Speed control of Permanent Magnet Synchronous Motor using Power Reaching Law based Sliding Mode Controller

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Abstract:- The Permanent Magnet Synchronous Motors (PMSM) are high-performance electromechanical motion devices essentially superseding traditional dc servomotors and fractional horsepower induction machines because of their high performance capability. The necessity for high performance in PMSM systems increases as the demand for precision controls. In order to optimize the speed-control performance of the PMSM system with different disturbances and uncertainties, a nonlinear speed-control algorithm for the PMSM servo systems using sliding-mode control is developed in this paper. A sliding-mode controller is designed based on conventional reaching law but the amount of chattering and reaching time are high. So a sliding mode controller is designed based on single power rate reaching law and double power rate reaching law methods. These methods improve the performance significantly. A comparison is made between PMSM servo system designed using Proportional Integral (PI) controller and sliding mode controllers designed based on different reaching law methods.

Key- Words: - PMSM, precision controls, sliding mode controller, chattering, reaching law, power rate reaching law.

1 Introduction

Permanent Magnet Synchronous Motor (PMSM) has better dynamic performance, smaller size and higher efficiency compared to other forms of motors. In recent years with the rapid development of power electronics and rare earth permanent magnetic materials there is increasingly sophisticated research in permanent magnet motors [1]. The classical Proportional Integral (PI) control technique is still popular due to its simple implementation [2]. However, in a practical PMSM system there are large quantities of the disturbances and uncertainties which may come internally or externally like unmodeled dynamics, parameter variation, friction force and load disturbances [3].

A Sliding Mode Control (SMC) evolved as a non linear control technique to counter the effect of load disturbances and parametric uncertainties on the system. SMC forces the state trajectory onto a stable manifold by continuous switching of the control input [4].The control algorithm relies on Lyapunov's second theorem of stability to prove that by forcing the state trajectory on to the sliding manifold and causes the system to stay there forever and causes the system to attain stability [5]-[6]. Power reaching law is a kind of common reaching law and its approaching speed decreases with distance when the system state is close to the sliding mode plane, which is benefit for weakening the chattering [7]. The load disturbances rejection capabilities of this control scheme made it ideally suitable for a non-linear time varying systems such as the PMSM despite of its major disadvantage which is chattering [8] – [10].

This work aims at the improvement of the power reaching law guaranteeing original merits of sliding mode controller and improving its reaching performance when the system state is away from the sliding mode plane.

2 PMSM Drive System

The PMSM has a permanent magnet round rotor, where the magnetic poles are aligned axially and distributed around the circumference of the solid rotor. In synchronous motors, the process of energy conversion is accomplished by producing an electromagnetic torque from a time-varying magnetic field developed in the machine air gap and stationary magnetic fields produced by the permanent magnets distributed on the rotor [1]. In order to achieve a desired speed, the phase voltages must be varied as a function of the rotor angular displacement.

The angular speed of the synchronous motor tracks the supplied voltage frequency to the stator windings. This necessitates the measurement or estimation of the angular displacement of the rotor, using encoders or estimators. The motor supply voltage is a DC source voltage which will be inverted using pulse width modulation (PWM) to provide a 3-phase AC source. The modulated voltages provide a variable amplitude and frequency sinusoidal source for the PMSM.

Detailed modeling of PM motor drive system is required for proper simulation of the system. The d-q model has been developed on rotor reference frame. At any time t, the rotating rotor d-axis makes an angle θr with the fixed stator phase axis and rotating stator mmf makes an angle α with the rotor d-axis.

The model of PMSM without damper winding has been developed on rotor reference frame using the following assumptions:

- Saturation is neglected.
- The induced EMF is sinusoidal.
- Eddy currents and hysteresis losses are negligible.
- There are no field current dynamics.

