# Design of Predictive Fractional Order PI Controller for the Quadruple Tank Process

G.PRAKASH[1], Dr.V.ALAMELUMANGAI[2]
Department of Electronics and Instrumentation Engineering[12]
Annamalai University[12]
42/66D, Jagadambal Illam, Therkkiruppu, Annamalai Nagar, Chidambaram
India
Prakash.gunasekar@gmail.com[1], almjai@yahoo.com[2]

---

*Abstract*: - Industrial processes are usually higher order processes and hence the effect of dead time is one of the important problems which may make the system unstable. Various control strategies have been introduced for the system with large dead time. Predictive PI control is one of the compensation technique which is a modified form of smith predictor algorithm. Introducing the concept of fractional order controller (in which the integrator and differentiator are of fractional order) in predictive control strategy the benefits of both can be derived. In this paper an attempt is made to design and analyze the scheme to compensate the effect of dead time present in a process through predictive fractional order PI controller. The Quadruple tank process taken to study the performance of the control strategy is a MIMO process which includes positive zero in its characteristics. The experimental model considered in this work is with large dead time. Therefore it is a challenging job to develop a control strategy for performance improvement. The performance of the process with developed scheme is evaluated using settling time, rise time, peak time, overshoot and ISE through simulation. Hagglund, Gainshaping, Amigo and Z-N tuning techniques are used to tune PI controller.

*Key-words:* - Predictive PI Controller, Fractional Order Controller, Hagglund, Gainshaping, Amigo, Z-N, Quadruple tank process

---

## 1 Introduction

Processes with only one output being controlled by a single manipulated variable are classified as S*ingle–Input Single-Output* (SISO) systems. Many processes, however, do not conform to such a simple control configuration. In the process industries any unit operation capable of manufacturing or refining a product cannot do so with only a single control loop. Each unit operation typically requires control over at least two control loops. System with more than one control loop is known as a Multi-Input Multi-Output (MIMO) or a multivariable system.

The quadruple tank process is a multivariable process, which has a multivariable zero. Control problem is simple for the minimum phase system, compared with non-minimum phase system. Multivariable control system could be with a decentralized [1], [2] or centralized controller [3]. The straight forward extension of controller tuning techniques used in SISO system can be used to design a decentralized controller for the multivariable process.

For processes with large dead times a predictive PI controller was introduced in [4]. Dead –time compensation with PI controller for industrial processes was presented in [5]. Robust tuning procedures of dead-time compensating controllers described in [6]. Various types of industrial process models with predictive PI controller were explained in [7]. Predictive PI controller's stability was analyzed in [8]. Fractional order Proportional Integral Derivative controller ($PI^{\lambda}D^{\mu}$) has been introduced in [9]. Speed control of DC motor using fractional order controller described in [10]. Ziegler–Nichols PID controller revisited in [11]. Performance of Amigo's tuning technique was analyzed in [12]. In this work to derive the benefits of flexibility in tuning from fractional order controller and to reduce the effect of dead time using Predictive controller with smith predictor algorithm a control strategy is developed by combining these two concepts and is known as Predictive Fractional order PI controller. The fractional order predictive PI strategy not addressed so far is developed to improve the performance of the Quadruple tank process and the results are presented. This paper is organized as follows: Section 2 presents the process description and modeling. Development of control strategy of Predictive fractional order PI Controller is presented in section 3. Simulation results are given in section 4 followed by conclusion in section 5.

## 2 Process Description and Modeling

The Quadruple tank process is a bench mark multivariable process. The unique characteristics of the quadruple tank process is that the location of multivariable zero is completely based on the manual valve (Mv1 and Mv2) position which is shown in Fig.1. It consists of four inter connected water tanks and two pumps. The quadruple tank process is shown in Fig.1. Its inputs are voltages to two pumps and the outputs are the water level in the lower two tanks. This process can easily be build by using two double-tank process, which are standard processes in many control laboratories [13], [14]. The setup is thus simple, but still the process can illustrate several interesting multivariable phenomena. The linearized model of the quadruple tank process has a multi- variable zero. The physical interpretation of positive zero makes quadruple tank process suitable to use in control education.

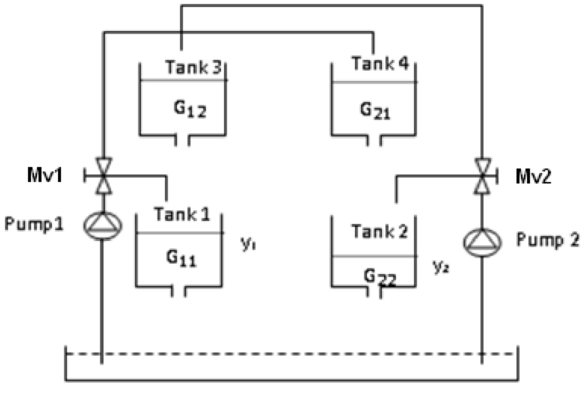

Fig.1.Schematic diagram of the quadruple-tank process

The dynamics of the process can be analyzed using the mathematical representation of the system. By applying first principles, the mathematical model is obtained as given below:

