)$, $g_i = g_i(x), \bar{g}_i = \bar{g}_i(x), h_i = h_i(x), K_i = K_i(x)$.

3.1 A family of adaptive H_∞ controllers

Assumption 3. There exits an $m \times n_1$ matrix Φ such that

$$\left[J_i(x,p) - R_i(x,p)\right]\Delta_{H_i}(x,p) = g_i \Phi_i \theta_i \tag{3}$$

where $\Delta_{H_i}(x,p) = \nabla H_i(x,p) - \nabla H_i$ and $\theta_i \in \mathbb{R}^{n_1}$ is a constant parameter perturbation vector depending on p.

Remark 3. Assumption 3 is the matched condition, a common assumption in adaptive control of Hamiltonian systems. In most cases, we can find Φ_i and θ_i such that (3) holds [24].

Theorem 1. Suppose Assumption 1 and 2 hold for system (1) and give a disturbance attenuation level $\gamma > 0$, to $\forall i \in \Lambda$

$$R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right) \ge 0 \tag{4}$$

$$\nabla H_i^T g_i K_i \le 0 \tag{5}$$

hold simultaneously. Then under an arbitrary switching law $\lambda(t)$, adaptive H_∞ control of system (1) can be realized by following controller which satisfies the rules R1 and R2.

$$\begin{cases} u = u\big|_{\lambda(t)=i} = -\frac{1}{2}\left(h_i^T h_i + \frac{1}{\gamma^2} I_m\right) g_i^T \nabla H_i + K_i - \Phi_i \hat{\theta}_i \\ \dot{\hat{\theta}}_i = Q_i \Phi_i^T g_i^T \nabla H_i \end{cases} \tag{6}$$

where $K_i \in \mathbb{R}^{m\times 1}$ is parameterized part of controller, I_m is an $m \times m$ unit matrix, $\hat{\theta}_i$ is an estimation of θ_i and Q_i is the adaptation gain matrix, which is constant, and note

$$\Psi_i(x) = \nabla H_i^T \left(R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i - \nabla H_i^T g_i K_i \ge 0.$$

Proof. Suppose $\lambda(t) = i \in \Lambda$, $t \in T_m$, $m \in Z_+$ is an arbitrary switching law. Substituting (6) into (1) and using condition (3) yield

$$\begin{cases} \dot{x} = \left[J_i(x,p) - R_i(x,p)\right]\nabla H_i - \frac{1}{2} g_i h_i^T h_i g_i^T \nabla H_i \\ \quad - \frac{1}{2\gamma^2} g_i g_i^T \nabla H_i + g_i K_i + g_i \Phi_i\left(\theta - \hat{\theta}\right) + \bar{g}_i \omega \\ \dot{\hat{\theta}} = Q_i \Phi_i^T g_i^T \nabla H_i \\ z = h_i g_i^T \nabla H_i \end{cases} \tag{7}$$

which can be rewritten as an augmented Hamiltonian system

$$\begin{cases} \dot{X} = \left[\bar{J}_i(X) - \bar{R}_i(X)\right]\nabla \bar{H}_i(X) + G_i \nu_i + \bar{G}_i(X)\omega \\ z = h_i G_i^T \nabla \bar{H}_i(X) \end{cases} \tag{8}$$

where $X = \left[x^T, \hat{\theta}^T\right]^T$, $G_i(X) = \left[g_i^T, 0\right]^T$, $\bar{G}_i(X) = \left[\bar{g}_i^T, 0\right]^T$, $\nu_i = \left[K_i, 0\right]^T$ are with proper dimensions; $\nabla \bar{H}_i(X) = \left[\nabla H_i^T, \nabla H_i\left(\hat{\theta}\right)^T\right]^T$,

$$\bar{J}_i(X) = \begin{bmatrix} J_i(x,p) & -\left(Q\Phi_i^T g_i^T\right)^T \\ Q\Phi_i^T g_i^T & 0 \end{bmatrix},$$

$$\bar{R}_i(X) = \begin{bmatrix} R_i(x,p) + \frac{1}{2} g_i h_i^T h_i g_i^T + \frac{1}{2\gamma^2} g_i g_i^T & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{H}_i(X) = H_i + \frac{1}{2}\left(\theta - \hat{\theta}\right)^T Q^{-1}\left(\theta - \hat{\theta}\right),$$

It is easy see that system (8) is still a class of dissipative Hamiltonian systems.

