The research and application of NMPC in Automatic Train Operation

XIAOJUAN LU, QI GUO, HAIYING DONG, BAOFENG MA
School of Automation and Electrical Engineering
Lanzhou Jiaotong University
No.88, Anning Road(west), Anning District, Lanzhou City, Gansu Province
China
luxj@mail.lzjtu.cn; 15209317494@163.com; hydong@mail.lzjtu.cn; mabf_lzjtu@163.com

Abstract:-According to the requirement of safety, punctuality, comfort and energy saving and environmental protection of high speed train, this paper presents that the terminal constrained nonlinear model predictive control based on Particle Swarm Optimization is applied to control of Automatic Train Operation(ATO)system. Firstly, according to the energy saving theory a target curve of saving energy operation of train was designed, namely speed distance curve. In terms of comfort and punctuality, acceleration and distance versus time curves were designed. secondly, through PSO algorithm and gradient descent method were drawn into NMPC, which formed hybrid optimization model, optimal control was achieved. In the end, PSO-NMPC was applied to the simulation analysis of the nonlinear parameter model of CRH2A(China Railway High-space), and then validity and feasibility of the algorithm applied in ATO was verified.

Key-words:-ATO；NMPC；Particle Swarm Optimization Algorithm；Energy saving optimization

1 introduction

Vigorous development of rail transportation which is large volume, low energy consumption, small pollution, punctual and reliable, fast and convenient is particularly important to promote the development of the national economy. The speed controller is the core unit of Automatic Train Operation(ATO) and is also important part of the train control system. Therefore, the advanced control theory applied to design the speed controller of ATO can improve the safety of high speed train, punctuality, comfort and energy saving and environmental protection requirements. In recent ten years, scholars at home and abroad use some advanced control algorithm to speed control of ATO, and most of control algorithms of the train ATO system use the proportional integral differential(PID) control[1]. Although the control precision achieved satisfactory results, the number of add and subtract of this algorithm is too many, which is not conducive to the smooth operation of the train and also affects the ride comfort[2].The combination of direct fuzzy control algorithm and the associative memory neural network control algorithm mentioned by railway science research institute and institute of automation chinese academy of sciences was used in long range predictive control for parking[3]. In 2007 Pretoria university used heavy haul train as object and spring mass model to control the robustness of the train[4], which successfully overcomed the influence of external environment on robustness of the train, put forward the formal modeling on dynamic characteristics of heavy haul train and opened up a new direction in train control. In 2008, Southwest

Jiao Tong University applied the maximum principle design and fuzzy control into train driving, generating the energy saving target curve and through the fuzzy control algorithm meet the comfort requirements of train operation[5]. C.S.Chang, Singapore scholar, according to various operation conditions applied genetic algorithm into the ATO system and calculated the optimal coasting points to reduce energy consumption before the train starting[6]. In 2014, Zhang Youpeng et al, LanZhou JiaoTong University professor, applied grey system theory to the simulation of high speed train in ATO speed controller[7],achieved the run the target curve under the high speed automatic train operation mode and applied the genetic algorithm to optimize. Results show that system performance was improved. In 2013, Luo Hengyu construct adaptive controller based on the augmented error ,applied to ATO speed control and achieved good effect[2]. The literature of [8] based on the Lyapunov stability theory proposed adaptive train speed and position control strategy and realized the high precision tracking control effect, but in the transition phase of operation condition the dramatic changes of control variables had a certain effect on comfort. Based on the literature research above, this paper present the application of nonlinear model predictive contro（NMPC） theory and applied to the ATO speed control, and analysis and simulation verified the effectiveness of the algorithm.