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} v_2$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} v_1$$

where

$A_i$ cross-section of tank i, i=1 to 4
$a_i$ cross-section of the outlet hole,i=1 to 4
$h_i$ water level in tank i ,i=1 to 4

The voltage applied to Pump j is $v_j$ (j=1 to 2) and the corresponding flow is $k_j v_j$. The parameters $\gamma_1$, $\gamma_2 \in (0,1)$ are determined from how the valves are set prior to an experiment. The flow to tank 1 is $\gamma_1$ $k_1$ $v_1$ and the flow to tank 4 is $(1-\gamma_1)$ $k_1 v_1$ and similarly for tank 2 and tank 3. The acceleration due to gravity is denoted by g. The measured level signals are $k_c$ $h_1$ and $k_c$ $h_2$. The parameters value of the laboratory process [15] are given in Table I as shown below.

Table 1. Parameters value of laboratory quadruple tank process

| Parameters | Values |
|---|---|
| $A_1$, $A_3$ ($cm^2$) | 176.71 |
| $A_2$, $A_4$ ($cm^2$) | 176.71 |
| $a_1$, $a_3$ ($cm^2$) | 0.3167 |
| $a_2$, $a_4$ ($cm^2$) | 0.3167 |
| $k_c$ (v/cm) | 0.667 |
| g ($cm/s^{2-}$) | 981 |

The transfer function model of the process can be experimentally found around the operating condition as shown in the Table 2.

Table 2. Nominal operating conditions of the quadruple tank process with Minimum Phase($P_-$) characteristics

| Parameters | Minimum Phase(P_) |
|---|---|
| $(h_1^0,h_2^0)$(cm) | (12,12) |
| $(h_3^0,h_4^0)$(cm) | (1.8,1.4) |
| $(u_1^0,u_2^0)$(mA) | (12,12) |
| $(k_1,k_2)$($cm^3$/v sec) | (27.78,27.78) |
| $(\gamma_1,\gamma_2)$ | (0.74,0.67) |

The Minimum Phase $G_-(s)$ transfer function model of the process is found to be

$$G_-(s) = \begin{bmatrix} \dfrac{3.06e^{-50s}}{(450s+1)} & \dfrac{2.353e^{-9s}}{(516s+1)} \\ \dfrac{1.6e^{-62.5s}}{(487.5s+1)} & \dfrac{4.4e^{-21.5s}}{(553.5s+1)} \end{bmatrix}$$

Table 3. Nominal operating conditions of the quadruple tank process with Non Minimum Phase($P_+$) characteristics

| Parameters | Non Minimum Phase($P_+$) |
|---|---|
| $(h_1^0,h_2^0)$(cm) | (14,14) |
| $(h_3^0,h_4^0)$(cm) | (5.2,4.8) |
| $(u_1^0,u_2^0)$(mA) | (12,12) |
| $(k_1,k_2)$($cm^3$/v sec) | (27.78,27.78) |
| $(\gamma_1,\gamma_2)$ | (0.243,0.473) |

The Non Minimum Phase $G_+(s)$ transfer function model of the process is found to be

$$G_+s = \begin{bmatrix} \dfrac{2.083e^{-63s}}{(525s+1)} & \dfrac{4.32e^{-10s}}{920s+1)} \\ \dfrac{2.45e^{-20s}}{(550s+1)} & \dfrac{1.411e^{-75s}}{(262.5s+1)} \end{bmatrix}$$

## 3 Development of control strategy of Predictive Fractional order PI Control

Predictive Proportional Integral (PPI) controller is developed based on the concept of smith predictor to compensate the effect of dead time present in an industrial process. The predictive feature of the smith predictor has been combined with controller to develop PPI strategy. The structure of PPI controller is shown in Fig.2.

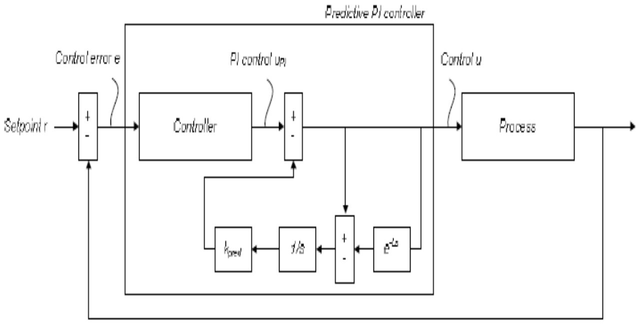

Fig. 2. Structure of Predictive PI controller (PPI)

The mathematical representation of the controller is given by

$$u(s) = u_{PI}(s) - \frac{K_p(t_i s+1)}{t_i s} g_o(s)(1-e^{-Ls})u(s)$$

Where

$u_{PI}$ = Controller output from the PI controller

$k_p$ = Proportional Gain

$k_i$ = Integral Gain

$g_0(s)$ = Process without a delay

L = Process dead time estimate

$k_{pred}$ = Predictive gain

This equation is used with FOPDT process and presented in Fig.2. The PI controller in PPI structure is assumed as a fractional order PI controller. The structure of fractional order controller is shown in Fig.3.