We choose the candidate Lyapunov function $V_i(X) = \bar{H}_i(X) - c \geq 0$ (c is some number such that $V_i(X)$ is positive definite near $X_0 = \left(x_0^T, \theta^T\right)^T$ and $\bar{H}_i(X)$ has a local minimum at X_0). For system (8), we have

$$\begin{aligned}
\dot{\bar{H}}_i(X) &= \nabla \bar{H}_i^T(X)\left[\bar{J}_i(X) - \bar{R}_i(X)\right]\nabla \bar{H}_i(X) \\
&\quad + \nabla \bar{H}_i^T(X) G_i \nu_i + \nabla \bar{H}_i^T(X)\bar{G}_i \omega \\
&= -\nabla H_i^T\left(R_i(x,p) + \frac{1}{2} g_i h_i^T h_i g_i^T + \frac{1}{2\gamma^2} g_i g_i^T\right)\nabla H_i \\
&\quad + \nabla H_i^T g_i K_i + \nabla H_i^T \bar{g}_i \omega \\
&= -\nabla H_i^T\left(R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i \\
&\quad + \nabla H_i^T g_i K_i - \frac{1}{2\gamma^2}\nabla H_i^T \bar{g}_i \bar{g}_i^T \nabla H_i \\
&\quad - \frac{1}{2}\nabla H_i^T g_i h_i^T h_i g_i^T \nabla H_i + \nabla H_i^T \bar{g}_i \omega \\
&= -\nabla H_i^T\left(R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i \\
&\quad + \nabla H_i^T g_i K_i + \frac{1}{2}\left(\gamma^2\|\omega\|^2 - \|z\|^2\right) \\
&\quad - \frac{1}{2}\left\|\frac{1}{\gamma}\bar{g}_i^T \nabla H_i - \gamma\omega\right\|^2
\end{aligned} \tag{9a}$$

We obtain

$$\begin{aligned}
&\dot{\bar{H}}_i(X) + \nabla H_i^T\left(R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i - \nabla H_i^T g_i K_i \\
&= \dot{\bar{H}}_i(X) + \Psi_i(X) \\
&= \frac{1}{2}\left(\gamma^2\|\omega\|^2 - \|z\|^2\right) - \frac{1}{2}\left\|\frac{1}{\gamma}\bar{g}_i^T \nabla H_i - \gamma\omega\right\|^2 \\
&\leq \frac{1}{2}\left(\gamma^2\|\omega\|^2 - \|z\|^2\right)
\end{aligned} \tag{9b}$$

where $\Psi_i(X) = \bar{R}_i(X) + \frac{1}{2\gamma^2}\left(G_i G_i^T - \bar{G}_i \bar{G}_i^T\right) - G_i \nu_i \geq 0$ is a scalar function.

So the rule R1 can be satisfied, which implies the L_2 gain of the closed-loop system (8) controlled by controller (6) (from ω to z) is bounded by γ .

Next, we prove that the closed-loop system is asymptotically stable at x_0 under the arbitrary switching laws $\lambda(t)$, when $\omega \equiv 0$.

When $\omega \equiv 0$, the closed-loop system can be expressed

$$\begin{aligned}
\dot{\bar{H}}_i(X) &= \nabla \bar{H}_i^T(X)\left[\bar{J}_i(X) - \bar{R}_i(X)\right]\nabla \bar{H}_i(X) \\
&\quad + \nabla \bar{H}_i^T(X) g_i \nu_i \\
&= -\nabla H_i^T\left(R_i(x,p) + \frac{1}{2} g_i h_i^T h_i g_i^T + \frac{1}{2\gamma^2} g_i g_i^T\right)\nabla H_i \\
&\quad + \nabla H_i^T g_i K_i \\
&= -\nabla H_i^T\left(R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i \\
&\quad - \frac{1}{2}\nabla H_i^T g_i h_i^T h_i g_i^T \nabla H_i - \frac{1}{2\gamma^2}\nabla H_i^T \bar{g}_i \bar{g}_i^T \nabla H_i + \nabla H_i^T g_i K_i \\
&= -\nabla H_i^T\left(R_i(x,p) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i \\
&\quad - \frac{1}{2}\left\|h_i g_i^T \nabla H_i\right\|^2 - \frac{1}{2\gamma^2}\left\|\bar{g}_i^T \nabla H_i\right\|^2 + \nabla H_i^T g_i K_i \leq 0
\end{aligned} \tag{10}$$

From Lemma 1, the rule R2 can be satisfied.