The function of the NMPC algorithm is to predict the future output based on the past input and output of the system, and the future input. In consideration of safety, energy saving, comfort and punctuality in the process of the train operation, the desired output trajectory of the system in the future is designed. Using the NMPC technology makes the future actual output of the system as close as to the the future expected output trajectory, design the optimal predictive control law of the system control according to the control task and then uses the feedback correction to modify predicted values[9][10]. According to line speed limit and the train speed restriction, ATO determines the current speed. NMPC algorithm converts the output constraints into the change interval of control incremental, so as to realize the system output constraint. In the process of rolling optimization, PSO is introduced to form hybrid optimization model to find the optimal control variables to realize the tracking control of target curve. Finally through the PSO-NMPC controller the simulation analysis is carried on to CRH2A. The NMPC controller of the hybrid optimization achieved good control effect based on energy saving.

2 Train operation model

According to the actual train running lines of data, the whole goal, total mileage information such as the running speed, Automatic train operation system generates a need to achieve the goal of the train speed curve and the speed curve in total mileage. Then the speed controller generates the target curve by the lines circumstance. Application of certain control means, to ensure the train safety, comfort and energy-saving operation.

2.1 The design of circuit model

In the train traveling process, the specific circumstances of the line will affect the actual operation of the train, so transmission line model should be as close as possible to the actual line. In the actual circuit there are straight section, on the downhill sections, curve sections in turn, the tunnel section and overlapping sections in which two or three sections of the actual line.

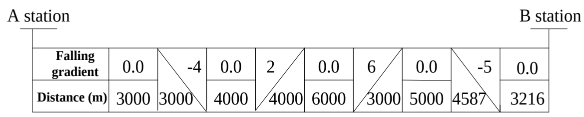

Fig.1 Transmission line mode

All these sections are switched to different acceleration to influence the running speed of the train. Therefore, in order to improve the calculation accuracy, the article simplified the circuit model which consists straight sections and downhill sections. Transmission line model as shown in figure 1.

2.2 Determination of train model

This paper cited train model in the literature [6]shown in formula (1).

$$G(s) = \frac{612}{s+0.34}\frac{1}{8621s+822.4} = \frac{0.07128}{8621s^2+3754s+280} \quad (1)$$

The train model has 12 control level as the system input, and the relationship between traction and train level is c/d=1800,so we can get the actual braking force and traction force of train. The actual running speed of the train is the system output. Set the sampling period T=2s, difference equation representation of the train model shown in (2).

$$\begin{aligned} y(k+1) &= 0.797y(k) + 0.9927y(k-1) - 0.8043y(k-2) \\ &\quad + 0.00829u(k) + 0.016u(k-1) + 0.007711u(k-2) \\ &\quad + \xi(k-1) - \xi(k-2) \end{aligned} \quad (2)$$

$\xi(k\text{-}1)$ 、 $\xi(k\text{-}2)$ are respectively the disturbance variable of k-1 and k-2.

In this paper, the model object is the CRH2A train. Referring to the train traction characteristics curve in literature[11] the relationship between train traction and speed can be seen as one curve formed by two linear function curve. Based on train parameters shown in Table 1,the relationship between Train traction f and speed v is obtained shown in (3).

$$f(v) = \begin{cases} \left(-\frac{9}{25}v + 175\right) \times \frac{100}{408.5} & v \le 125km/h \\ \left(-\frac{13}{25}v + 195\right) \times \frac{100}{408.5} & v > 125km/h \end{cases} \quad (3)$$

Table1 parameters of CRH2A train

Type of train	CRH2
Quality weight / capacity/t	360/408.5
Marshalling length /m	201.4
Unit resistance w_0(N/t)	w_0=8.63+0.07295v+0.00112v^2
Rotary inertia coefficient γ	0.05

Table2 train braking deceleration of CRH2A

Speed V	0-70km/h	70-118km/h	118-260km/h
1N	0.1667	0.2072-0.0005787v	0.1765-0.0003185v
4N	0.4583	0.5758-0.0016782v	0.4780-0.0008493v
7N	0.7472	0.9336-0.0026620v	0.7844-0.0013977v
High-speed	1.1222	1.4017-0.0039931v	1.1790-0.0021054v

Train braking acceleration of CHR2A can be obtained from table 2. Considering the characteristics of the energy saving of target curve, This paper uses the 7N as the braking deceleration and braking.