The electrical and mechanical equations of PMSM in rotor (d-q) frame is

$$u_d = R_s I_d + \frac{d\varphi}{dt} - \omega_s \varphi_q \quad (1)$$

$$u_q = R_s I_q + \frac{d\varphi}{dt} - \omega_s \varphi_d \quad (2)$$

$$\varphi_d = L_d I_q + \varphi_v \quad (3)$$

$$\varphi_q = L_q I_q \quad (4)$$

Where,
I_d and I_q are dq stator currents,
L_d and L_q are dq axes inductances,
φ_d and φ_q are dq stator flux linkages,
φ_v is the rotor flux.

$$\frac{dI_d}{dt} = \frac{1}{L}\left(V_d - rI_q + \frac{P}{2}\omega_r\left(L_q I_q\right)\right) \quad (5)$$

$$\frac{dI_q}{dt} = \frac{1}{L}\left(V_d - rI_d + \frac{P}{2}\omega_r\left(L_q I_d + \gamma_m\right)\right) \quad (6)$$

$$\frac{d\omega}{dt} = \frac{1}{J}\left(T_e - T_l - B\omega_r\right) \quad (7)$$

$$\frac{d\theta_r}{dt} = \omega_r \quad (8)$$

Where

$$T_e = \frac{3}{2} p \left(\gamma_{ds} I_q - \gamma_{qs} I_d\right), \quad (9)$$

$$\gamma_{ds} = LI_d + \gamma_m \quad (10)$$

γ_m is the flux linkage
T_e is the electromagnetic torque
T_l is the load torque
r is the stator resistance

From above equations, the model of the PMSM system in state space is written as

$$\dot{X} = AX + BU \quad (11)$$

$$Y = CX + DU \quad (12)$$

$$\text{Where } X = \begin{bmatrix} I_d \\ I_q \\ \omega_r \\ \theta_r \end{bmatrix},\; U = \begin{bmatrix} V_q \\ V_d \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{-r}{L_d} & \frac{P}{2}\omega_{ro} & \frac{P}{2} I_{qo} & 0 \\ -\frac{P}{2}\omega_{ro} & \frac{-r}{L_q} & -\frac{p}{2}\left(I_{do} + \frac{\gamma_m}{L_q}\right) & 0 \\ 0 & \frac{K_t}{J} & \frac{-B}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = [1 \quad 1 \quad 0 \quad 0]$$

$$D = [0]$$

3 Controller Design

Speed control of motors mainly consists of two loops, the inner loop for current and the outer loop for speed. Speed controller calculates the difference between the reference speed and the actual speed producing an error, which is fed to the inner loop current controller. Since the PMSM is operated using field oriented control, it can be modeled like a dc motor.

PI controllers are used widely for motion control systems. They consist of a proportional gain that produces an output proportional to the input error and an integration to make the steady state error zero for a step change in the input [2].

The Sliding-Mode controller (SMC) will be used as a tracking controller for the speed of a PMSM to investigate its utility. The control objective is to track a reference speed with the actual rotor speed. The error signal between the reference and actual speeds will represent the sliding surfaces. Since the speed control loop of the PMSM is essentially a first order system, the SMC design is conventional in its derivation, and is based on the Lyapunov stability concept.

SMC is more insensitive to internal parameter variations and external disturbance once the system trajectory reaches and stays on the sliding surface. However, designing SMC with reduced chattering is crucial, which motivates the researches for a new reaching law. In general, SMC design involves two steps, the first is to choose the sliding-mode surface, and the next is to design the control input such that the system trajectory is forced toward the sliding-mode surface, which ensures the sliding-mode reaching condition. Second- order nonlinear model is generally used to describe the SMC system adopting conventional reaching law method:

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f(x) + g(x) + b(x)u\end{aligned}\right\} \tag{13}$$

