Observer design for nonlinear systems with interval time-varying delay

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Abstract: This paper investigates the problem of observer design for a class of nonlinear systems with time-delay and uncertain nonlinearity. Firstly, using the Mean-value Theorem and combining constructing the Lyapunov-Krasovskii functional, the convergence conditions of nonlinear observer for a class of nonlinear systems with time-varying delay and uncertain nonlinearity are established in terms of a linear matrix inequality. Then the new sufficient conditions are derived to ensure the convergence of the observer for a class of nonlinear systems with constant time-delay and uncertain nonlinearity. The simulation results are presented to show the effectiveness of the proposed method.

Key-Words: Nonlinear systems; Observer; Asymptotic stability; Time-varying delay

1 Introduction

State estimation for nonlinear systems is a long standing problem that has been addressed with different looks. In practice, all state variables are rarely available for direct on-line measurement. In all circumstances, there is an abundant need for a reliable estimation of the unmeasurable state variable. For this special task, a state observer is usually used. In a general way, there are many major design methods in both linear and nonlinear systems. In the case of linear systems, early result can be found in [1, 2], and many people are still doing further research. In the case of nonlinear systems, observer design for Lipschitz systems was first considered by Thau in [3], and a sufficient condition to ensure the asymptotic stability of the observer was obtained. Thau's condition is a very useful analysis tool but does not address the fundamental design problem. Encouraged by Thau's result, several authors studied the observer design problem for Lipschitz systems. The use of Lyapunov function and the Bellman-Gronwall lemma for this design problem with application in feedback stabilization were considered by Zak in [4]. An observer design methodology for a class of nonlinear systems in which the nonlinearity was assumed to be Lipschitz was presented in [5]. When the system fails to be put in certain form of observability, high-gain observer design reveals as a powerful method that is often used to reconstruct the system states under the assumption that the vector nonlinearity is globally or locally Lipschitz, see [6-7]. By the coordinate transformation approach, a new constant gain observer design methodology for a class of multi-output nonlinear systems was proposed in [8].

Time-delays are inherent in many engineering systems, and such time-delays can limit and degrade the achievable performance of controlled systems, and even induce instability. Delay terms lead to infinite dimensionality in the characteristic equations, making time-delay systems difficult to control with classical control methods. Hence stability analysis and observer design for time-delay systems has been investigated in recent years [9-22]. In [9], a geometric study of reduced order observer design for discrete-time nonlinear systems was given. Using the center manifold theory for maps, the error convergence for the reduced order estimator for discrete-time nonlinear systems was established. In [10], a new approach to the nonlinear observer design problem in the presence of delayed output measurements was presented. The proposed nonlinear observer possesses a state-dependent gain which was computed from the solution of a system of first-order singular partial differential equations.

In [11], an adaptive observer was developed for single-input single-output nonlinear systems that can be transformed into a certain observable canonical form. In [12], a discrete-time observer design procedure based on linear matrix inequalities was presented for a class of nonlinear system and measurement models. A common framework was provided to design observers according to a variety of performance criteria. In [15], the problem of observer design for a class of nonlinear discrete-time systems with time-delay was considered. A new approach of nonlinear observer design was proposed for the class of systems. By using differential mean value theorem, which allows transforming a nonlinear error dynamics into a linear parameter varying system, and based on Lyapunov stability theory, an approach of observer design for a class of nonlinear systems with time-delay was proposed in [17]. The sufficient conditions, which guarantee the estimation error to asymptotically converge to zero, were given.

This paper considers the problem of observer design for a class of nonlinear systems with time-delay and uncertain nonlinearity. Using a novel Lyapunov-Krasovskii functional, the convergence conditions of nonlinear observer for a class of nonlinear systems with time-varying delay and uncertain nonlinearity are established, which are expressed in terms of a linear matrix inequality. Then the new sufficient conditions are derived to ensure the convergence of the observer with constant time-delay and uncertain nonlinearity. And what is more, the proposed LMI-based results are computationally efficient as they can be solved numerically by employing the LMI toolbox in Matlab. The simulation results are presented to show the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, a class of nonlinear systems with time-delay is studied, and the corresponding observer is introduced. Using a novel Lyapunov-Krasovskii functional, the sufficient conditions that guarantees the observer error converges asymptotically to zero expressed as matrix inequalities are established. In Section 3, a numerical example is given to show the performances of our method. Finally, some conclusions and remarks are drawn in Section 4.