2.3 Generation of train's target curve

When the train is under coasting conditions , there is neither traction nor braking force, its energy consumption is equal to the locomotive self-consumption. Energy consumption of the train under uniform conditions is equal to the energy necessary to overcome the resistance plus with locomotive self-consumption. Therefore, in the process of moving, let the train as many under coasting or uniform conditions can achieve the target of energy-saving.

The train remains constant speed or traction-coasting cycle model in addition to the outside of starting and braking interval. These two kinds of operation model are the ideal energy saving operation model, but it is hard to realize in the process of train’s operation. In this paper, when generating the target curve, let the train traveling close to the actual conditions and make the train operate under the uniform and coasting conditions as much as possible, in order to achieve the purpose of save energy. By contrast with the power consumption under constant speed operation model, energy-saving of target curve generated in this paper is illustrated.

In this article the limit of inbound and outbound speed is set for 75km/h, interval speed limit is 200km/h, and the distance of inbound and outbound is 1000 meters. Actual operation speed of train is lower than the speed limit, the maximum speed of inbound and outbound speed is 70km/h, and maximum speed of driving in the interval is 195Km/h. Train’s speed controller controls the train to follow the target curve in order to save energy.

Figure2 is curve of uniform energy-saving model based on line model of Figure1. Figure3 is energy saving target curve which is generated from line model of Figure 1. Target curve uses maximum pulling acceleration to start the train and maximum braking deceleration to stop the train. In interval, transform is accomplished by the condition principles of pulling- coasting - pulling- constant velocity - coasting - braking.

