Adaptive Stabilization of Nonholonomic Mobile Robots with Unknown Kinematic Parameters

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Abstract: - This paper investigates the problem of adaptive stabilization of nonholonomic mobile robots with nonholonomic constraints under the condition that the kinematic parameters are unknown and no known constants can (lower and upper) bound them. By defining a new unknown parameter which need dynamic updating, and also by using input-state-scaling transformation and backstepping technique, an adaptive state-feedback stabilizing controller is designed. The asymptotical stability of the control system is proved with Lyapunov stability theory. A simulation example is provided to show the effectiveness of the proposed method.

Key-Words: -Nonholonomic mobile robots, Unknown kinematic parameters, Adaptive control

1 Introduction

In recent years, nonholonomic systems, which can be modeled with constraints concerning velocity or acceleration as well as coordinates and position angle, have become a hot research topic of the mechanical systems. As a class of typical nonholonomic systems, the mobile robots have caused the extensive concern. Nonholonomic mobile robots have good flexibility, since they could realize autonomous movement in the case of nobody involving. However, due to the limitations imposed by Brockett's condition[1], this class of nonlinear systems cannot be stabilized by stationary continuous state-feedback, although it is controllable. There are currently several effective control methodologies that overcome the topological obstruction. The idea of using time-varying smooth controllers was first proposed in [2], in order to stabilize a mobile robot. For driftless systems in chained form, several novel approaches have been proposed for the design of periodic, smooth, or continuous stabilizing controllers [3] [4]. Most of the time-varying control scheme suffer from a slow convergence rate and oscillation. However, it has been observed that a discontinuous feedback control schemes usually results in a fast convergence rate. An elegant approach to constructing discontinuous feedback controllers was developed in [5]. The drawback is that there is a restriction on the initial conditions of the controlled system. This limitation has been overcome by a switching state or output control scheme [6]. Subsequently, [7-9]further developed the discontinuous feedback control strategy based on different control targets, respectively.

However, those aforementioned constructive methods are considered in ideal cases. Because of the possible modeling errors and external disturbance, uncertainties do exist in any real world systems, which can degrade a system's performance and even cause system instability. Therefore, from a practical point of view, when designing controller for a system, uncertainties should be taken into account. As is known, adaptive control is one of the effective ways to deal with control systems with parametric uncertainty. Although it is not easy to propose adaptive control strategies for general nonlinear systems, a great deal of efforts have been made in this area and some well-known adaptive design methods are proposed for nonlinear systems with uncertain parameters (referring to [10] and the other references therein). Particularly, when the bounedness of unknown kinematic parameters are known, adaptive control technique was successfully applied to nonholonomic mobile robots in [11]. As its natural extension, the adaptive control for nonholonomic mobile robots with more uncertainties, i.e., the bounedness of unknown kinematic parameters are unknown, should be attention to. However, to the authors' knowledge, there is no result for such the problem.

In the paper, by flexibly combining input-state-scaling transformation and switching control strategy [6, 11], motivated by the adaptive control technique in [12], we successively overcome the technical obstacle caused by the unknown kinematic parameters, and furthermore, a new adaptive controller is explicitly developed such that the closed-loop system is globally asymptotically regulated at origin.

The rest of this paper is organized as follows. In Section 2, the problem formulation and preliminaries are given. Section 3 presents the input-state-scaling transformation and the backstepping design procedure, while Section 4 provides the switching control strategy and the main result. Section 5 gives simulation result to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 6.

2 Problem Formulation

The model of a mobile robot with two independently drivable wheels considered in this paper is shown in Fig.1, where XOY is the world coordinate system, $X_aO_aY_a$ is the coordinate system fixed the mobile robot body, O_a is the center of the axle of two driving wheels, (x, y) indicates the coordinate of the robot in world coordinate system, θ is the angle of moving direction (right angle to the wheel axis), v is the linear velocity of the robot and ω is its angular velocity.