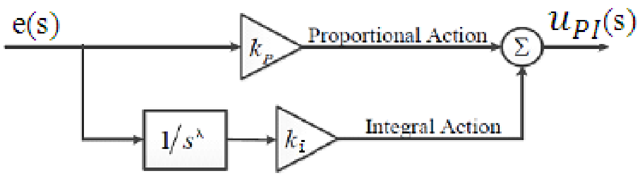

Fig 3. Structure of Fractional order PI controller

The mathematical representation is given by

$$u_{PI}(s) = k_p + \frac{k_i}{s^\lambda}$$

Where

e = Error

$k_p$= Proportional Gain

$k_i$ = Integral Gain

λ = Tuning parameter

The Fractional order PI controller is more flexible and gives an opportunity to better adjust the dynamical properties of a control system. The parameters of the fractional order PI controller are obtained using Hagglund [5], gainshaping [16], Amigo [11], and Z-N tuning techniques. Therefore the control strategy developed could be used to derive the benefits of both the concepts. The block diagam of the minimum phase and non- minimum phase characteristics of the Quadruple tank process with developed control strategy are presented in Fig.4. and Fig.5 respectively.

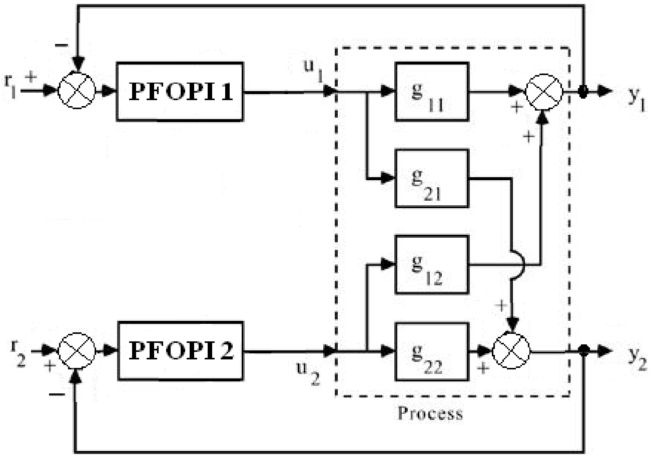

Fig 4. Structure of multiloop Predictive fractional order PI control strategy for minimum phase process

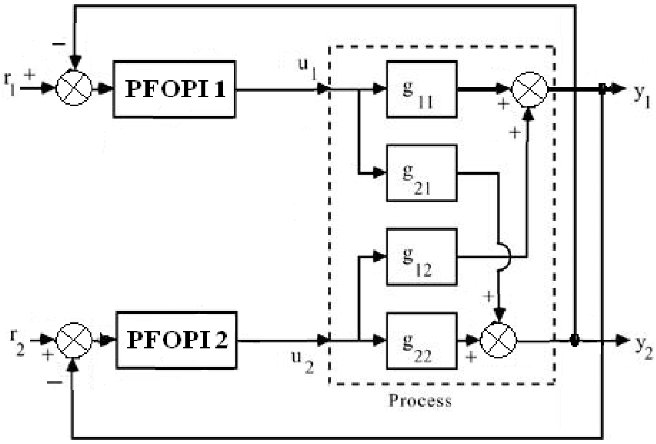

Fig. 5. Structure of multiloop Predictive fractional order PI control strategy for non-minimum phase process

## 4 Simulation Results

The servo and regulatory responses of the process with minimum phase as well as non–minimum phase characteristics are obtained and presented from Fig. 6 to 21. The parameters of the controller are calculated and presented in Tables 4 and 5. The performance indices are evaluated and presented in Tables 6 and 7.

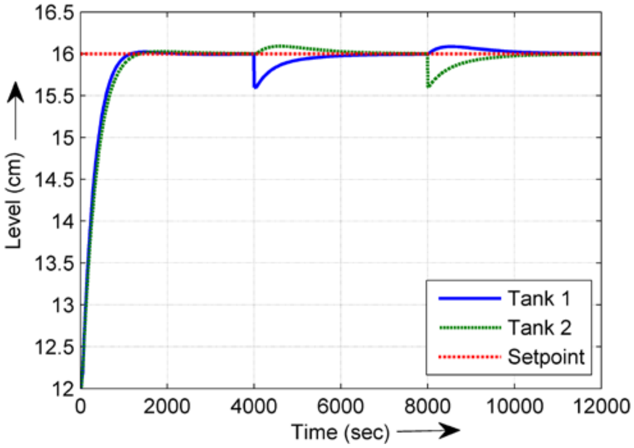

Fig. 6. Servo and regulatory responses of the process (minimum phase characteristics) with Predictive Hagglund tuned Fractional order PI control strategy