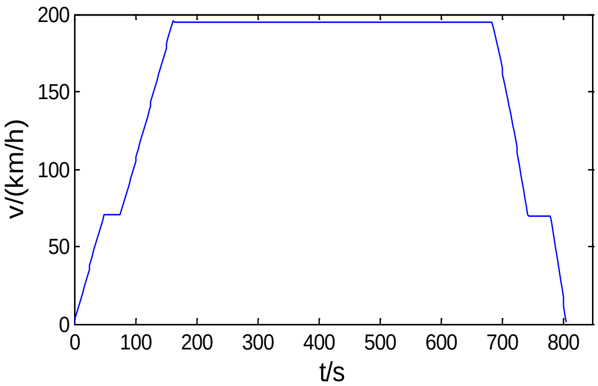

Fig.2 Uniform operation mode of v-t

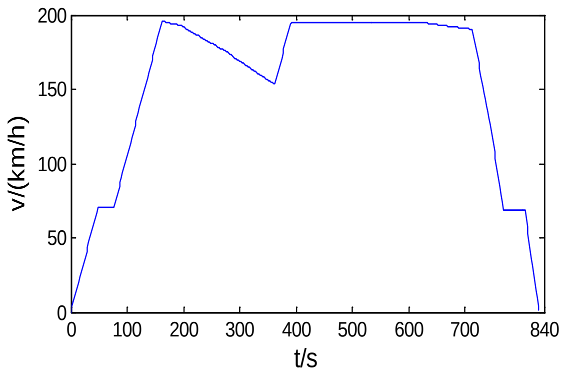

Fig.3 target curve of v-t

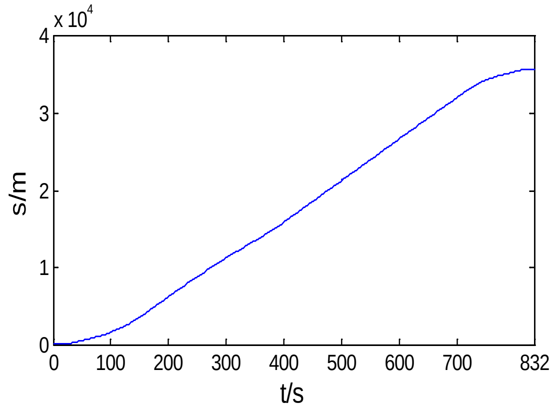

Fig.4 target curve of s-t

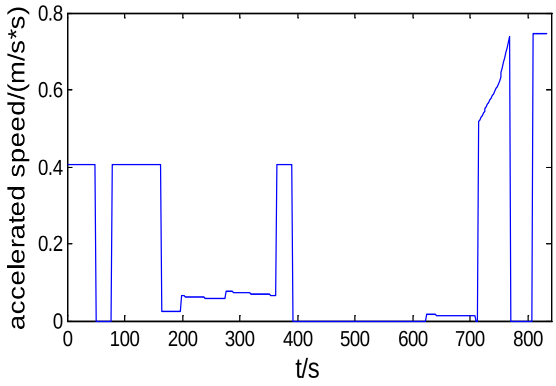

Fig.5 The acceleration curve of target curve

Figure 4 is the s-t target curve of train operation, and train finished 35,807 meters within the stipulated 832 seconds.

Figure 5 is the target acceleration curve corresponding to the target curve. It can be known from the figure that in the whole operation process the acceleration is less than $1m/s^2$, so that the comfort for passengers is ensured.

2.4 Energy saving calculation of objective curve

When the train is moving, the main external forces of train are separately traction, resistance of train and braking force. The resultant of unit weight forces is obtained from analysis, which can be indicated by C with N/kN. It is expressed by the following formula.

$$c = \frac{C}{Mg} = \frac{F_x - B - W_o - W_j}{Mg} = f_x - b - w_0 - w_j \qquad (4)$$

Where C is the train resultant force of unit weight, f_x is the train traction force of unit weight, b is the braking force of the unit weight, w_0 is the unit resistance force of the train, and w_j is the unit additional resistance force of the train. The unit of the forces mentioned above is N/kN. M is the total weight of the train, and the unit is *t*. *g* is the acceleration of gravity, and the unit is m/s^2. w_0=A+Bv+Cv^2, where A,B,C are the empirical constants which are connected with train type.

Consisted the the operation process of train, it can be seen as three kinds of working conditions including traction, coasting and braking.

(1) Traction working conditions

When the train is under traction working conditions, the main force of train are separately locomotive traction, basic resistance of the train operation. There are also some additional resistances because of curve, ramps and tunnel. The resultant force of the train is expressed by the following formula.

$$c = f_x - w_0 - w_j \qquad (5)$$

At present, the energy consumption of the train are self-consumption and traction consumption during operation of the locomotive.

(2) Coasting working conditions

When the train is coasting working conditions, the main force of train is the basic resistance of the train operation. There are also some additional resistances. The resultant force of the train is expressed by the following formula.

$$c = -w_0 - w_j \qquad (6)$$

At present, the energy consumption of the train is only self-consumption.

(3) Braking working conditions

When the train is braking working conditions, the main force of train are separately braking force, basic resistance of the train operation. There are also some additional resistances. The resultant force of the train is expressed by the following formula.

$$c = -w_0 - w_j - b \qquad (7)$$

At present, the energy consumption of the train are self-consumption and the energy consumption which is used to produce braking force.

There are different force analyses with different working conditions. The energy consumption with different working conditions can be calculated by the formula W=Fs. The energy consumption curve which are expressed by the following figure2 with uniform operation energy-saving optimized mode is 193.74×10^{7}w through calculation. The energy consumption of aim curve in this paper which are expressed by the figure 3 is 184.33×10^{7}w. Through the comparison of two energy consumption, the target curve generated in this paper can save 5% energy compared with fast energy-saving operation mode. The result of energy consumption comparison is expressed by the following figure6.