So the closed-loop system (1) controlled by controller (6) is asymptotically stable under the arbitrary switching laws $\lambda(t)$. This completes the proof. □

Remark 4. (1)The condition (4) in Theorem is not restrictive, and can be easily satisfied in many systems.

(2) K_i is a polynomial vector with parameters. We can obtain the parameters of K_i via solving condition (5).

(3)The proposed parameterization method can be used for general nonlinear switched systems, and of course the first step in applying the method is to express the nonlinear systems as DSHSs based on dissipative Hamiltonian realization methods [25, 26].

3.2 Solving parameters algorithm (SP)

From condition (4), we can obtain the γ_i^*. Let $\gamma \geq \max\{\gamma_i^*\}$ such that condition (4) holds. Then we propose an algorithm to find parameters ranges of controller (6) via solving the parameters of K_i in condition (5), respectively. Note $N(x) = K_i$. The SP algorithm now proceeds as follows.

S1. Set $N(x) = \begin{bmatrix} N_1(x) & N_2(x) & \cdots & N_m(x) \end{bmatrix}^T$ and suppose a positive integer r, which is the degree of polynomial vector $N(x)$. Write $N_i(x) = \sum_{j=1,r}^{j=l} a_{ij} p_r(x)$, where $l = \sum_r c(n+r-1, r)$, $p_r(x) = \prod_{i=1}^n x_i^{r_i}$ and n is the number of state variable.

S2. Let $S = -\nabla H_i^T g_i N(x)$.

S3. The influence of high order items can be ignored because this paper consider locally asymptotically stable for system. Choose all terms of $\deg(S) \geq 3$ and $\deg(S) = 1$ from S and let the coefficients of these terms be zero. So obtain a set of equations A.

S4. Obtain a set of parameters solution U_1 via solving A by using cylindrical algebraic decompositions (CAD) algorithm [27].

S5. Substitute U_1 into S and obtain a new polynomial S', which is a quadratic form.

S6. Rewrite S' as coefficient matrix M, and all principal minors of M must be positive semi-definite [28]. Choose all principal minors of M and obtain inequalities B.

S7. Obtain a set of parameters solution U_2 via solving B by using CAD algorithm.

S8. Let $U = U_1 \bigcup U_2$ and substitute U into controller (4), thus obtain the polynomial parameterized controller. This completes the algorithm. □

Remark 5. (1)The SP algorithm starts from $r = 1$ normally.

(2)The CAD algorithm is given by Semi-Algebraic-Set-Tools of Regular-Chains in Maple 16.

(3)The set of parameters solution obtained by SP algorithm is a subset of solutions of results by using CAD algorithm.

4 Numerical experiment

Consider a SDHS with disturbances and parameter perturbations,

$$J_1(x,p) = \begin{bmatrix} 0 & x_1^2 - \sin(x_2 x_3) & -1 \\ -x_1^2 + \sin(x_2 x_3) & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}$$

$$g_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},\ g_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},\ R_1(x,p) = \mathrm{Diag}\{1,2,1\},$$

$$H_1(x,p) = x_1^2 + x_2^2 + (1+p_1)x_3^2,$$

$$J_2(x,p) = \begin{bmatrix} 0 & -x_3^2 & -2 \\ x_3^2 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$g_{21} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},\ g_{22} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 1 & 0 \end{bmatrix},\ R_2(x,p) = \mathrm{Diag}\{1,3,2\}$$

$$H_2(x,p) = \frac{1}{2}x_1^2 + 2x_2^2 + (1+p_2)x_3^2$$

where $|p_1| \leq 1$ and $|p_2| \leq 1$. (11)

4.1 Controller design and solving parameters

From system (11), it is easy to get

$$\mathrm{Hess}(H_1(x_0)) = Diag\{2,2,2\} > 0,$$

$$\mathrm{Hess}(H_2(x_0)) = Diag\{1,4,2\} > 0$$

So Assumption 1 holds.

Then, we check that condition (4) holds for all x and given γ. From system (11), we have

Let $\gamma_1^* = 1$, $\gamma_2^* = \sqrt{2}$. To ensure that condition (4) holds, the following statement should be satisfied.