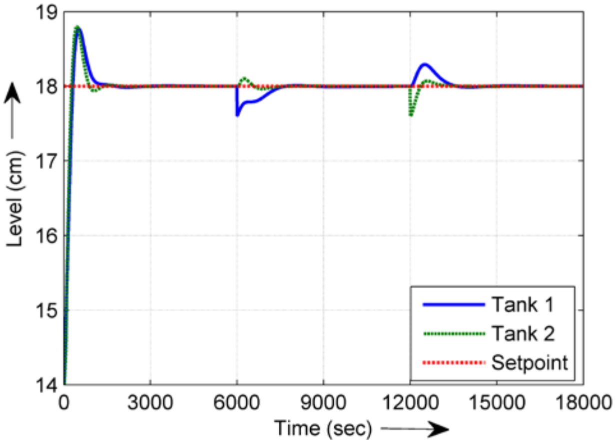

Fig. 8. Servo and regulatory responses of the process (non- minimum phase characteristics) with Predictive Hagglund tuned Fractional order PI control strategy

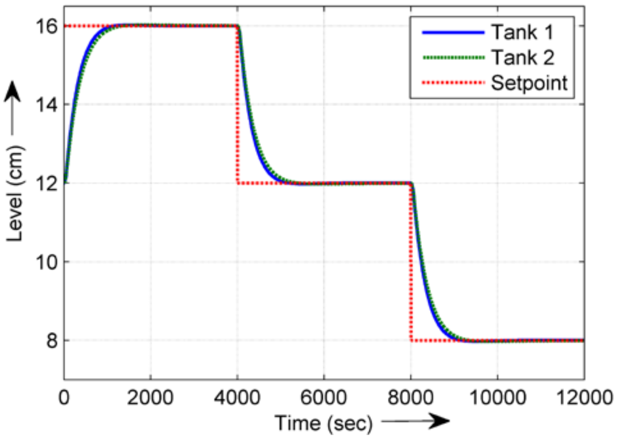

Fig.7. Servo responses of the process (minimum phase characteristics) for multiple change in setpoint with Predictive Hagglund tuned Fractional order PI control strategy

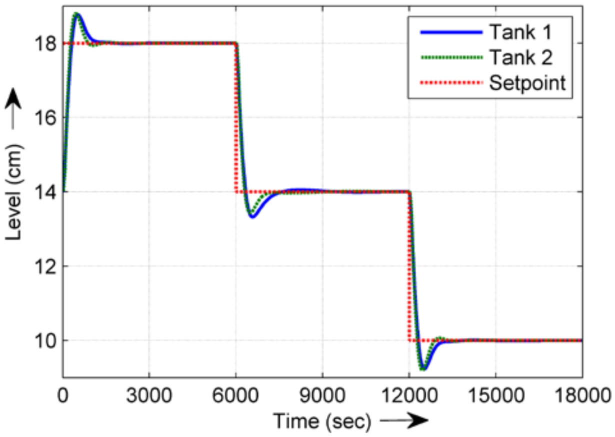

Fig. 9. Servo responses of the process (non-minimum phase characteristics) for multiple change in setpoint with Predictive Hagglund tuned Fractional order PI control strategy

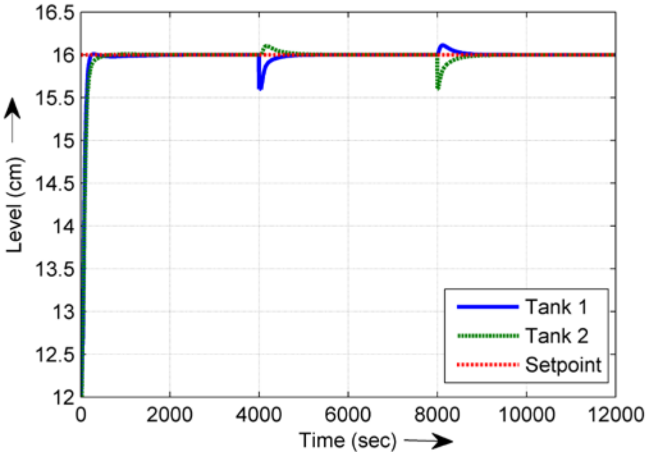

Fig. 10. Servo and regulatory responses of the process (minimum phase characteristics) with Predictive Gainshaping tuned Fractional order PI control strategy