Where, $x = [x_1, x_2]^T$ is the system parameters g(x) represents the system disturbances b(x) is not zero, u is control input. First, the typical sliding-mode surface is chosen as follows:

$$s_1 = cx_1 + x_2 \tag{14}$$

Next, the control input u should be designed in such a way that the sliding-mode reaching condition is met. Thus, equal reaching law is chosen as follows:

$$\dot{s}_1 = -k_1.sgn(s_1) \tag{15}$$

Substituting (14) in (15) yields

$$c\dot{x}_1 + \dot{x}_2 = -k_1.sgn(s_1) \tag{16}$$

Next, substituting (16) into (13) yields

$$cx_2 + f(x) + g(x) + b(x)u = -k_1.sgn(s_1) \tag{17}$$

$$u = -b^{-1}(x)[cx_2 + f(x) + g(x) + k_1.sgn(s_1) \tag{18}$$

Here, it can be found that the discontinuous term is contained in the control input, which leads to the occurrence of chattering. The time required to reach sliding-mode surface can be derived by integrating (15) with respect to time as follows:

$$t_1 = \frac{|s(0)|}{k_1} \tag{19}$$

The following second-order nonlinear model is generally used to describe the SMC system adopting single power rate reaching law method:

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f(x) + g(x) + b(x)u\end{aligned}\right\} \tag{20}$$

Where, $x = [x_1, x_2]^T$ is the system parameters g(x) represents the system disturbances b(x) is not zero, u is control input. First, the typical sliding-mode surface is chosen as follows:

$$s_1 = cx_1 + x_2 \tag{21}$$

Next, the control input u should be designed in such a way that the sliding-mode reaching condition is met. Thus, single power rate reaching law is chosen as follows:

$$\dot{s_1} = -\mathrm{k}.\varepsilon \mathrm{s_1}^{\alpha} \cdot \mathrm{sgn}(s_1)$$
(22)

Substituting (21) in (22) yields

$$c\dot{x}_1 + \dot{x}_2 = -k_1.\varepsilon \mathrm{s_1}^{\alpha} \cdot \mathrm{sgn}(s_1) \tag{23}$$

Next, substituting (23) into (20) yields

$$cx_2 + f(x) + g(x) + b(x)u = -\mathrm{k}.\varepsilon \mathrm{s_1}^{\alpha} \cdot \mathrm{sgn}(s_1) \tag{24}$$

$$\mathrm{u} = -\mathrm{b}^{-1}(\mathrm{x}).$$

$$\left[\mathrm{c\,x_2} + \mathrm{f(x)} + \mathrm{g(x)} + \mathrm{k}.\varepsilon \mathrm{s_1}^{\alpha} \cdot \mathrm{sgn}(s_1)\right](19) \tag{25}$$

In the power rate reaching law approach, the speed at which the trajectory converges onto the sliding surface when it is far away from the surface is fast but when the state is nearer to the surface the rate of convergence reduces. The time required to reach sliding-mode surface can be derived by integrating (22) with respect to time as follows:

$$t_{reach} \leq \frac{1}{(1-\alpha)\varepsilon} s_0^{(1-\alpha)} \tag{26}$$

A new double power reaching law is based on the analysis of the common reaching law, and this new law could not only guarantee the fast reaching movements, but also greatly weakened the chattering and improved the anti-interference ability. The following second-order nonlinear model is generally used to describe the SMC system adopting single power rate reaching law method:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x) + b(x)u \end{aligned}\right\} \tag{27}$$

Where, $x = [x_1, x_2]^T$ is the system parameters g(x) represents the system disturbances b(x) is not zero, u is control input. First, the typical sliding-mode surface is chosen as follows:

$$s_1 = cx_1 + x_2 \tag{28}$$

Next, the control input u should be designed in such a way that the sliding-mode reaching condition is met. Thus, double power rate reaching law is chosen as

$$\dot{s_1} = -k.\varepsilon_1.s_1^{\alpha}\text{sgn}(s_1) - k.\varepsilon_2.s_1^{\beta}\text{sgn}(s_1) \tag{29}$$

Substituting (28) in (29) yields

$$c\dot{x}_1 + \dot{x}_2 = -k.\varepsilon_1.s_1^{\alpha}\text{sgn}(s_1) - k.\varepsilon_2.s_1^{\beta}\text{sgn}(s_1) \tag{30}$$