Though out this paper, we denote by R the set of real numbers, $R^{n\times m}$ denotes the space of $n\times m$ real matrix and I denotes an identity matrix with appropriate dimension. The notation $A>0$ (resp. $A<0$) means that the matrix A is positive definite (resp. negative definite). A^T is the matrix transpose of A. The symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix.

2 Problem statement and main result

In this section, the analysis of the observer is given. Under the appropriate assumptions, a nonlinear observer is developed.

Let us consider the nonlinear system with time-varying delay

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h(t)) + Gf_1(x(t),u(t)) \\ \qquad + G_h f_2(x(t-h(t)),u(t)), \\ x(t) = \phi(t), \quad t\in[-h_2,0], \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x\in R^n$ is the state vector, $u\in R^m$ is the system input, $y\in R^p$ is the output, $A, A_h \in R^{n\times n}$, $G, G_h \in R^{n\times n}$, $C\in R^{p\times n}$ are all constant matrices. In the systems (1), $h(t)$ is the time-delay satisfying

$$\begin{aligned} &0\le h_1 \le h(t) \le h_2, \\ &h_{12} = h_2 - h_1, \\ &\dot{h}(t) \le \mu < 1, \end{aligned}$$

and $\phi(t)\in C([-h,0],R^n)$ is the initial function. We assume that (A, C) is observable.

To complete the system description the following assumption is taken into consideration.

Assumption 1 The nonlinearity vectors $f_i(x(t),u(t)), i=1,2,$ are globally Lipschitz with respect to $x(t)$, uniformly to $u(t)$, and $\forall x(t)\in \mathrm{T}\subset R^n$ and $\forall u(t)\in \mathrm{K}\subset R^m$ there exist constant matrices $E_i\in R^{n\times n}$ and $N_i\in R^{n\times n}, i=1,2,$ such that the Jacobian of uncertain nonlinearity verify

$$\left.\frac{\partial f_i(s,u(t))}{\partial s}\right|_{s=x(t)} = E_i M(x(t),u(t)) N_i, \quad i=1,2, \tag{2}$$

where $M(x(t),u(t))$ is unknown matrix satisfying $M^T(x(t),u(t))M(x(t),u(t)) \le I.$

We introduce the following lemmas which will be used in setting the proofs of the next statements.

Lemma 1 [23] Given constant symmetric matrices S_1, S_2, S_3, and $S_1 = S_1^T < 0$, $S_3 = S_3^T > 0$, then $S_1 + S_2 S_3^{-1} S_2^T < 0$ if and only if

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & -S_3 \end{bmatrix} < 0.$$

Lemma 2 [24] Let D, E and F be real matrices of appropriate dimensions with $F^T F \le I$, then for any scalar $\varepsilon > 0$ we have the following inequality:

$$DFE + E^T F^T D^T \le \varepsilon^{-1} DD^T + \varepsilon E^T E.$$

Lemma 3 [25] For any constant symmetric matrix $R > 0,$ scalar $h > 0,$ and vector function $\dot{x}(\cdot):[-h,0] \to R^n$ such that the following integral is well defined, then

$$-h\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le z^T(t)\begin{bmatrix} -R & R \\ R & -R \end{bmatrix} z(t),$$

where $z(t) = \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix}^T$.