$$\gamma \geq \max\{\gamma_1^*, \gamma_2^*\} \tag{12}$$

Next, we consider condition (5) such that system (11) satisfies robustness in H_∞ control. Set the Lyapunov function $V_1(x) = H_1(x)$, $V_2(x) = H_2(x)$. It follows from controller (6) that

$$\begin{cases} u_1 = \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = -\frac{1}{2}\left[h_1^T h_1 + \frac{1}{\gamma^2} I_m \right] g_{11}^T \nabla H_1 + K_1 - \Phi_1 \hat{\theta}_1 \\ \dot{\hat{\theta}}_1 = Q_1 \Phi_1^T g_{11}^T \nabla H_1 \end{cases} \tag{13}$$

$$\begin{cases} u_2 = \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = -\frac{1}{2}\left[h_2^T h_2 + \frac{1}{\gamma^2} I_m \right] g_{21}^T \nabla H_2 + K_2 - \Phi_2 \hat{\theta}_2 \\ \dot{\hat{\theta}}_2 = Q_2 \Phi_2^T g_{21}^T \nabla H_2 \end{cases} \tag{14}$$

where $K_1 = \begin{bmatrix} K_{11} & K_{12} \end{bmatrix}^T$, $K_2 = \begin{bmatrix} K_{21} & K_{22} \end{bmatrix}^T$.

We know $n = 4$ in system (11) and let $r = 1$. We have

$K_{11} = a_{11}x_1 + a_{12}x_2 + a_{13}x_3$, $K_{12} = b_{11}x_1 + b_{12}x_2 + b_{13}x_3$,
$K_{21} = a_{21}x_1 + a_{22}x_2 + a_{23}x_3$, $K_{22} = b_{21}x_1 + b_{22}x_2 + b_{23}x_3$,
where $a_{ij}, b_{ij}, i = 1,2,\ j = 1,2,3$ are the parameters.

From system (11), we obtain

$$\nabla H_1(x) = \begin{bmatrix} 2x_1 & 2x_2 & 2x_3 \end{bmatrix}^T$$
$$\nabla H_2(x) = \begin{bmatrix} x_1 & 4x_2 & 2x_3 \end{bmatrix}^T$$

Let $S_1 = -\nabla H_1^T g_{11} K_1$, $S_2 = -\nabla H_2^T g_{21} K_2$, we have

$$\begin{aligned} S_1 &= -2a_{11}x_1^2 - (2a_{12} + 2b_{11})x_1x_2 - (2a_{11} + 2a_{13})x_1x_3 \\ &\quad - 2b_{12}x_2^2 - (2a_{12} + 2b_{13})x_2x_3 - 2a_{13}x_3^2 \\ S_2 &= -b_{21}x_1^2 - (4a_{21} + b_{22})x_1x_2 - (2b_{21} + b_{23})x_1x_3 \\ &\quad - 4a_{22}x_2^2 - (4a_{23} + 2b_{22})x_2x_3 - 2b_{23}x_3^2 \end{aligned},$$

S_1 and S_2 are quadratic form and can be rewritten as coefficient matrix M_1 and M_2 (multiply constant 2 for simplifying computation) , respectively.

$$M_1 = \begin{bmatrix} -4a_{11} & -2a_{12} - 2b_{11} & -2a_{11} - 2a_{13} \\ -2a_{12} - 2b_{11} & -4b_{12} & -2a_{12} - 2b_{13} \\ -2a_{11} - 2a_{13} & -2a_{12} - 2b_{13} & -4a_{13} \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -2b_{21} & -4a_{21} - b_{22} & -2b_{21} - b_{23} \\ -4a_{21} - b_{22} & -8a_{22} & -4a_{23} - 2b_{22} \\ -2b_{21} - b_{23} & -4a_{23} - 2b_{22} & -4b_{23} \end{bmatrix}.$$

All principal minors of M_1 and M_2 must be positive semi-definite, we can obtain a set of parameters solution U_1 and U_2 using CAD algorithm, respectively.

$$U_1 = \{a_{11} \le 0, a_{12} = 0, \mathrm{a}_{13} = 0, \mathrm{b}_{11} = 0, b_{12} \le 0, b_{13} = 0\} \tag{15}$$

$$U_2 = \{a_{21} = 0, a_{22} \le 0, \mathrm{a}_{23} = 0, \mathrm{b}_{21} \le 0, b_{22} = 0, b_{23} = 2b_{21}\} \tag{16}$$

Substitute U_1 and U_2 into controller (13) and (14), we obtain

$$\begin{cases} u|_{\lambda(t)=1} = \begin{bmatrix} -\left(h_{11}^2 + \frac{1}{\gamma^2}\right)(x_1 + x_3) + a_{11}x_1 \\ -\left(h_{12}^2 + \frac{1}{\gamma^2}\right)x_2 + b_{12}x_2 \end{bmatrix} - \Phi_1\hat{\theta}_1 \\ \dot{\hat{\theta}}_1 = Q_1\left(-2x_1x_3 + 4x_2x_3 - 2x_3^2\right) \end{cases} \tag{17}$$

where $a_{11} \le 0$, $b_{12} \le 0$.