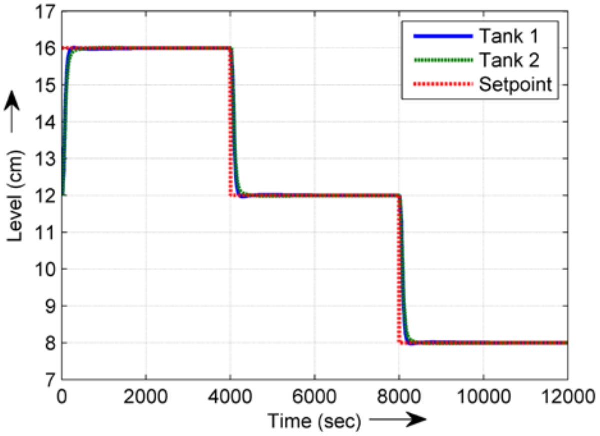

Fig. 11. Servo responses of the process (minimum phase characteristics) for multiple change in setpoint with Predictive Gainshaping tuned Fractional order PI control strategy

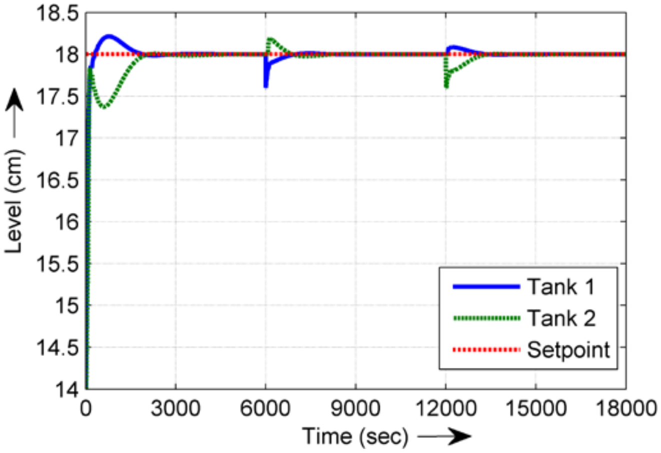

Fig. 12. Servo and regulatory responses of the process (non- minimum phase characteristics) with Predictive Gainshaping tuned Fractional order PI control strategy

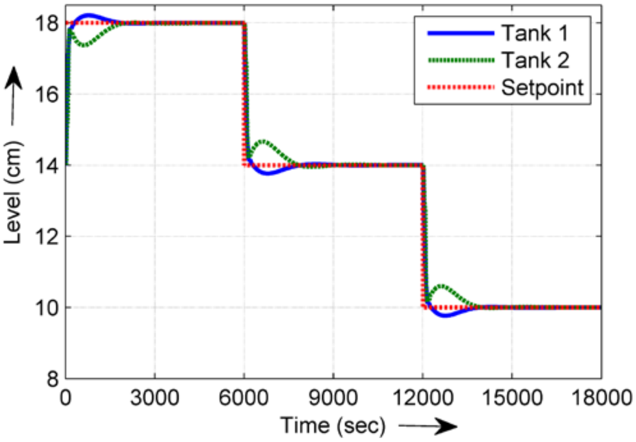

Fig. 13. Servo responses of the process (non-minimum phase characteristics) for multiple change in setpoint with Predictive Gainshaping tuned Fractional order PI control strategy

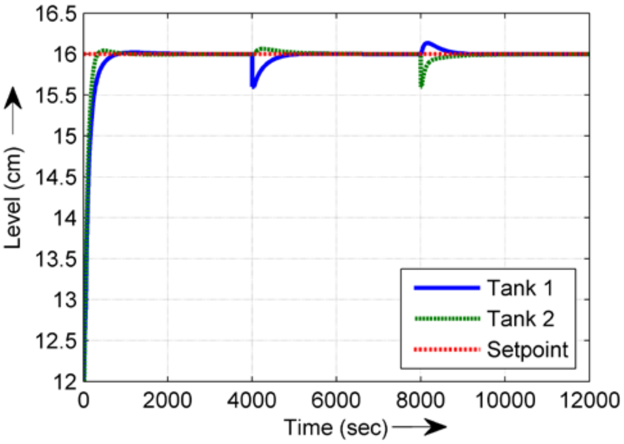

Fig. 14. Servo and regulatory responses of the process (minimum phase characteristics) with Predictive Amigo tuned Fractional order PI control strategy

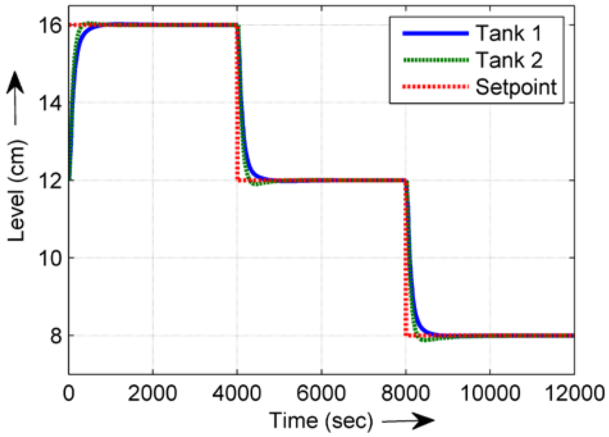

Fig. 15. Servo responses of the process (minimum phase characteristics) for multiple change in setpoint with Predictive Amigo tuned Fractional order PI control strategy