Consider the following nonlinear observer dynamical equation

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + A_h\hat{x}(t-h(t)) + Gf_1(\hat{x}(t),u(t)) \\ &\quad + G_h f_2(\hat{x}(t-h(t)),u(t)) + L(y(t) - C\hat{x}(t)).\end{aligned} \tag{3}$$

Let $\tilde{x}(t) = x(t) - \hat{x}(t).$ The estimation error dynamic is given by

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x} + A_h\tilde{x}(t-h(t)) + G\Delta f_1 + G_h\Delta f_2, \tag{4}$$

where

$$\Delta f_1 = f_1(x(t),u(t)) - f_1(\hat{x}(t),u(t)),$$
$$\Delta f_2 = f_2(x(t-h(t)),u(t)) - f_2(\hat{x}(t-h(t)),u(t)).$$

Theorem 1 Suppose that Assumption 1 is satisfied. Then the observer error dynamic (4) is asymptotically stable, if there exist a matrix L, positive definite matrices $Q_1, Q_2, Q_3, R_1, R_2,\ P \in R^{n\times n}$ and positive scalars $\varepsilon_i > 0, i = 1,2,3,4,$ such that the following linear matrix inequality holds:

$$\begin{pmatrix} \Omega & R_1 & 0 & PA_h & \Omega_{15} & \Omega_{16} & \Omega_{17} & 0 & 0 \\ * & \Omega_{22} & R_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Omega}_{44} & \Omega_{45} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\Upsilon I & 0 & 0 & \Omega_{58} & \Omega_{59} \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_4 I \end{pmatrix} < 0, \tag{5}$$

where

$$\Omega_{15} = (A - LC)^T \Upsilon, \Omega_{16} = PGE_1, \Omega_{17} = PG_hE_2,$$
$$\Omega_{22} = -R_1 - R_2 - Q_1, \quad \Omega_{33} = -R_2 - Q_2,$$
$$\Omega_{45} = A_h^T\Upsilon, \quad \Omega_{58} = \Upsilon GE_1,\ \Omega_{59} = \Upsilon G_hE_2,$$
$$\bar{\Omega}_{44} = -(1-\mu)Q_3 + \varepsilon_2 N_2^T N_2 + \varepsilon_4 N_2^T N_2,$$
$$\Omega = \Omega_0 - R_1 + \varepsilon_1 N_1^T N_1 + \varepsilon_3 N_1^T N_1,$$
$$\Omega_0 = (A - LC)^T P + P(A - LC) + Q_1 + Q_2 + Q_3,$$
$$\Upsilon = h_1^2 R_1 + h_{12}^2 R_2.$$

Proof. Consider the following Lyapunov-Krasovskii functional for the system (4)

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t),$$

where

$$\begin{aligned} V_1(t) &= \tilde{x}^T(t)P\tilde{x}(t), \quad P > 0, \\ V_2(t) &= \int_{t-h_1}^{t} \tilde{x}^T(s)Q_1\tilde{x}(s)ds, \quad Q_1 > 0, \\ V_3(t) &= \int_{t-h_2}^{t} \tilde{x}^T(s)Q_2\tilde{x}(s)ds, \quad Q_2 > 0, \\ V_4 &= h_1\int_{-h_1}^{0}\int_{t+v}^{t} \dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)dsdv, \quad R_1 > 0, \\ V_5(t) &= h_{12}\int_{-h_2}^{-h_1}\int_{t+v}^{t} \dot{\tilde{x}}^T(s)R_2\dot{\tilde{x}}(s)dsdv, \quad R_2 > 0, \\ V_6(t) &= \int_{t-h(t)}^{t} \tilde{x}^T(s)Q_3\tilde{x}(s)ds, \quad Q_3 > 0. \end{aligned} \tag{6}$$

The time derivative of $V_1(t)$ along the trajectory of the error dynamic (4) is

$$\begin{aligned} \dot{V}_1 &= \tilde{x}^T(t)[(A - LC)^T P + P(A - LC)]\tilde{x}(t) \\ &\quad + \Delta f_1^T G^T P\tilde{x}(t) + \tilde{x}^T(t)PG\Delta f_1 + \Delta f_2^T G_h^T P\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)PG_h\Delta f_2 + \tilde{x}^T(t)PA_h\tilde{x}(t-h(t)) \\ &\quad + \tilde{x}^T(t-h(t))A_h^T P\tilde{x}(t). \end{aligned} \tag{7}$$