$$\begin{cases} u|_{\lambda(t)=2} = \begin{bmatrix} -2\left(h_{21}^2 + \frac{1}{\gamma^2}\right)x_2 + a_{22}x_2 \\ -\left(h_{22}^2 + \frac{1}{\gamma^2}\right)\left(\frac{1}{2}x_1 + x_3\right) + b_{21}x_1 + 2b_{21}x_3 \end{bmatrix} - \Phi_2\hat{\theta}_2 \\ \dot{\hat{\theta}}_2 = Q_2\left(-2x_1x_3 - 4x_2x_3 - 4x_3^2\right) \end{cases} \tag{18}$$

where $a_{22} \le 0$, $b_{21} \le 0$.

So we have a family of controllers of system (11), which have parameters.

4.2 **Simulations and results**

In order to evaluate the robustness of the controller (17) and (18), we set the parameters of system (11) as: $\gamma = 2$, $h_{11} = 2$, $h_{12} = 3$, $h_{21} = 3$, $h_{22} = 2$ and the parameters of controller as: $a_{11} = -10$, $b_{12} = -10$, $a_{22} = -10$, $b_{21} = -10$. Choose $\Phi_1 = \begin{bmatrix} -x_3 & 2x_3 \end{bmatrix}^T$, $\Phi_2 = \begin{bmatrix} -x_3 & -2x_3 \end{bmatrix}^T$, $\hat{\theta}_1 = 2p_1$, $\hat{\theta}_2 = 2p_2$. We have

$$\begin{aligned} &\left[J_1(x,p) - R_1(x,p)\right]\Delta_{H_1}(x,p) = g_{11}\Phi_1\hat{\theta}_1 \\ &= \begin{bmatrix} -2p_1x_3 & 4p_1x_3 & -2p_1x_3 \end{bmatrix}^T \\ &\left[J_2(x,p) - R_2(x,p)\right]\Delta_{H_2}(x,p) = g_{21}\Phi_2\hat{\theta}_2 \\ &= \begin{bmatrix} -4p_2x_3 & -2p_2x_3 & -4p_2x_3 \end{bmatrix}^T \end{aligned},$$

So the condition (6) holds for system (11). Let $Q_1 = 1$, $Q_2 = 1$, $p_1 = 0.8$, $p_2 = 0.8$, we obtain the controller (19) and (20),

$$\begin{cases} u|_{\lambda(t)=1} = \begin{bmatrix} -\frac{57}{4}x_1 - \frac{17}{4}x_3 + x_3\hat{\theta}_1 \\ -\frac{77}{4}x_2 - 2x_3\hat{\theta}_1 \end{bmatrix} \\ \dot{\hat{\theta}}_1 = -2x_1x_3 + 4x_2x_3 - 2x_3^2 \end{cases} \tag{19}$$

$$\begin{cases} u|_{\lambda(t)=2} = \begin{bmatrix} -\dfrac{57}{2}x_2 + x_3\hat{\theta}_2 \\ -\dfrac{97}{8}x_1 - \dfrac{97}{4}x_3 + 2x_3\hat{\theta}_2 \end{bmatrix} \\ \dot{\hat{\theta}}_2 = -2x_1x_3 - 4x_2x_3 - 4x_3^2 \end{cases} \tag{20}$$

Suppose that $x(0) = (0,1,0.4)^T$ is the pre-assigned operating point of system (11), we impose an external disturbance $\omega = [4,4;4,4]^T$ on system (11) during the time period 0.6~0.9s and 3~3.6s. The simulation results are shown in Figure. 1 and Figure. 2.

Figure. 1 and Figure. 2 are the response of the state, the controller (19) and (20) and estimate $\hat{\theta}$ of system (11) under the switching law $\lambda_1(t)$, respectively.

$$\lambda_1(t) = \begin{cases} 2, t \in [t_{2k}, t_{2k+1}), t_{2k+1} - t_{2k} = 0.15, \\ 1, t \in [t_{2k+1}, t_{2k+2}), t_{2k+2} - t_{2k+1} = 0.15, \end{cases}$$