Next, substituting (30) into (27) yields

$$cx_2 + f(x) + g(x) + b(x)u = -k.\varepsilon_1.s_1^{\alpha}\text{sgn}(s_1) - k.\varepsilon_2.s_1^{\beta}\text{sgn}(s_1) \tag{31}$$

$$u = -b^{-1}(x)\begin{bmatrix} cx_2 + f(x) + g(x) + \\ k.\varepsilon_1.s_1^{\alpha}\text{sgn}(s_1) + k.\varepsilon_2.s_1^{\beta}\text{sgn}(s_1) \end{bmatrix} \tag{32}$$

The next step is to design the control input such that the system trajectory is forced toward the sliding-mode surface, which ensures the system to satisfy the sliding-mode reaching condition. Speed controller is designed based on different reaching laws. Consider,

$$s = e = \omega_{ref} - \omega \tag{33}$$

(33) is called linear sliding-mode surface. Taking the time derivative of the sliding-mode surface yields

$$\dot{s} = \dot{e} = \dot{\omega_{ref}} - \dot{\omega} \tag{34}$$

The dynamic equation of the motor can be expressed as follows:

$$\dot{\omega} = ai_q - bT_L - c\omega \tag{35}$$

$$= a_n i_q - b_n T_L - c_n\omega + \Delta a i_q - \Delta bgT_L - \Delta\, c\omega$$

$$= a_n i_q - c_n\omega + r(t) \tag{36}$$

Where,

$$a = a_n + \Delta a = \frac{3p^2\varphi_a}{2J}$$

$$b = b_n + \Delta b = \frac{p}{J}$$

$$c = c_n + \Delta c = \frac{B}{J}$$

$$r(t) = \Delta a i_q - \Delta\, c\omega - bT_L \quad \& \; |r(t)| \leq t$$

Therefore

$$\dot{s} = \dot{\omega_{ref}} + c_n\omega - r(t) - a_n i_q = -k\,.sgn(s) \tag{37}$$

Therefore, the control input is designed as follows:

$$i_q{}^* = a_n^{-1}\left\{\begin{matrix} \dot{\omega_{ref}} + c_n\omega - \\ r(t) - a_n i_{q-} + k\,.\text{sgn}(s) \end{matrix}\right\} \tag{38}$$

The lumped disturbances r(t) is unknown in this control input. Thus, it is not yet completive. To deal with this problem, the lumped disturbances r(t) is replaced by the upper bound l, and then the following control input is designed:

$$i_q{}^* = a_n^{-1}\{\dot{\omega_{ref}} + c_n\omega + [l + k].sgn(s)\} \tag{39}$$

Proceeding similarly the control input for single power rate and double power rate reaching law can be written as

$$i_q{}^* = a_n{}^{-1}\{\dot{\omega_{ref}} + c_n\omega + [l + k].\varepsilon s_1{}^{\alpha.\text{sgn}(s)}\} \tag{40}$$

$$i_q^* = a_n{}^{-1}\left\{\begin{matrix} \dot{\omega_{ref}} + \\ c_n\omega + [l + k][\varepsilon_1 s_1{}^{\alpha.\text{sgn}(s_1)}. + \varepsilon_2 s_1{}^{\beta.\text{sgn}(s_1)}] \end{matrix}\right\} \tag{41}$$

4 Simulation Results

The simulation is preformed in MATLAB/SIMULINK with the parameters given in the Table 1.

Table 1 PMSM parameters

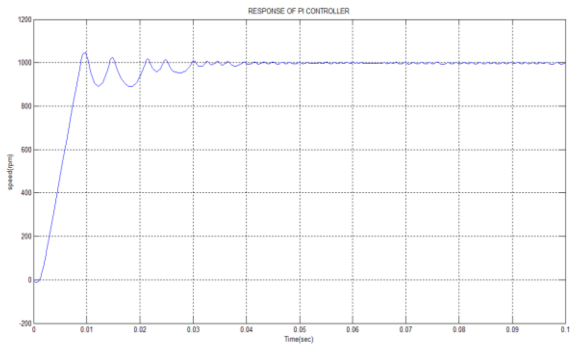
PARAMETERS	PARAMETER VALUES
Inductance L_d	0.0066 H
Inductance L_q	0.0058 H
Rotor Flux constant Ψ_a	0.1546 V/rad/s
Moment of inertia J	0.00176 kgm^2
Friction Vicious gain B	0.00038818 Nm/ rad/s
Number of Poles P	6
Nominal parameter a_n	0.000008
Nominal parameter c_n	0.0227
Sliding mode gain K	22
Gain adjustment factor ε	0.2
Gain adjustment factor for DPSMRL ε_1 & ε_2	0.3 & 0.4
Convergence constant α, β	0.15, 0.32