Using the Mean-value Theorem and combining Assumption 1, we write that

$$\begin{aligned} \Delta f_1 &= f_1(x(t),u(t)) - f_1(\hat{x}(t),u(t)) \\ &= \int_0^1 \frac{\partial f_1(\eta,u(t))}{\partial \eta}\bigg|_{\eta = x(t) - \lambda(x(t)-\hat{x}(t))} (x(t) - \hat{x}(t))d\lambda \\ &= \int_0^1 E_1 M(\eta(\lambda,t),u(t))N_1 d\lambda\tilde{x}(t), \end{aligned} \tag{8}$$

and

$$\begin{aligned} \Delta f_2 &= f_2(x(t-h(t)),u(t)) - f_2(\hat{x}(t-h(t)),u(t)) \\ &= \int_0^1 \frac{\partial f_2(\beta,u(t))}{\partial \beta}\bigg|_{\beta = x(t-h(t)) - \lambda(x(t-h(t))-\hat{x}(t-h(t)))} \\ &\quad \times (x(t-h(t)) - \hat{x}(t-h(t)))d\lambda \\ &= \int_0^1 E_2 M(\beta(\lambda,t),u(t))N_2 d\lambda\tilde{x}(t-h(t)), \end{aligned} \tag{9}$$

where

$$\eta(\lambda,t) = x(t) - \lambda(x(t) - \hat{x}(t)),$$
$$\beta(\lambda,t) = x(t-h(t)) - \lambda(x(t-h(t)) - \hat{x}(t-h(t))).$$

From (8) and (9), we have

$$\begin{aligned} \dot{V}_1 &= \tilde{x}^T(t)[(A - LC)^T P + P(A - LC)]\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)\int_0^1 N_1^T M^T(\eta(\lambda,t),u(t))E_1^T G^T P d\lambda\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)\int_0^1 PGE_1 M(\eta(\lambda,t),u(t))N_1 d\lambda\tilde{x}(t) \\ &\quad + \tilde{x}^T(t-h(t))\int_0^1 N_2^T M^T(\beta(\lambda,t),u(t))E_2^T G_h^T P d\lambda \\ &\quad \times \tilde{x}(t) + \tilde{x}^T(t)\int_0^1 PG_hE_2 M(\beta(\lambda,t),u(t))N_2 d\lambda \\ &\quad \times \tilde{x}(t-h(t)) + \tilde{x}^T(t)PA_h\tilde{x}(t-h(t)) \\ &\quad + \tilde{x}^T(t-h(t))A_h^T P\tilde{x}(t). \end{aligned} \tag{10}$$

And we can write

$$\begin{aligned}\dot{V}_1 &= \tilde{x}^T(t)\int_0^1[(A-LC)^T P + P(A-LC)\\ &\quad + N_1^T M^T(\eta(\lambda,t),u(t))E_1^T G^T P\\ &\quad + PGE_1 M(\eta(\lambda,t),u(t))N_1]d\lambda\, \tilde{x}(t)\\ &\quad + \tilde{x}^T(t)\int_0^1[PG_h E_2 M(\beta(\lambda,t),u(t))N_2 + PA_h]\\ &\quad \times d\lambda \tilde{x}(t-h(t)) + \tilde{x}^T(t-h(t))\int_0^1[A_h^T P +\\ &\quad N_2^T M^T(\beta(\lambda,t),u(t))E_2^T G_h^T P]d\lambda \tilde{x}(t).\end{aligned} \tag{11}$$

We have

$$\begin{aligned}\dot{V}_2(t) &= \tilde{x}^T(t)Q_1\tilde{x}(t) - \tilde{x}^T(t-h_1)Q_1\tilde{x}(t-h_1),\\ \dot{V}_3(t) &= \tilde{x}^T(t)Q_2\tilde{x}(t) - \tilde{x}^T(t-h_2)Q_2\tilde{x}(t-h_2),\\ \dot{V}_6(t) &= -(1-\dot{h}(t))\tilde{x}^T(t-h(t))Q_3\tilde{x}(t-h(t))\\ &\quad + \tilde{x}^T(t)Q_3\tilde{x}(t)\\ &\le -(1-\mu)\tilde{x}^T(t-h(t))Q_3\tilde{x}(t-h(t))\\ &\quad + \tilde{x}^T(t)Q_3\tilde{x}(t).\end{aligned} \tag{12}$$