$k = 0,1,2,\cdots,$

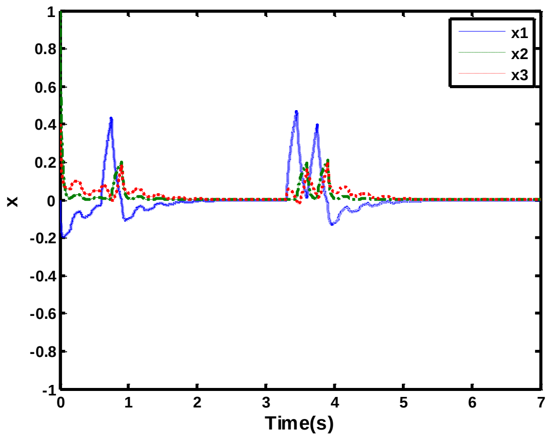

Figure. 1: Swing curves of x under $\lambda_1(t)$

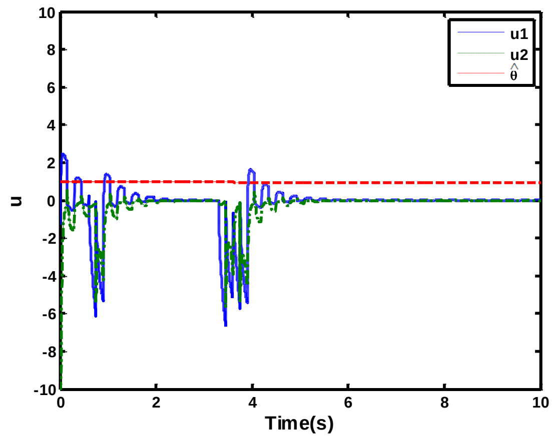

Figure. 2: Swing curves of u and $\hat{\theta}$ under $\lambda_1(t)$

When the conditions and the parameters of system (11) are not changing, the parameters of controller as: $a_{11} = -1$, $b_{12} = -1$, $a_{22} = -1$, $b_{21} = -1$. We obtain the other controller:

$$\begin{cases} u|_{\lambda(t)=1} = \begin{bmatrix} -\dfrac{21}{4}x_1 - \dfrac{17}{4}x_3 + x_3\hat{\theta}_1 \\ -\dfrac{41}{4}x_2 - 2x_3\hat{\theta}_1 \end{bmatrix} \\ \dot{\hat{\theta}}_1 = -2x_1x_3 + 4x_2x_3 - 2x_3^2 \end{cases} \tag{21}$$

$$\begin{cases} u|_{\lambda(t)=2} = \begin{bmatrix} -\dfrac{39}{2}x_2 + x_3\hat{\theta}_2 \\ -\dfrac{25}{8}x_1 - \dfrac{25}{4}x_3 + 2x_3\hat{\theta}_2 \end{bmatrix} \\ \dot{\hat{\theta}}_2 = -2x_1x_3 - 4x_2x_3 - 4x_3^2 \end{cases} \tag{22}$$

Figure. 3 and Figure. 4 are the response of the state, the controller (21) and (22) and estimate $\hat{\theta}$ of system (11) under the switching law $\lambda_1(t)$, respectively.

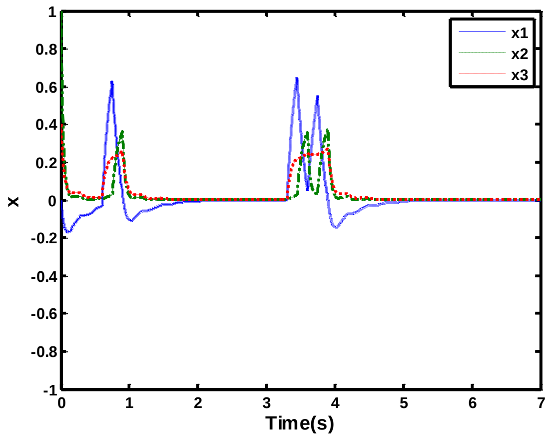

Figure. 3: Swing curves of x under $\lambda_1(t)$

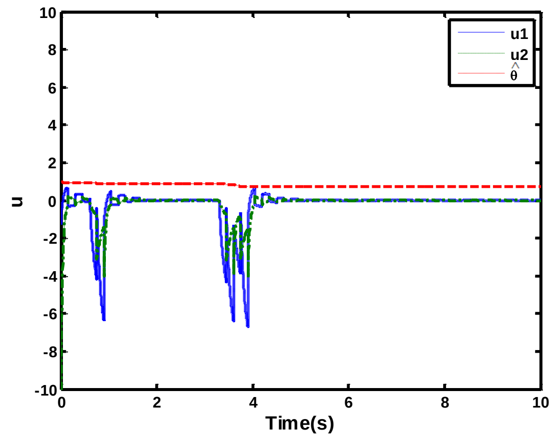

Figure. 4: Swing curves of u and $\hat{\theta}$ under $\lambda_1(t)$

From Figure. 3 and Figure. 1, we know that we can adjust the parameters value in controller (17) and (18) to insure that the system (11) is adaptive H_∞ control and optimize the robustness of system (11) under $\lambda_1(t)$.