4.1 Response of the Controllers under Steady State

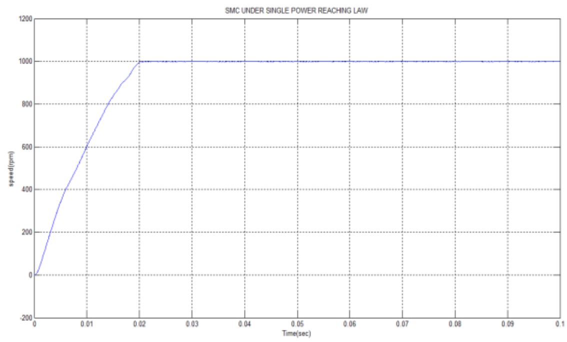
The response of the controllers without any disturbances and uncertainties are described in this section. The behavior of PI controller, SMC designed using conventional sliding mode reaching law, single power rate reaching law method and double power rate reaching law methods are analyzed.

Fig 1 (a) shows response of PMSM with PI controller with $K_p = 20$ and $K_i = 50$ for the given reference of 1000 rpm and it is evident that the overshoot is high and also the steady state error. Fig 1 (b) shows sliding mode controller designed for PMSM system using conventional reaching law and it can be seen that the overshoot and steady state error is reduced for a conventional sliding mode controller as compared to a PI controller. Sliding mode controller is designed using single power rate reaching law and is shown in Fig 1 (c). The convergence rate is improved to a greater extent and better reaching time is obtained.

A new double power reaching law is designed, and this new law not only guarantees the fast reaching movements, but also greatly weakened the chattering and improved the anti-interference ability as shown in Fig 1 (d). The comparative merits are tabulated and given in Table 2.



(a)



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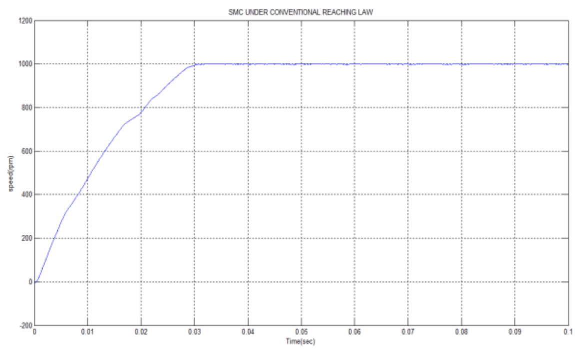

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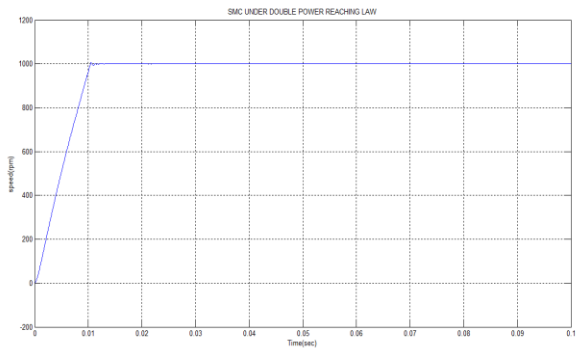


(d)

Fig 1 (a) Response of PI controller, (b) Response SMC under conventional reaching law, (c) Response of SMC under single power rate reaching law (d) Response of SMC using double power rate reaching law.