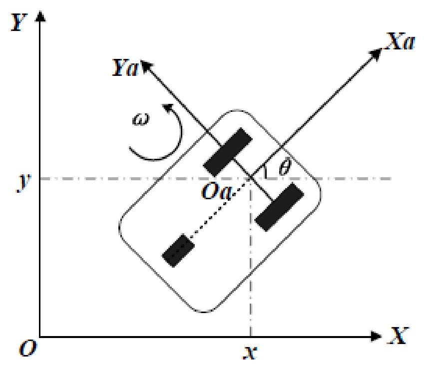


Fig. 1. The planar graph of a mobile robot

Although the model is the simplest one which has constrained by velocity, it has inherent difficulty of the nonholonomic system. Suppose that the wheels of the robot rotate without slipping. Thus, the constraint of the mobile robot motion is denoted by

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \tag{1}$$

And the model of a nonholonomic mobile robot can be obtained

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{2}$$

As stated in [6], the presupposition of the modeling (2) is based on the commonly admitted pure rolling without slipping condition and the assumption that the masses and inertias of the wheels are negligible. In other words, (2) only represents the modeling of the robot in the ideal case. A realistic description of the robot motion in the presence of uncertainties will give rise to far more complex equations. J.Hespanha et al.[13] introduced the mobile robot with parametric uncertainties

$$\begin{cases} \dot{x} = p_1 v\cos\theta \\ \dot{y} = p_1 v\sin\theta \\ \dot{\theta} = p_2\omega \end{cases} \tag{3}$$

where p_1 and p_2 are unknown positive parameters determined by the radius of the rear wheels and the distance between them.

For system (3), by taking the following state and input transformation

$$\begin{cases} x_0 = \theta \\ x_1 = x\sin\theta - y\cos\theta \\ x_2 = x\sin\theta + y\cos\theta \\ u_0 = \omega \\ u_1 = v \end{cases} \tag{4}$$

we obtain

$$\begin{cases} \dot{x}_0 = p_2 u_0 \\ \dot{x}_1 = p_2 u_0 x_2 \\ \dot{x}_2 = p_1 u_1 - p_2 u_0 x_1 \end{cases} \tag{5}$$

which belongs to the class of nonholonomic chain systems introduced in [6].

Remark 1. Though p_1 and p_2 taking values in a known interval $[p_{\min}, p_{\max}]$, the asymptotically stabilization of (3) or (5) was solved in [11], when the boundeness of p_1 and p_2 are unknown, the controller design for such system will be more difficult and cannot be solved by simply by general existing methods. It is precisely our intention of this paper.For details, in this paper, the main objective is, under the condition that the boundeness of kinematic parameters are unknown, to design an adaptive state-feedback controller $u_0 = u_0(x_0)$, $u_1 = u_0(x_0, x, \mu)$, $\dot{\mu} = \mu(x_0, x, \mu)$ such that all signals of the closed-loop system are bounded. Furthermore, global asymptotic regulation of the states are achieved, i.e. $\lim_{t\to\infty}(|x_0| + |x|) = 0$ .In order to achieve the above control objective, throughout the paper, the following assumption regarding system (5) is imposed.

Assumption 1 The signs of p_i , i=1,2 are known, and there exist unknown positive constants a and b such that

$$a \le p_i \le b \tag{6}$$

3. Adaptive controller design

In this section, we proceed to design a robust adaptive controller based on backstepping technique for the case $x_0(0) \ne 0$. While the case that the initial $x_0(0) = 0$ is dealt in next section. The inherently triangular structure of system (5) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

For x_0 – subsystem, we take the following control law

$$u_0 = -x_0 \tag{7}$$

. Take Lyapunov function $V_0 = x_0^2/2$ and then we have

$$\dot{V}_0 = -x_0^2 = -2V_0 \tag{8}$$

As a result, the first result of this paper is obtained.

Lemma 1. For any initial condition $x_0(t_0) \ne 0$,where $t_0 \ge 0$, the corresponding solution $x_0(t)$ exists

and globally exponentially converges to zero. Furthermore, the control $u_0(t)$ given by (7) also exists and does not cross zero.