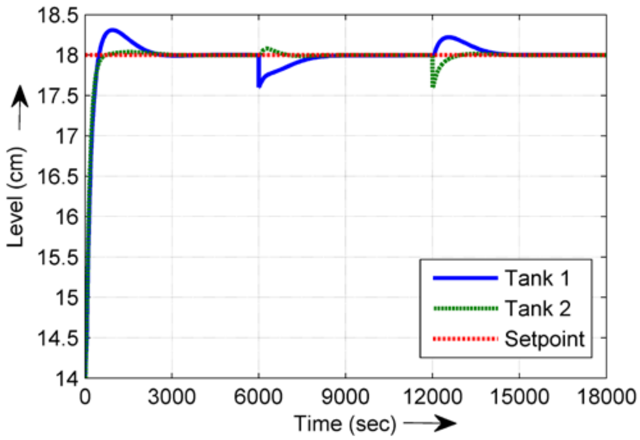

Fig. 16. Servo and regulatory responses of the process (non- minimum phase characteristics) with Predictive Amigo tuned Fractional order PI control strategy

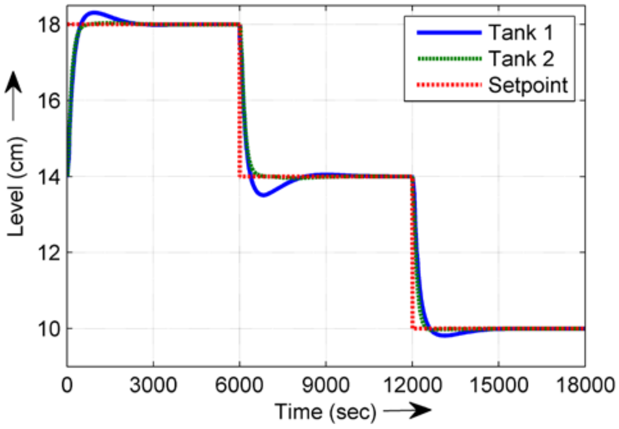

Fig. 17. Servo responses of the process (non-minimum phase characteristics) for multiple change in setpoint with Predictive Amigo tuned Fractional order PI control strategy

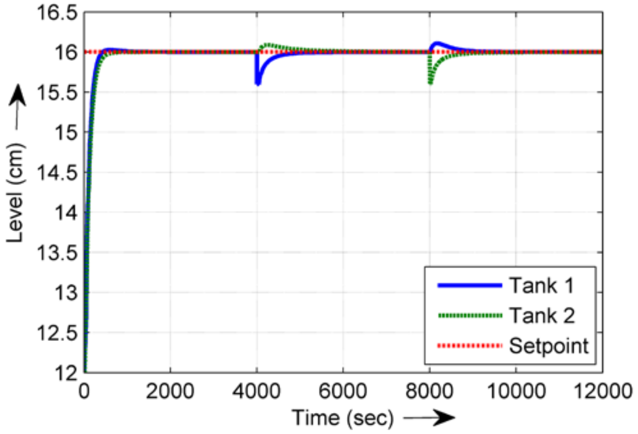

Fig. 18. Servo and regulatory responses of the process (minimum phase characteristics) with Predictive Z-N tuned Fractional order PI control strategy

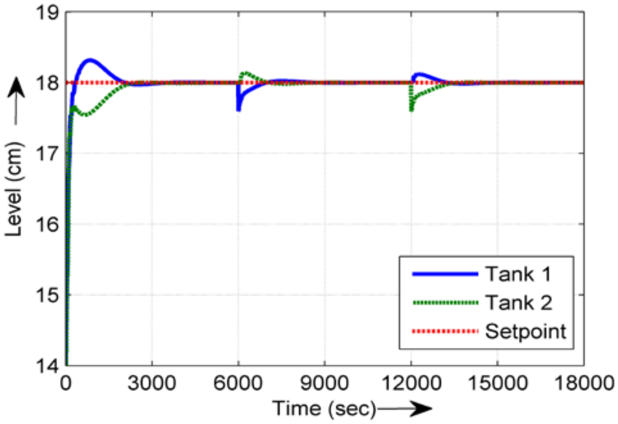

Fig. 20. Servo and regulatory responses of the process (non- minimum phase characteristics) with Predictive Z-N tuned Fractional order PI control strategy

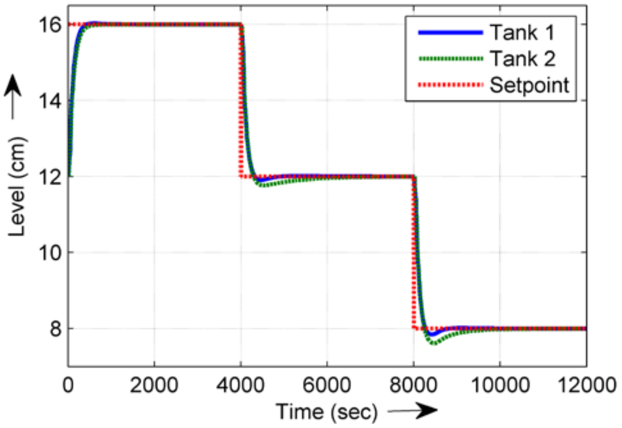

Fig. 19. Servo responses of the process (minimum phase characteristics) for multiple change in setpoint with Predictive Z-N tuned Fractional order PI control strategy