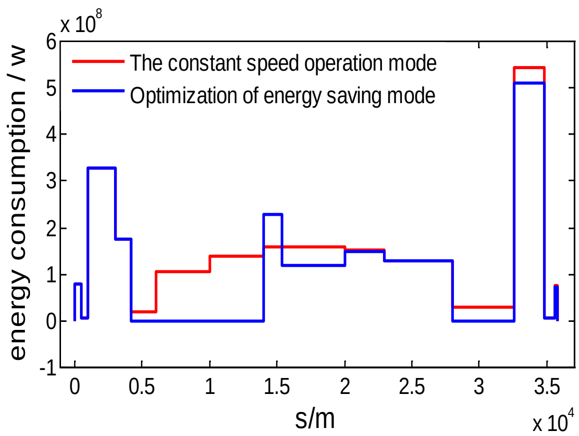

Figure 6 the comparison of the energy consumption

3 Controller Design

3.1 Nonlinear system predict model

Considering the following nonlinear system[12][13]

$$y(k+1|k) = F(y(k|k), u(k-1)) + G(y(k|k))\Delta u(k|k) \tag{8}$$

where, $y(k+1|k)$ is the predictive value of the system output.

$$u(k|k) = u(k-1) + \Delta u(k|k)$$

$F(y(k|k), u(k-1))$ is the composition of the known information of the system before the moment.

$G(y(k|k))\Delta u(k|k)$ is the increment part of control variable which is unknown and to be solved in system future output prediction.

3.2 NMPC controller design

The model predict is that under the action of control variable $u(k|k)$ the output prediction value $y(k+1|k)$ at the next moment $k+1$ is as close as possible to the given expected value $y_r(k+1)$. The optimization performance index at k moment is expressed by the following formula.

$$\begin{aligned} J(k) &= \frac{1}{2}\|y(k+1|k) - y_r(k+1)\|_Q^2 + \frac{1}{2}\|\Delta u(k|k)\|_R^2 \\ &= \frac{1}{2}\Delta u(k|k)^T H \Delta u(k|k) + \frac{1}{2}A_1\Delta u(k|k) + \frac{1}{2}\Delta u(k|k)^T A_2 \\ &\quad + \frac{1}{2}J_0 \end{aligned} \tag{9}$$

Where,

$$\begin{aligned} H &= (G(y(k|k)))^T Q(G(y(k|k))) + R \\ A_1 &= (F(y(k|k), u(k-1)) - y_r(k+1))^T QG(y(k|k)) \\ A_2 &= (G(y(k|k)))^T Q(F(y(k|k), u(k-1)) - y_r(k+1)) \\ J_0 &= (F(y(k|k), u(k-1)) - y_r(k+1))^T Q(F(y(k|k), u(k-1)) - y_r(k+1)) \end{aligned} \tag{10}$$

$y_r(k+1)$ is the setting value at the *k+1* moment, Q is the output weighting matrix, which is the $n_y \times n_y$ dimensional positive definite matrix, R is the weight matrix of increment part of control variable, which is $n_u \times n_u$ dimensional semi-positive definite matrix.

3.2.1 Predictive control without constraints

For the unconstrained system, let the formula (6) to obtain the minimum value, so it is equivalent to:

$$\frac{\partial J(K)}{\partial \Delta u(k|k)} = H\Delta u(k|k) + \frac{1}{2}A_2 + \frac{1}{2}A_1^T = 0 \tag{11}$$

From the expressions of the A_1, A_2 and a diagonal matrix Q, equation can be obtained:

$$A = A_2 = A_1^T \tag{12}$$

The control law can be obtained from the formula(11)

$$\Delta u(k|k) = -H^{-1}A \tag{13}$$

3.2.2 Control algorithm with output limited

Consider the control target of the control object and the performance of equipment, the upper and lower bounds of output variables of system can be determined. Set the upper and lower limit of output variables of (4) as $y_{\max}$, $y_{\min}$,so

$$y_{\min} \le y(k+1|k) \le y_{\max} \tag{14}$$

(14) into(8):

$$\begin{cases} \Delta u(k|k) \ge \mathbf{G}^{-1}(y(k|k))y_{\min}(k+1|k) - F(y(k|k), u(k-1)) \\ \Delta u(k|k) \le \mathbf{G}^{-1}(y(k|k))y_{\max}(k+1|k) - F(y(k|k), u(k-1)) \end{cases} \tag{15}$$

So, condition of output constraints is converted to optimization problems of limiting condition of control increment. when there are constraints of output , in this paper, particle swarm optimization algorithm is employed to find an optimal control increment to satisfy the limited conditions of output.