From the above analysis, we can see the x_0 -state in (5) can be globally exponentially regulated to zero via u_0 in (7) as $t \to \infty$. However, it is troublesome in controlling the x −subsystem via the control input u_1 , because, in the limit (i.e. $u_0 = 0$), the x −subsystem is uncontrollable. This problem can be avoided by utilizing the following discontinuous input-state-scaling transformation:

$$z_1 = \frac{x_1}{u_0}, \quad z_2 = x_2 \tag{9}$$

According to (5) and (9), we get

$$\begin{cases} \dot{z}_1 = p_2 z_2 + p_2 z_1 \\ \dot{z}_2 = p_1 u_1 + p_2 z_1 x_0^2 \end{cases} \tag{10}$$

In the next, the controller u_1 will be recursively constructed by applying backstepping technique to the system (10). Before the beginning of the recursive design steps, we need to define the following unknown parameter

$$\Theta = (1 + b^2)(b/a, a^{-2}, a^2) \tag{11}$$

Step 1: Begin with z_1 -subsystem of (10), where z_2 is regarded as a virtual control. Introducing the transformation

$$e_1 = z_1, \quad e_2 = z_2 - z_2^* \tag{12}$$

and choosing Lyapunov function

$$V_1 = \frac{1}{2} z_1^2 + \frac{a}{2}\tilde{\Theta}^2 \tag{13}$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$ and $\hat{\Theta}$ is an estimate of Θ , it comes from (10), (12) and (13) that

$$\begin{aligned} \dot{V}_1 &= e_1(p_2 z_2 + p_2 z_1) - a\tilde{\Theta}\dot{\hat{\Theta}} \\ &\le -\frac{2}{a}e_1^2 + \frac{2}{a}e_1^2 + e_1(p_2 z_2 + p_2 z_1) - a\tilde{\Theta}\dot{\hat{\Theta}} \\ &\le -\frac{2}{a}e_1^2 + 2a\Theta e_1^2 + p_2 e_1(e_2 + z_2^*) + a\Theta e_1^2 - a\tilde{\Theta}\dot{\hat{\Theta}} \\ &\le -\frac{2}{a}e_1^2 + p_2 e_1 e_2 + a e_1(\frac{p_2}{a}z_2^* + 3e_1\hat{\Theta}) - a\tilde{\Theta}(\dot{\hat{\Theta}} - 3e_1^2) \end{aligned} \tag{14}$$

Obviously, the first virtual controller

$$z_2^* = -3e_1\hat{\Theta} \tag{15}$$

leads to

$$\dot{V}_1 \le -\frac{2}{a}e_1^2 + p_2 e_1 e_2 - a\tilde{\Theta}(\dot{\hat{\Theta}} - 3e_1^2) \qquad 16)$$

Step 2: Consider the candidate Lyapunov function

$$V_2 = V_1 + \frac{1}{2}e_2^2 \qquad (17)$$

In view of (10), (14), (16) and (17), we have

$$\begin{aligned}\dot{V}_2 &\le -\frac{2}{a}e_1^2 + p_2 e_1 e_2 - a\tilde{\Theta}(\dot{\hat{\Theta}} - 3e_1^2) + e_2\dot{e}_2 \\ &= -\frac{2}{a}e_1^2 + p_2 e_1 e_2 - a\tilde{\Theta}(\dot{\hat{\Theta}} - 3e_1^2) \qquad (18)\\ &\quad + e_2(p_1 u_1 + p_2 z_1 x_0^2) + 3e_2\hat{\Theta}(p_2 z_2 + p_2 z_1) + 3e_1 e_2 \dot{\hat{\Theta}}\end{aligned}$$

Using (11) and the Young's Inequality, we have following estimations:

$$\begin{aligned}&+p_2 e_1 e_2 \le \frac{1}{6a}e_1^2 + a\Theta e_2^2\gamma_{21}\\ &+p_2 e_2 z_1 x_0^2 \le \frac{1}{6a}e_1^2 + a\Theta e_2^2\gamma_{22}\\ &+3p_2 e_2 z_2\hat{\Theta} \le \frac{1}{6a}e_1^2 + a\Theta e_2^2\gamma_{23}\\ &+3p_2 e_2 z_1\hat{\Theta} \le \frac{1}{6a}e_1^2 + a\Theta e_2^2\gamma_{24} \qquad (19)\end{aligned}$$

where $\gamma_{21} = 3$, $\gamma_{22} = 3x_0^4$, $\gamma_{23} = 3 + 243\hat{\Theta}^4$ and $\gamma_{24} = 27\hat{\Theta}^2$.