Table 2. Comparison of controllers under steady state

S. No	Parameters	PI Controller	Sliding Mode Controller		
			Conventional reaching law	Single power rate reaching law	Double power rate reaching law
1	Rise Time (s)	0.008	0.115	0.015	0.0095
2	Settling Time(s)	0.04	0.035	0.033	0.031
3	Overshoot (%)	5	0.4	0.31	0.25
4	Stead state error (rpm)	4	2	1.6	1.2

4.2 Servo Response of the Controllers

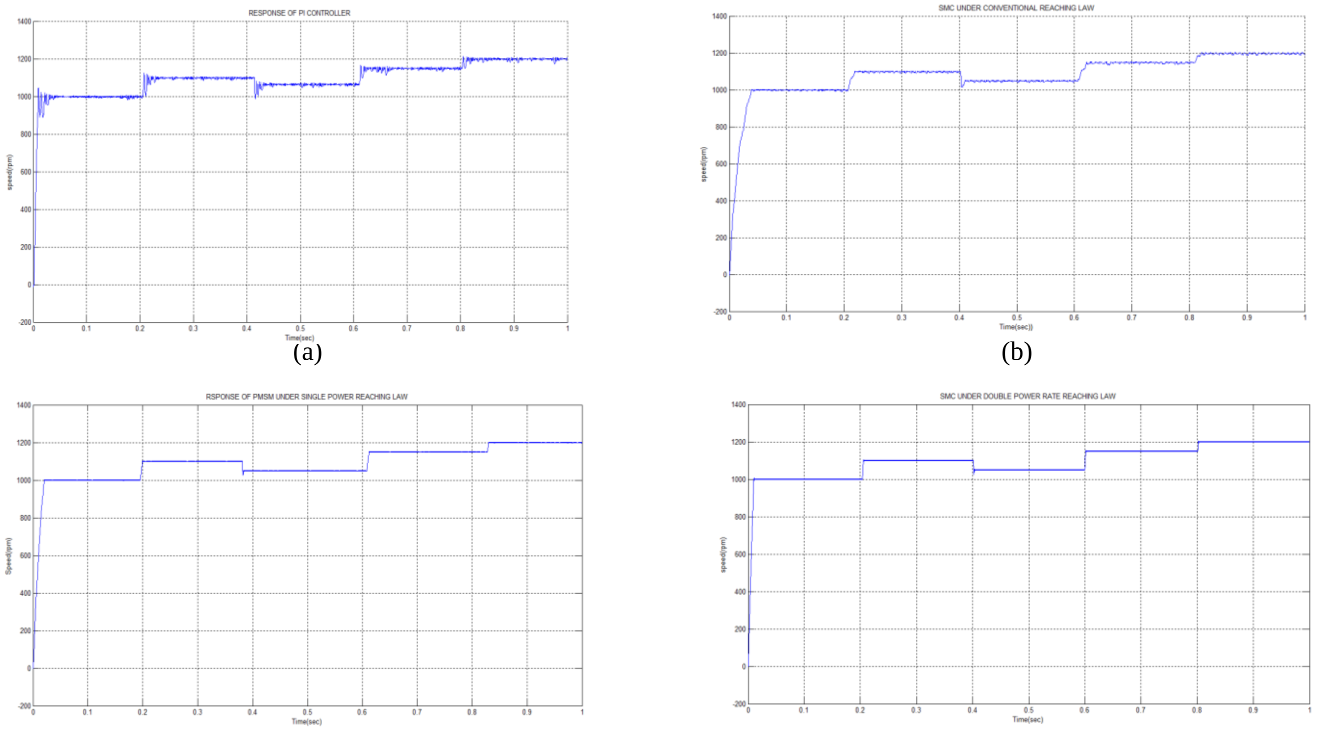
Response of PMSM for different set point changes are shown in Fig 2 .

From Fig 2 (b) it is evident that the overshoot is the steady state error is less compared to a PI controller which is given in Fig 2 (a). But the amount of chattering is little high, although the tracking performance is better. It is evident from Fig 2 (c) that the single power rate reaching law produces lower amount of chattering as compared to SMC design using conventional reaching law.