3.3 PSO-NMPC algorithm

3.3.1 PSO algorithm

Particle swarm optimization algorithm [14-16] is described as follows: If the dimension of search space is n, the population consists of m particles, $x = (x_1, x_2, \cdots, x_m)^T$ and $x_i = (x_{i,1}, x_{i,2} \cdots x_{i,n})^T$ is the position of particle i ,velocity of which is $v_i = (v_{i,1}, v_{i,2}. \cdots v_{i,n})^T$. Individual extreme value of the particle is $p_i = (p_{i,1}, p_{i,2} \cdots p_{i,n})^T$,and the global extreme value of the population is $p_g = (p_{g,1}, p_{g,2} \cdots p_{g,n})^T$.When particles find individual and global extreme value , they update the velocity and position according following two formula.

$$v_{i,d}^{k+1} = v_{i,d}^{k} + c_1 r_1 (p_{i,d}^{k} - x_{i,d}^{k}) + c_2 r_2 (p_{g,d}^{k} - x_{i,d}^{k}) \quad (16)$$

$$x_{i,d}^{k+1} = x_{i,d}^{k} + v_{i,d}^{k} \quad (17)$$

where, c_1 and c_2 are weight coefficient of acceleration, r_1 and r_2 are random function between (0,1), and $v_{i,d}^{k}$ and $x_{i,d}^{k}$ are respectively the velocity and position of *d* dimension of particle i in the *k* iteration. $p_{i,d}^{k}$ is the position of individual extreme value of *d* dimension of particle i. $p_{g,d}^{k}$ is the position of global extreme value of *d* dimension of population.

According to the evolution equations of particles above, c_1 regulates the step which can make particle fly to the best position of itself and c_2 regulates the step which can make particle fly to the best position of overall situation. At the same time in order to reduce the possibility in the evolutionary process of particles leaving the search space, $v_{i,d}$ is usually restricted in a certain range $v_{i,d} \in [-v_{\max}, v_{\max}]$. If the search space of question is restricted in $[-x_{\max}, x_{\max}]$, $v_{\max}$ can be set $v_{\max} = kx_{\max}$, $0 \le k \le 1$.

3.3.2 PSO-NMPC algorithm

When control variables have constraints, use the PSO algorithm to optimize the performance index (5), and directly solve predictive control law. Specific steps are:

The performance index (5) as a fitness function of PSO, Parameters to be optimized of PSO algorithm is the sequence of variable quantity of the predictive control variable $\Delta u(k+j-1|k), j=1,...,N$. The range of optimized parameters is $[\Delta u_{\min}, \Delta u_{\max}]$. The number of optimized parameters is N. The PSO algorithm parameter space dimension is $D = N$, for the *j*-th position of the particles can be expressed as an N-dimensional vectors:

$$X_j = [\Delta u_j(k|k), \Delta u_j(k+1|k), ...\Delta u_j(k+N-1|k)] \quad (j=1,...,M)$$

Update each element, then calculate and optimize performance indicators (5). If the performance indicators meet the end conditions, then the position of the particle of which the optimal prediction fitness function value is the smallest in swarm particles is the required optimal control sequence. Then the first element of the present time is acted as the control input of the system, the next time the process is repeated. You can get the optimal control equation as follows:

$$u^*(k) = u^*(k-1) + \Delta u^*(k) \quad (18)$$

When there are constraints in controlled object, gradient optimization and PSO optimization are used with each other. When there are no constraints in controlled object, gradient optimization is used to obtain optimal control inputs. The predictive control will continue to carry out the optimization process to achieve rolling optimization. Figure 7 is structure diagram of nonlinear model predictive control of hybrid optimization.