Substituting (19) into (18) yields

$$\begin{aligned}\dot{V}_2 &\le -\frac{4}{3a}e_1^2 - a\tilde{\Theta}(\dot{\hat{\Theta}} - 3e_1^2) + e_2 p_1 u_1 + 3e_1 e_2\dot{\hat{\Theta}} + a\Theta e_2^2\sum_{j=1}^{4}\gamma_{2j}\\ &\le -\frac{4}{3a}e_1^2 - \frac{1}{a}e_2^2 - (a\tilde{\Theta} - 3e_1e_2)(\dot{\hat{\Theta}} - 3e_1^2 - e_2^2(1+\sum_{j=1}^{4}\gamma_{2j}))\\ &\quad + e_2 p_1 u_1 + a\hat{\Theta}e_2^2(1+\sum_{j=1}^{4}\gamma_{2j}) + 3e_1e_2[3e_1^2 - e_2^2(1+\sum_{j=1}^{4}\gamma_{2j})]\end{aligned}$$

(20)

To finally get the explicit expression of u_1, furthermore treatment should be taken for the last term on the right hand side of the second inequality in (20). By the Young's Inequality, we have

$$+3e_1e_2[3e_1^2 - e_2^2(1+\sum_{j=1}^{4}\gamma_{2j})] \le \frac{1}{3a}e_1^2 + ae_2^2\gamma_{25} \qquad (21)$$

where $\gamma_{25} = 27[3e_1^2 - e_2^2(1+\sum_{j=1}^{4}\gamma_{2j})]^2/2$.

Putting (20) and (21) together, we obtain

$$\begin{aligned}\dot{V}_2 &\le -\frac{1}{a}e_1^2 - \frac{1}{a}e_2^2 - (a\tilde{\Theta} - 3e_1e_2)(\dot{\hat{\Theta}} - 3e_1^2 - e_2^2(1+\sum_{j=1}^{4}\gamma_{2j}))\\ &\quad + ae_2\left[\frac{p_1}{a}u_1 + e_2(\hat{\Theta}(1+\sum_{j=1}^{4}\gamma_{2j}) + \gamma_{25})\right]\end{aligned}$$

(22)

Clearly, the smooth actual control u_1 and update law for $\hat{\Theta}$ can be easily chosen as

$$u_1 = -e_2(\hat{\Theta}(1+\sum_{j=1}^{4}\gamma_{2j}) + \gamma_{25}) \qquad (23)$$

$$\dot{\hat{\Theta}} = 3e_1^2 + e_2^2(1+\sum_{j=1}^{4}\gamma_{2j}) \qquad (24)$$

from which and (22) , it follows that

$$\dot{V}_2 \le -\frac{1}{a}e_1^2 - \frac{1}{a}e_2^2 \qquad (25)$$

We have thus far completed the controller design procedure for $x_0(t_0) \ne 0$. Without loss of generality, we can assume that $t_0 = 0$ ..

4. Switching controller and main result

In the preceding section, we have given controller design for $x_0(0) \ne 0$. Now, we discuss how to select the control laws u_0 and u_1 when $x_0(0) = 0$.

In the absence of the disturbances, most of the commonly used control strategies use constant control $u_0 = u_0^* \ne 0$ in time interval $[0, t_s)$. In this paper, we also use this method when $x_0(0) = 0$, with u_0 chosen as follows:

$$u_0 = u_0^*, \quad u_0^* > 0 \qquad (26)$$

During the time period $[0, t_s)$, using u_0 defined in (26), new control law $u_1 = u_1^*(x_0, x)$ can be obtained by the control procedure described above to the original x-subsystem in (5). Then we can conclude that the x-state of (5) cannot blow up during the time period $[0, t_s)$. Since $x_0(t_s) \ne 0$ at $t = t_s$, we can switch the control input u_0 and u_1 to (7) and (23), respectively.

We are now ready to state the main theorem of this paper.

Theorem 1. Under Assumption 1 , if the proposed control design procedure together with the above switching control strategy is applied to system (5), then, for any initial conditions in the state space $(x_0, x) \in R^3$, the states of the original system converge to the origin, and the other signals of the closed-loop system are bounded..

