Abstract: - Under limited environment, PRM (probabilistic roadmap method) algorithm is used to plan initial path in mobile robot path planning, a path optimization algorithm is proposed based on the improved node method and geometric smooth strategy. The node enhancing method is used to optimize the initial planning with the base PRM algorithm, the original path nodes are gradually substituted by some new nodes, and the number of the inflection points will be reduced greatly in the path, thus the length of path will be shorten. At the same time, a new strategy of geometric optimization is used to smooth the optimized path, the path smoothing is achieved. The simulation result shows that the algorithm can reduce the length of the searched path, the smoothness of the path is greatly improved.

Key-Words: mobile robot, path planning, PRM algorithm, node enhancing, routine optimization

1 Introduction

Mobile robot path planning is a foundation to complete a variety of tasks, it has been a hot issue in the field of robotics research. A variety of effective planning algorithms had been proposed, such as artificial potential field method [1,2], the cell decomposition method [3], probabilistic roadmap method (PRM) in robot system [4-7], quick search tree method [8]. Because accurate modeling is required for obstacles of pose space in artificial potential field method and the cell decomposition method, when the environment obstacles is more complex, it will result in a larger amount of computation to the plan algorithm. And based on random sampling technique, PRM method can effectively avoid model accurately of obstacles in pose space, it can effectively solve the high-dimensional space constraints and complex path planning problems. However, when roadmap is built by this method, sampling for landmark nodes is entirely based on random sampling technology, the search path is random, so it easily leads that the eventual search path is not always the best path.

In response to these shortcomings of basic PRM method, add the path optimization phase is added based on a two-stage PRM method. New node methods are adopted to replace the node of the original path by the enhanced node, the number of inflection points is reduced in the initial path, thus the path length is shortened. Meanwhile, according to the knowledge of geometry, a circular arc is used to replace the dogleg path between the primary inflection point and the nodes of their adjacent sides, thus it effectively improves the smoothness of the search path.

2 PRM Fundamentals

A robot operating environment is a two-dimensional enclosed flat space C, which contains a number of obstacles. The robot moving freely region is called free space, i.e., the obstacle and the edge are removed in the region, it is denoted $C_{\text{free}}$. The mobile robot is as the particle within C, and it does not consider the motion dynamics constraints.

The basic idea of PRM algorithm is that the random network is used to represent the free space $C_{\text{free}}$ of robot system running, the random network is the probability map which is established by robot system[4]. Probability map is an undirected cycle diagram, it is represented by $G = (G_a, G_e)$. Which $G_a$ is node set, elements in $G_a$ are randomly the selected position-shape in free planning space $C_{\text{free}}$ for robot, elements of edge set $G_e$ represent connections between nodes, it represents the local feasible path between position-shapes. Then, a search algorithm is used to search for a feasible path for robot system in the random network. The algorithm can be accomplished through two
stages: offline learning phase (learning phase) and online inquiry stage (query phase).

1) **Offline:** The main task at this learning stage is to establish a probability map $G = (G_a, G_e)$ for the robot in $C_{free}$.

   First, robot pose points are sampled in a random manner in $G_{free}$, they constitute a set of nodes $G_a$ in the map $G$; secondly, a local planner potential energy is used for each node to find their neighbor nodes, and a connection is established to them, thus the map edge set $G_e$ is constituted. In learning phase, roadmap $g$ is established to use for online inquiries.

2) **Online:** The main task at this inquiry stage is that under a given initial position and orientation point $s$ and destination point $g$ condition, according to the offline phase roadmap, a feasible path is searched out, it is a connection with the $s$ and $g$. First, $s$ and $g$ are respectively connected to the roadmap with the local planner, and its nearest two nodes are $\tilde{s}$ and $\tilde{g}$; and then, heuristic search algorithms (such as breadth-first search algorithm, A* search algorithm or Dijkstra search algorithm) is used to search out a feasible path from roadmap, which is connected with the $\tilde{s}$ and $\tilde{g}$.

### 3 PRM shortcomings

From the basic principles of PRM, it is not difficult to find that the roadmap is built by random sampling point, its quality plays a decisive role. In the conventional method, sampling is usually done by completely random sampling strategy, sampling points are completely randomly distributed throughout $C_{free}$, although this withdrawing method is very convenient, but there are two drawbacks: 1) In the free space $C_{free}$, the selected node is not the ideal uniform distribution; 2) As the search path is dependent on sampling points and the limited number of the sampling point, it can not completely cover the entire free space, and their distribution is random, in the end, it was not the best route path, but it is only sub-optimal path. Although the number of points is enough for a long time, these two defects can be overcome, but PRM algorithm is mainly to reduce the amount of calculation, too many points will greatly increase the computational algorithm so that the algorithm loses its fundamental purpose.