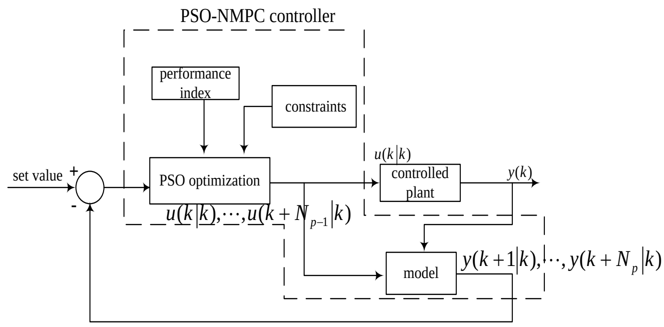

Figure7 PSO-NMPC Control structure

4. Simulation analysis

In this paper, we set the optimized target curve as the system input and use Matlab7.0 as platform for simulation. PSO-NMPC controller is used to follow and control the target curve. The parameters in the simulation are c_1=2, c_2=2, $\Delta U \in$[-35,35], *n*=6, *m*=2, λ=0.9 ,*α*=0.3.

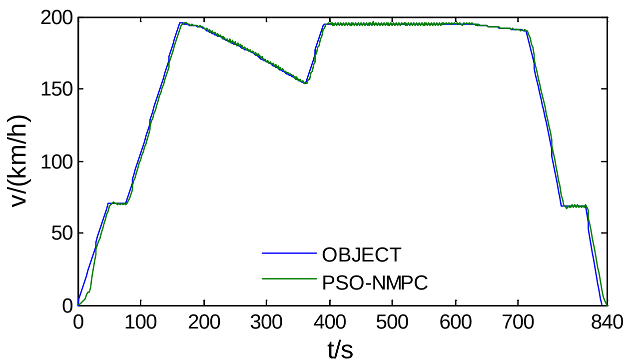

Fig.8 PSO-NMPC follow the V-t curve

Figure 8 is the target curve simulation map of PSO-NMPC controller following speed-time, in which the travel time of the target curve is set as

832s , the true following time of the PSO-NMPC controller is 840s, and the error of the following time is 8s.

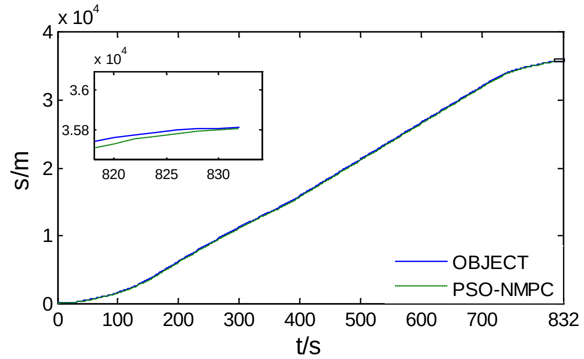

Fig.9 PSO-NMPC follow the S-t curve

Figure 9 is the target curve simulation map of PSO-NMPC controller following distance-time in which the traveling distance of the target curve is set as 35807m, the following distance is 35805m, and the following error is 2m. In the train station parking, the error range within 25m is feasible.

5.Conclusions

In order to improve control accuracy of the train ATO system, this paper use energy saving optimization theory to generate energy-saving optimization target curve. Comparing with fast operation mode, energy-saving mode can save energy 5% when the train running in the set environment. PSO-NMPC controller is designed to track the energy-saving target curve based on the trains model. The results indicate that PSO-NMPC applied to the ATO speed control is effective and feasible. Hybrid particle swarm optimization can reduce errors of the system dynamic process and improve the robustness and convergence speed of the system.

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