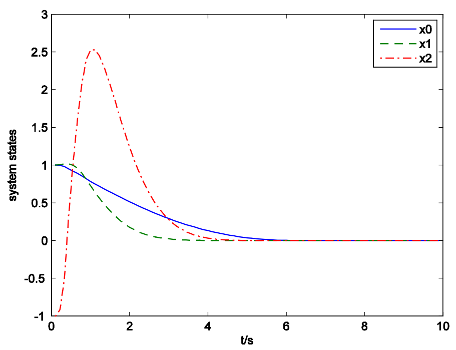
Proof. According to the above analysis, it suffices to prove the statement in the case where $x_0(0) \neq 0$.

From the Section 3, we know that x_0 can be globally regulated to zero as $t \to \infty$. In the z-coordinates, from the last step in the recursive backstepping design, we obtained (25), which implies that $\hat{\Theta}$ is bounded and $z \to \infty$ as $t \to \infty$. From the input-state-scaling transformation (11), we conclude that $x \to \infty$ as $t \to \infty$ ..This completes the proof of Theorem 1.

Remark 2. As seen from (23) and (7), the control law u_1 may exhibit extremely large value when $x_0(0) \neq 0$ is sufficiently small. This is unacceptable from a practical point of view. It is therefore recommended to apply (26) in order to enlarge the initial value of x_0 before we appeal to the finite-time converging controllers (7)and (23).

5. Simulation result

The simulation is implemented for the controllers defined in Sections 3-4. T. The simulation results for initial conditions $(x_0(0), x_1(0), x_2(0), \hat{\Theta}(0)) = (1,1,-1,1)$ are shown in Fig.2, while the results for $(x_0(0), x_1(0), x_2(0), \hat{\Theta}(0)) = (0,1,-1,1)$ are in Fig.3. From the figures, it is clear to see that the global asymptotic regulation of closed-loop system states are achieved.



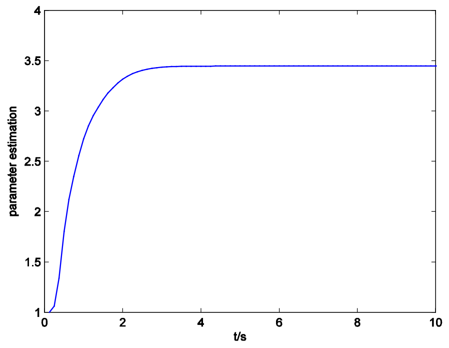


Fig.2. State trajectories of the closed-loop system with $(x_0(0), x_1(0), x_2(0), \hat{\Theta}(0)) = (1,1,-1,1)$

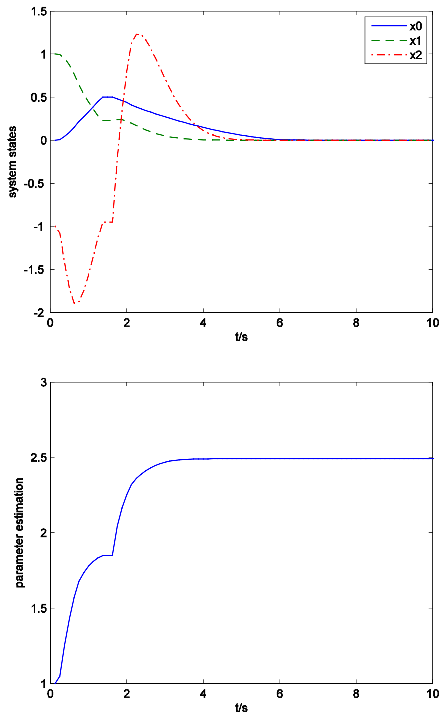


Fig.3. State trajectories of the closed-loop system with $(x_0(0), x_1(0), x_2(0), \hat{\Theta}(0)) = (0,1,-1,1)$

6. Conclusion

In this paper, the problem of adaptive stabilization of nonholonomic mobile robots with unknown kinematic parameters. By using input-state-scaling transformation and backstepping technique, an adaptive state-feedback controller is obtained. Based on switching strategy to eliminate the phenomenon of uncontrollability, the proposed controller can guarantee that the system states globally asymptotically converge to the origin. Simulation results demonstrate the effectiveness of the proposed control design.

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