Then, we consider the robustness of system (11) against external disturbance and parameter perturbations under the arbitrary switching law.

$$\lambda_2(t)=\begin{cases}1, & t\in[t_{2k},t_{2k+1}),t_{2k+1}-t_{2k}=0.4\times rand,\\ 2, & t\in[t_{2k+1},t_{2k+2}),t_{2k+2}-t_{2k+1}=0.4\times rand,\end{cases}$$

$k=0,1,2,\cdots,$

where $0<rand<1$.

Figure. 5 and Figure. 6 are the response of the state, the controller (21) and (22) and estimate $\hat{\theta}$ of system (11) under the switching law $\lambda_2(t)$, respectively.

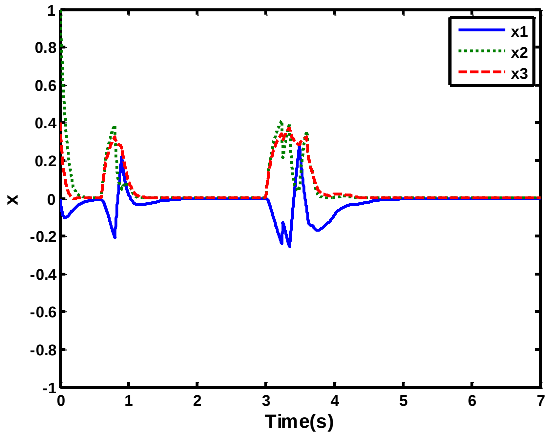

Figure. 3: Swing curves of x under $\lambda_2(t)$

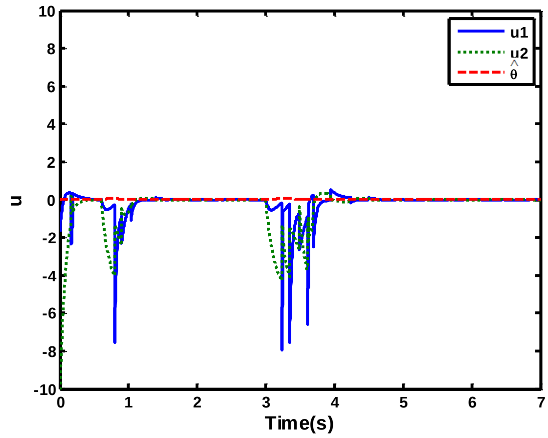

Figure. 4: Swing curves of u and $\hat{\theta}$ under $\lambda_2(t)$

From Figure.1 to Figure. 6, we can see that our adaptive controllers have stronger robustness against external disturbance and parameter perturbations under the fixed switching law or the arbitrary switching law.

5. Conclusions

In this paper, we consider adaptive H_∞ control problem for SDHSs under arbitrary switching laws. When there are external disturbances and parameter perturbations in such systems, a family of controllers has been obtained using the MLFs method for such systems and an algorithm for solving parameters of the controller has been proposed with symbolic computation. The numerical experiment and simulations show that the controllers have efficient in adaptive H_∞ control. The method proposed in this paper suggests a framework to solve adaptive H_∞ control problem of general nonlinear switched systems.

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References:

[1] D. Liberzon, A.S. Morse, Basic problems in stability and design of switched systems, *IEEE Control System Magazine*, 19(5), 1999, pp. 59-70.

[2] Zamani I, Shafiee M, Ibeas A, Exponential stability of hybrid switched nonlinear singular systems with time-varying delay, *Journal of the Franklin Institute*, 350(1), 2013, pp. 171–193.

[3] Xiong J, Lam J, Shu Z, et al., Stability analysis of continuous-time switched systems with a random switching signal, *IEEE Transactions on Automatic Contro*, 59(1), 2014, pp. 180-186.

[4] Ma D, Zhao J, Stabilization of networked switched linear systems: An asynchronous switching delay system approach, *Systems & Control Letters*, 77, 2015, pp. 46-54.

[5] Vu L, Liberzon D, Common Lyapunov functions for families of commuting nonlinear systems, *Systems & Control Letters*, 54(5), 2005, pp. 405-416.