In response to these shortcomings, although it is based on a pseudo-random sampling method, it is between deterministic and completely random, it has been able to ensure that the sampling points are distributed more evenly throughout the $C_{free}$, but it still has locally on randomness, that is, under the premise of the path exists, it still contains optimum path, as shown in Figure 1.

![Figure 1 Local Path](image)

In Figure 1, A, B, C are the adjacent three path nodes, when the robot moves from point A to point C, its local path AB, BC are shown in Figure 1. However, if the local path along the dotted line is shown in AD, DC motion, the path length will be shortened, the resulting path will be closer to the optimal path, so finding a point $D$ is necessary on the path optimization.

When PRM algorithm is used to search partial path, whether the path between adjacent nodes are communicated, and it need to be determined. As a result of binary detection method, when there is no feasible path to be communicated between neighboring nodes, there is a relatively small amount of calculation; while if there is a feasible path, the need is to traverse the entire partial path, its amount of calculation is large. Therefore, the threshold is used to limit the length of the partial path to be detected, i.e. all the paths exceeding the threshold value is considered unfeasible, so that the computation of search algorithm is reduced. And this in turn makes that the algorithm itself gives up some possible paths, and they may just be part of the optimal path or near optimal path, so to some extent, so that the path may get away from the optimal path, but since the limited of the threshold, the resulting path contains a number of inflection.

In addition, the robot is impossible suddenly turns in practice, it always needs curvature to achieve a turning, therefore the smoothing is needed for an inflection point, but there is too much inflection, path smoothness may become very difficult to deal with the late, or can not be implemented in reality.
4 Route optimization based on node enhancement method

According PRM shortcomings which are described in the previous section, an optimization strategy is proposed based on path node enhancement method. The method can be implemented by two steps: 1) the original path optimization; 2) optimizing path is smoothed.

4.1 Optimization

Since this part is planned to optimize the path, this path contains relatively few nodes, thus the threshold qualification is removed, and the local path is re-searched between adjacent path nodes. According node enhancement method, additional node is used successively to replace the original path node, in order to reduce the number of inflection points in the path, so as to shorten the path length. The basic principle of this method is shown in Figure 2.

![Figure 2 Schematic of Node Enhancement Method](image)

1) "dichotomy" is used to detect a line segment AC collision, if the point of collision is encountered, testing is stopped, and the collision point B is recorded.

2) B is as Pedal, AC perpendicular is done, and then along this vertical line and on both sides of AC, extend point D is looked in turns with distance h, point D is the first collision detection points, which is able to meet CD and AD. Here note that h is smaller, then it takes some more ideal, but at the same time, the greater the amount of calculation, h should be appropriate to the size of the selection. Similarly, when the collision of the CD and AD is detected, the method of detecting is detected alternately, i.e., the CD and AD is inspected in turn. So that a desired angle will reduce the amount of calculation.

In the basis of the base node enhance, it is modified as follows, the specific implementation is shown in Figure 3.

![Figure 3 Improved Node Enhancement Method](image)

The path of the dotted line in Figure 3 shows the initial path before optimization, seven nodes are denoted as A1, A2, A3, A4, A5, A6, A7 from left to right. From the A7 now, to start searching forward, whether it is feasible in turn to judge A7, A6, A5, ... Until the first one is not feasible node (shown as A4), in this time, a node B2 is extended between the A7 and A4 by the described method above, the nodes between A7 and A4 are discarded (including A4), and A7 is noted for B1. Then the endpoint is B2, the search is started farther from A4, B2A4, B2A3 ... are checked to be viability, until it encounters one point (shown as A3) which is not feasible, then the above method is used to amplify point B3, the same nodes between A3 and A7 are discarded (including A7). And so on, until the starting point, and the starting point is referred to as Bn+1 (Bn is the last one amplification points).

The new road map in the path is shown by solid black line, it is for B1B2B3B4 after it was optimized by this method.

4.2 Smoothing

After the above steps, inflection point is a significant reduction in the new path, which in turn create the condition for route smoothing, a circular arc is mainly configured to replace the inflection point of the original path and the adjacent fold line, so as to achieve the purpose of a smooth path. Following a turning point, for example, the smoothing process is described.

This inflection point is noted A, which the near two points on the path are denoted B and C, the high of the side BC is AD, AD and BC...
intersect at D. In this case extend DA length $h$ (Step 1) in the amplification points, to give A’ point. To construct such a point O, it is tangent to A’B and A’C, and tangent point is line segment between A’B and A’C, so the radius must be first determined in O. For this reason, the bisector A’E is done first for $\angle BA’C$, the center O should be in A’E. $\angle BA’C$ is noted $\theta$, $\angle \triangle OAB'$ radius is set to $R$, $\phi = \angle DA’C$, $\angle DA’E = \frac{\theta}{2} - \phi = \rho$. The principle is shown in Figure 4, the symbol circle with dot is a key point.