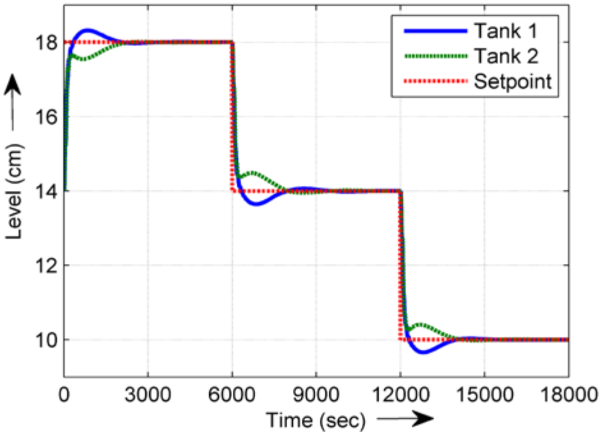

Fig. 21. Servo responses of the process (non-minimum phase characteristics) for multiple change in setpoint with Predictive Z-N tuned Fractional order PI control strategy

Table 4. Controller parameters for the minimum phase process

| Controller | Hagglund Controller Parameters | | | |
|---|---|---|---|---|
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 0.32679 | 450 | 0.002222 | 1.01 |
| **Loop2** | 0.22727 | 553.5 | 0.001806 | 1.02 |
| | | | | |
| **Controller** | **Gainshaping Controller Parameters** | | | |
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 1.635 | 553.5 | 0.0090390 | 1.03 |
| **Loop2** | 1.135 | 450 | 0.0110977 | 0.96 |
| | | | | |
| **Controller** | **Amigo Controller Parameters** | | | |
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 0.8137 | 286.122 | 0.0087023 | 0.97 |
| **Loop2** | 1.871241 | 196.80 | 0.0418366 | 0.805 |
| | | | | |
| **Controller** | **Z-N Controller Parameters** | | | |
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 2.647 | 166.5 | 0.048647 | 0.86 |
| **Loop2** | 5.266 | 71.595 | 0.32363 | 0.67 |

Table 5. Controller parameters for the non- Minimum phase process

| Controller | Hagglund Controller Parameters | | | |
|---|---|---|---|---|
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 0.48007 | 525 | 0.001904 | 1.3 |
| **Loop2** | 0.70871 | 262.5 | 0.003809 | 0.8 |
| | | | | |
| **Controller** | **Gainshaping Controller Parameters** | | | |
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 2.4 | 262.5 | 0.019044 | 0.86 |
| **Loop2** | 3.543 | 525 | 0.009522 | 0.88 |
| | | | | |
| **Controller** | **Amigo Controller Parameters** | | | |
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 1.08961 | 344.38 | 0.0065905 | 1.02 |
| **Loop2** | 0.54577 | 221.25 | 0.0034805 | 0.78 |
| | | | | |
| **Controller** | **Z-N Controller Parameters** | | | |
| | $k_p$ | $t_i$ | $k_{pred}$ | λ |
| **Loop1** | 3.598 | 209.79 | 0.035724 | 0.83 |
| **Loop2** | 2.232 | 249.75 | 0.012610 | 0.83 |

Table 6. Performance comparison of the minimum phase process with Predictive fractional order PI control strategy

| Controller | Hagglund Controller | | | | |
|---|---|---|---|---|---|
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 1042 | 0.5 | 1400 | 1167 | 2852 |
| **Loop2** | 1215 | 0.65 | 1750 | 1366.5 | 3145 |
| | | | | | |
| **Controller** | **Gainshaping Controller** | | | | |
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 212 | 0.21 | 289 | 245.5 | 953.9 |
| **Loop2** | 414 | 0.12 | 1000 | 687 | 1025 |
| | | | | | |
| **Controller** | **Amigo Controller** | | | | |
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 658 | 0.487 | 1120 | 808.5 | 1176 |
| **Loop2** | 621 | 1.03 | 480 | 339.3 | 1028 |
| | | | | | |
| **Controller** | **Z-N Controller** | | | | |
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 381 | 0.675 | 590 | 430.5 | 1105 |
| **Loop2** | 532 | 0 | 0 | 1220 | 1199 |