[6] Sun Y G, Wang L, Xie G, Stability of switched systems with time-varying delays: delay-dependent common Lyapunov functional approach, *Proceeding of the 2006 American Control Conference*, American, 2006.

[7] M.S. Branicky, Multiple Lyapunov functions and other analysis tools for switched and hybrid

systems, *IEEE Transactions on Automatic Control*, 43(4), 1998, pp. 475-482.

[8] El-Farra N H, Mhaskar P, Christofides P D, Output feedback control of switched nonlinear systems using multiple Lyapunov functions, *Systems & Control Letters*, 54(11), 2005, pp. 1163-1182.

[9] X. Sun, J. Zhao, D. J. Hill, Stability and L_2-gain analysis for switched delay systems, *Automatica*, 42(10), 2006, pp. 1769-1774.

[10] R. Wang, M. Liu, J Zhao, Reliable H_∞ control for a class of switched nonlinear systems with actuator failures, *Nonlinear Analysis: Hybrid Systems*, 1(3), 2007, pp. 317-325.

[11] Escobar G, van der Schaft A J, Ortega R, A Hamiltonian viewpoint in the modeling of switching power converters, *Automatica*, 35(3), 1999, pp. 445-452.

[12] Haddad W M, Nersesor S G, Chellaboina V S, Energy-based control for hybrid port-controlled Hamiltonian systems, *Automatica*, 39(7), 2003, pp. 1425-1435.

[13] Zhu L, Wang Y, Study on the stability of switched dissipative Hamiltonian systems, *Science in China Series F: Information Sciences*, 49(5), 2006, pp. 578-591.

[14] J.J. E. Slotine and W. Li, Applied Nonlinear Control, Prentice-Hall, Inc., New Jersey, 1991.

[15] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*, New York, Wiley, 1995.

[16] A. Astolfi, D. Karagiannis, R. Ortega, Towards applied nonlinear adaptive control, *Annual Reviews in Control*, 32(2), 2008, pp. 136-148.

[17] Lu WM, Dolye JC, H_∞ control of nonlinear systems via output feedback: controller parameterization, *IEEE Transactions on Automatic Control*, 39(12), 1994, pp. 831-846.

[18] Astolfi A, Parameterization of output feedback controller that satisfy an H_∞ norm bound, *Proceedings of the 2nd European Control Conference*, Groningen, 1993, pp. 74-78.

[19] C.F. Yung, Y. P. Lin, F. B. Yeh, A family of nonlinear H_∞ output feedback Controllers, *IEEE Transactions on Automatic Control*, 41(2), 1996, pp. 232-236.

[20] C.F. Yung, J. L. Wu, T. T. Lee, Parameterization of nonlinear H_∞ state-feedback Controllers, *Automatica*, 33(7), 1997, pp. 1587-1590.

[21] Y.S. Fu, Z. H. Tian, S. J. Shi. A family of reliable nonlinear H_∞ state-feedback controllers. *IET Control Theory and Application*, 18(3), 2001, pp. 447-452.

[22] S. Xu, X. Hou, A family of H_∞ controllers for dissipative Hamiltonian systems, *International Journal of Robust and Nonlinear Control*, 22(10), 2012, pp. 1258-1269.

[23] S. Xu, X. Hou, A family of adaptive H_∞ controllers with full information for dissipative Hamiltonian systems, *International Journal of Automation and Computing*, 8(2), 2011, pp. 209-214.

[24] Shen. T, Ortega. R, Lu. Q, Mei. S, Tamura. K, Adaptive L_2 disturbance attenuation of Hamiltonian systems with parameter perturbations and application to power systems, *Proceedings of the 39th IEEE Conference on Decision and Control*, (5), 2000, pp. 4939-4944.

[25] Wang Y, Chen D, Li C, Ge Y, Dissipative Hamiltonian realization and energy-based L_2-disturbance attenuation control of multimachine power systems, *IEEE Transactions on Automatic Control*, 48(7), 2003, pp. 1428-1433.

[26] Wang Y, Chen D, Ge S, Approximate dissipative Hamiltonian realization and construction of local Lyapunov function, *Systems & Control Letters*, 56(2), 2007, pp. 141-149.

[27] Caviness, B. F., et al. Quantifier Elimination and Cylindrical Algebraic Decomposition, Vienna,*Springer Verlag*, 1998.

[28] K.X. Swamy, On Sylvester's criterion for positive semidefinite matrices, *IEEE Transactions on Automatic Control*, 18(3), 1973, pp. 306.