A new double power rate reaching law is designed so as to reduce the amount of chattering as well as to reduce overshoot as shown in Fig 2 (d). Various performance criteria is tabulated and given in Table 3.

Table 3. Performance of various controllers for set point tracking

S. No	Parameters	PI Controller	Sliding Mode Controller		
			Conventional reaching law	Single power rate reaching law	Double power rate reaching law
1	Settling time(s)	0.04	0.035	0.033	0.031
2	Over shoot (%)	5	0.4	0.31	0.25
3	Stead state error (rpm)	4	2	1.6	1.2



(a) (b) (c) (d)

Fig 2 (a) Tracking response of PI controller (b) Tracking response of SMC under conventional reaching law method (c) Tracking response of SMC under single power rate reaching law method (d) Tracking response of SMC using double power rate reaching law method.

4.3 Regulatory Response of the Controllers

The performance of the motor when a load is applied (regulatory problem) is checked. A step load of 1.1 Nm is added to the motor at time 0.5s to check the load rejection capability.

Fig 3 (b) shows response of PMSM with conventional SMC and there is an under shoot because of the application of the step load but it is better as compared to a PI controller's response as shown in Fig 3 (a).

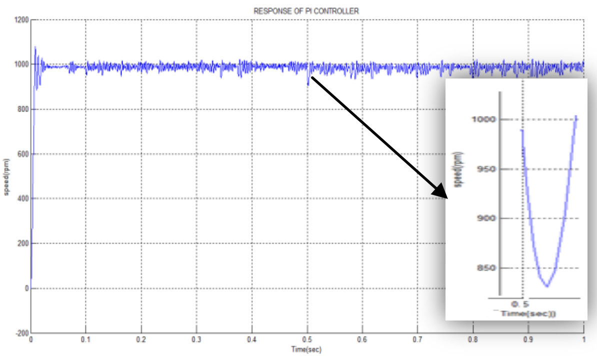
Fig 3 (c) and 3 (d) shows performances of controllers for regulatory response of PMSM with single power rate reaching law and double power rate reaching law.

A small under shoot of 2.5% is measured in SMC designed using double power rate reaching law. This shows that the disturbance rejection capability of double power rate SMC is better compared to a PI controller, conventional SMC and single power rate SMC.

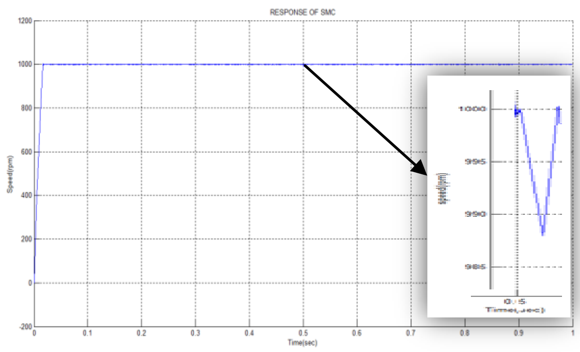
Various performance criteria is tabulated and given in Table 4.

Table 4 Performance of various controllers for disturbance rejection

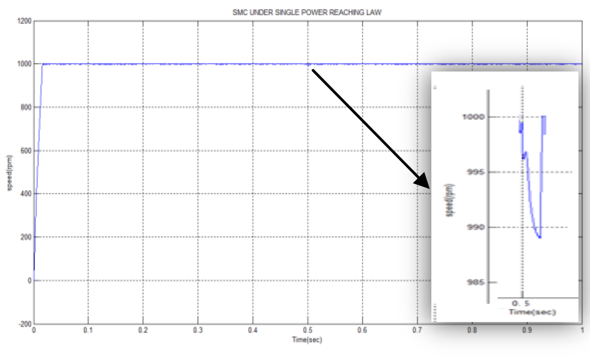
S.No	Parameters	PI Controller	Sliding Mode Controller		
			Conventional reaching law	Single power rate reaching law	Double power rate reaching law
1	Settling Time(s)	0.04	0.035	0.033	0.031
2	Under shoot (%)	16.5	1.2	1.1	0.3
3	Stead state error (rpm)	20	5	4	1