Table 7. Performance comparison of the non-minimum phase process with Predictive fractional order PI control strategy

| Controller | Hagglund Controller | | | | |
|---|---|---|---|---|---|
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 1183.7 | 19.18 | 517 | 313 | 1902 |
| **Loop2** | 1208.5 | 20 | 453 | 274.5 | 1894 |
| | | | | | |
| **Controller** | **Gainshaping Controller** | | | | |
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 1614 | 5.325 | 770 | 313 | 1277 |
| **Loop2** | 1839 | 0.15 | 2300 | 2120 | 670.8 |
| | | | | | |
| **Controller** | **Amigo Controller** | | | | |
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 2250 | 7.75 | 950 | 485 | 1615 |
| **Loop2** | 1770 | 0.95 | 1560 | 726 | 1858 |
| | | | | | |
| **Controller** | **Z-N Controller** | | | | |
| | $t_s$(sec) | $M_p$(%) | $t_p$(sec) | $t_r$(sec) | ISE |
| **Loop1** | 1850 | 7.9 | 820 | 323 | 1219 |
| **Loop2** | 2084 | 0.1 | 2735 | 2500 | 976.3 |

## 5 Conclusions

The quadruple tank process is a multivariable laboratory process that consists of four interconnected water tanks. The quadruple tank process is used to study the multivariable control concepts. Minimum phase as well as non-minimum phase characteristics of the process were considered in this paper. Fractional order PI controller is tuned using Hagglund, Gain shaping, amigo and Z-N techniques. The performance indices considered to evaluate the performance of the process are settling time**,** rise time, peak time, overshoot and ISE. It is observed from the results that the predictive fractional order PI control structure with Gainshaping tuning technique performs better than the other strategies with respect to settling time, overshoot and ISE for minimum phase system. Predictive fractional order PI control structure with Hagglund tuning technique performs better than the other strategies with respect to settling time, peak time and rise time for non-minimum phase system.

*References:*

[1] K.H.Johnson and A.Rantzer, "Multiloop control of Minimum Phase processes," in Proc.16th Amer.Contr.conf, Albuquerque, NM,1997.

[2] K.H.Johnson and A.Rantzer, "Decentralized Control of Sequentially minimum phase Systems," IEEE Trans. Automat.Contr,Vol.44, pp.1909-1913, 1999.

[3] R.Nunes, "Modeling and control of the quadruple-tank process," Master thesis Dept.Automat.contr., Lund Inst.Technol.Lund,Sweeden,1997.

[4] Hägglund, T, "A predictive PI controller for Processes with long dead times," Control Systems Magazine, IEEE Vol. 12, Issue 1, pp. 57-60, 1992.

[5] Hägglund, T, "An industrial dead-time Compensating PI controller," Control Engineering Practice, Vol. 4, No.6, pp.749- 756, 1996.

[6] Ari Ingimundarson and Tore Hägglund, Robust tuning procedures of dead-time compensating controllers, Control Engineering Practice,Volume 9, Issue 11, Pages 1195–1208, November 2001.

[7] Pasi Airikka ,"Extended Predictive Proportional –Integral controller for typical industrial processes,"Metso Corporation, Tampere, Finland. Preprints of the 18th IFAC World Congress , Milano (Italy) August 28 - September 2, 2011.

[8] Pasi Airikka ,Stability Analysis of a Predictive PI Controller," Metso Corporation, Tampere, P.O Box 237, FIN- 33101,Finland.

[9] Fractional-Order Systems and $PI^{\lambda}D^{\mu}$- Controllers, Igor Podlubny, IEEE Transactions on Automatic Control,Vol.44,No.1, January 1999.

[10] Rinku Singhal, Subhransu Padhee, Gagandeep Kaur, Design of Fractional Order PID Controller for Speed Control of DC Motor, International Journal of Scientific and Research Publications, Volume 2, Issue 6, June 2012.

[11] K.J. Astrom and T. Hagglund, "Revisiting the Ziegler –Nichols step response method for PID control, Journal of Process Control 14, 635–650, 2004.

[12] Mazidah Tajjudin, Mohd Hezri Fazalul Rahiman, Norhashim Mohd Arshad, and Ramli Adnan, "Robust Fractional-Order PI Controller with Ziegler-Nichols Rules," World Academy of Science, Engineering and Technology Vol:7 2013-07-29.

[13] K.J. Astrom and M.Lundh. "Lund control program combines theory with hands on Experience," IEEE contr. Syst. Mag., vol. 12, No. 3, pp.22-30,1992,DOI:10.1109/37.165511.

[14] K.J.Astrom ,A.B.Ostberg. "A teaching laboratory for process control," IEEE contr.Syst. Mag.; Vol. 6, pp.37-42,1986, DOI: 10.1109/MCS.1986.1105142.

[15] V. Alamelumangai and S.P. Natarajan. "Tuning methods of multi- loop controller for a multivariable process," Indian Journal of science and Technology, Vol.3, No. 2, ISSN:0974- 6846, Feb 2010.

[16] V.Alamelumangai and S.P.Natarajan, "Multiloop gain shaping controller for a multivariable process," UPA International journal of Power Systems and Power Electronics, Vol. 3,No.1,November 2010.