Figure 4 Smoothing schematics

In order to make the smoothed line not collide with the obstacle.

1) In order to ensure A meet $|AO| \leq R$, $|AA'| = h$ in O. And easy to know $|EA'O| = \frac{R}{\sin\left(\frac{\theta}{2}\right)}$, it can be obtained from the law of cosines:

$$|AO|^2 = |AA'|^2 + |A'O|^2 - 2 |AA'| \cdot |A'O| \cos(\rho) \leq R^2$$

Simplification was as follow:

$$\cot^2\left(\frac{\theta}{2}\right)R^2 \frac{2hR \cos(\rho)}{\sin\left(\frac{\theta}{2}\right)} + h^2 \leq 0$$

Obtained:

$$h(1 - \sqrt{\cos^2(\rho) - \cos^2\left(\frac{\theta}{2}\right)}) \leq \frac{\cot^2\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}{2}$$

$$2) \text{ if } R < \frac{h(1 - \sqrt{\cos^2(\rho) - \cos^2\left(\frac{\theta}{2}\right)})}{\cot^2\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}, \text{ At this point, A'A and O'O intersect at two points, though A point is in O'O outside, but the new route calculation includes in the original path, these two is integrated to show:}$$

$$0 \leq R \leq \frac{h(1 + \sqrt{\cos^2(\rho) - \cos^2\left(\frac{\theta}{2}\right)})}{\cot^2\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)} = b$$

Cut-off point should be ensured in A'B and A'C at the same time, so there should be as follow:

$$R \leq |A'B| \tan\left(\frac{\theta}{2}\right), R \leq |A'C| \tan\left(\frac{\theta}{2}\right)$$

In summary record

$$c = \min(b, |A'B| \tan\left(\frac{\theta}{2}\right), |A'C| \tan\left(\frac{\theta}{2}\right))$$

According to the actual situation $R \leq \frac{c}{2}$, which is to prevent that two consecutive inflection corresponding to c are the same, and there is exactly the same angle $\theta$, in addition, c also just is $L \tan\left(\frac{\theta}{2}\right)$ (L is the line segment length between two inflection). Circle radius R is determined according to the value of h and the previous equation. Here $R = \frac{c}{4}$, the position of the center O can be calculated to determine the arc.

It should be noted that the selection of the arc radius is very important. Obviously when very small radius is no limit, at this time the arc can be constructed, but this time, the resulting arc can be short, it is likely to become a straight line in the pixel map, thus it loses the smoothing processing mind. It is hoped to have a certain radius R size, but if radius R is a large and arc is very long, the robot can turn a long time in the state, which is not conducive to the operation of the robot. Therefore, the R should not exceed the upper limit in the case, a moderate value is taken, the upper limit of R is estimated in step 2), which has been given.

5 Simulation results and analysis

For the correctness and validity of the proposed optimization method, the Vc ++ 6.0 is used to establish a system simulation
experiments and analysis. Simulation program is
ihat an area with 600 × 600 pixels is the
simulated robot working area, 100 sampling
points are selected in the plan space, Dmax
value is set to 80 pixels in size, the resulting
simulation results are shown in Figure 5 and
Figure 6.

6 Conclusion

PRM is proposed in this study, it is based
on enhancement method with the improved
node and geometric smoothing strategy, the
planned path can be optimized by PRM
effectively. Enhancement method is used with
the improved node. The node in the initial paths
is alternative, the number of inflection points is
reduced in the search path, and the length of the
path is greatly reduced. And based on the
geometric smoothing strategies, the dogleg path
of a neighbor is asked to approximate with arc,
thereby the smoothness of the whole path is
improved. The principle of the method is simple,
it is easy to implement, and it can be well
applied to the motion control of the robot.

In addition, it can be noted that while the
expansion point, the robot also has a preliminary
understanding on the location of obstructions.
While expansions are searched, the robot can
record the distance between the expand point
and the obstacle. If this distance is very small, it
indicates that the corresponding arc distance of
the inflection point is close to the distance of
obstacles, when the robot is cornering, other
measures can be taken such as slowing down to
their own protection.

References:

[1] FAVEP, JON B, TOURNASSOUD P. A
practical approach to motion planning
for manipulator with many degrees of
freedom [M]. Cambridge: MIT Press,

Guochang, Potential grid based global
path planning for robots[J], JOURNAL
OF HARBIN ENGINEERING
UNIVERSITY, 2003, 24(2):
013

[3] KATEVAS NI, TZAFESTAS SG,
PNEVMATIKATOS CG. The
approximate cell decomposition with
local node refinement global path
planning method: path nodes
refinement and curve parametric
interpolation [J]. Journal of Intelligent
and Robotic Systems: Theory and