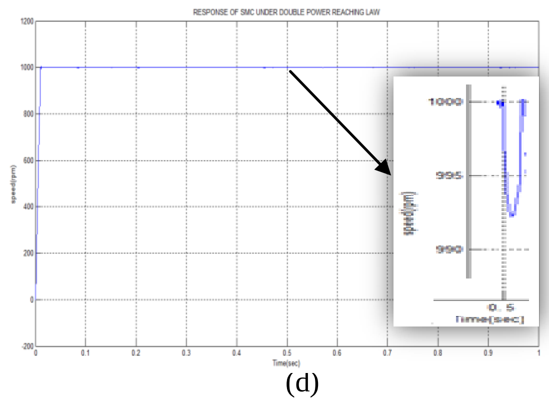
(a)



(b)



(c)



(d)

Fig 3 (a) Regulatory response of PI controller, (b) Regulatory response SMC under conventional reaching law, (c) Regulatory response of SMC under single power rate reaching law (d) Regulatory response of SMC using double power rate reaching law

5 Conclusions

A detailed modeling of PMSM has been performed. A PI controller for PMSM is designed in the speed loop. To obtain robust system performances a sliding-mode controller (SMC) is designed based on conventional, single power rate, double power rate reaching law. The performances of different controllers were evaluated under steady state and SMC designed using double power rate reaching law greatly reduces the amount of chattering in the response. While evaluating the tracking performance of different controllers, SMC designed using double power rate reaching law gives lesser overshoot and undershoot than other SMC designs. The regulatory performance is analyzed to show its insensitivity to load changes.

Thus it can be concluded that SMC designed using double power reaching law gives a better performance compared to SMC design based on conventional and single power rate reaching laws and also PI controller.

References

1. R. Krishnan, "Electric Motor Drives Modelling, Analysis and Control", Prentice Hall, 2001.
2. Y. X. Su, C. H. Zheng, and B. Y. Duan, "Automatic disturbances rejection controller for precise motion control of permanent-magnet synchronous motors," IEEE Trans. Ind. Electron., vol. 52, no. 3, pp. 814–823, Jun. 2005.
3. S. Li and Z. Liu, "Adaptive speed control for permanent magnet synchronous motor system with variations of load inertia," IEEE Trans. Ind. Electron., vol. 56, no. 8, pp. 3050–3059, Aug. 2009.
4. K. Zhao, X. G. Zhang, L. Sun, and C. Cheng, "Slidingmode control of high-speed PMSM based on precision linearization control," inProc. Int.Conf. Electr. Mach. Syst., 2011, pp.1–4.
5. J. Liu and F. Sun, "Research and development on theory and algorithms of sliding mode control," Control Theory &Applications, vol. 24, no. 3, Feb 2007,pp. 407–418.
6. S.Li, M.Zhou, and X.Yu "Design and Implementation of Terminal Sliding Mode Control Method for PMSM Speed Regulation System."IEEE Trans.on Industrial Informatics,vol. 9, no. 4,Nov.2013, pp. 1879-1891.
7. C J. Fallaha and M. Saad "Sliding-Mode Robot Control With Exponential Reaching Law." IEEE Trans.on Industrial Electronics, vol. 58, no. 2,Feb.2011,pp 679-691.
8. G. H. B. Foo and M. F. Rahman, "Direct torque control of an ipmsynchronous motor drive at very low speed using a sliding-mode statorflux observer,"IEEE Trans. Power Electron., vol. 25, no. 4,Apr. 2010, pp. 933–942.
9. L. Wang, T. Chai, and L. Zhai, "Neural-network-based terminal slidingmode control of robotic manipulators including actuator dynamics,"IEEETrans. Ind. Electron., vol. 56, no.9, Sep. 2009,pp.3296–3304.
10. J.Y.Rath and B.J Yesh, "Effective Speed Control in 3-Phase BLDC Motor by Reaching Law based Sliding Mode Technique",International Journal of Computer Applications,Vol. 43, no.16, April 2012,pp.975 – 8887.