According Lemma 3, we get

$$\begin{aligned}\dot{V}_4 &= h_1^2\dot{\tilde{x}}^T(t)R_1\dot{\tilde{x}}(t) - h_1\int_{t-h_1}^{t}\dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)ds\\ &\le h_1^2\dot{\tilde{x}}^T(t)R_1\dot{\tilde{x}}(t) + \begin{bmatrix}\tilde{x}^T(t) & \tilde{x}^T(t-h_1)\end{bmatrix}\\ &\quad \times\begin{bmatrix}-R_1 & R_1\\ R_1 & -R_1\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-h_1)\end{bmatrix},\end{aligned}$$

and

$$\begin{aligned}\dot{V}_5(t) &= h_{12}^2\dot{\tilde{x}}^T(t)R_2\dot{\tilde{x}}(t) - h_{12}\int_{t-h_2}^{t-h_1}\dot{\tilde{x}}^T(s)R_2\dot{\tilde{x}}(s)ds\\ &\le h_{12}^2\dot{\tilde{x}}^T(t)R_2\dot{\tilde{x}}(t) + \begin{bmatrix}\tilde{x}^T(t-h_1) & \tilde{x}^T(t-h_2)\end{bmatrix}\\ &\quad \times\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}\tilde{x}(t-h_1)\\ \tilde{x}(t-h_2)\end{bmatrix}.\end{aligned}$$

So, we get

$$\begin{aligned}\dot{V} &\le \tilde{x}^T(t)\int_0^1[(A-LC)^T P + P(A-LC)\\ &\quad + N_1^T M^T(\eta(\lambda,t),u(t))E_1^T G^T P + PG\\ &\quad \times E_1 M(\eta(\lambda,t),u(t))N_1]d\lambda \tilde{x}(t)\\ &\quad + \tilde{x}^T(t)Q_1\tilde{x}(t) - \tilde{x}^T(t-h_1)Q_1\tilde{x}(t-h_1)\\ &\quad + \tilde{x}^T(t)Q_2\tilde{x}(t) - \tilde{x}^T(t-h_2)Q_2\tilde{x}(t-h_2)\\ &\quad + h_1^2\dot{\tilde{x}}^T(t)R_1\dot{\tilde{x}}(t) + \tilde{x}^T(t)Q_3\tilde{x}(t)\\ &\quad - (1-\mu)\tilde{x}^T(t-h(t))Q_3\tilde{x}(t-h(t))\\ &\quad + \begin{bmatrix}\tilde{x}^T(t) & \tilde{x}^T(t-h_1)\end{bmatrix}\begin{bmatrix}-R_1 & R_1\\ R_1 & -R_1\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-h_1)\end{bmatrix}\\ &\quad + h_{12}^2\dot{\tilde{x}}^T(t)R_2\dot{\tilde{x}}(t) + \begin{bmatrix}\tilde{x}^T(t-h_1) & \tilde{x}^T(t-h_2)\end{bmatrix}\\ &\quad \times\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}\tilde{x}(t-h_1)\\ \tilde{x}(t-h_2)\end{bmatrix}\\ &\quad + \tilde{x}^T(t)\int_0^1[PG_h E_2 M(\beta(\lambda,t),u(t))N_2 + PA_h]\\ &\quad \times d\lambda \tilde{x}(t-h(t)) + \tilde{x}^T(t-h(t))\int_0^1[A_h^T P +\\ &\quad N_2^T M^T(\beta(\lambda,t),u(t))E_2^T G_h^T P]d\lambda \tilde{x}(t).\\ &= \tilde{x}^T(t)\int_0^1[(A-LC)^T P + P(A-LC)\\ &\quad + N_1^T M^T(\eta(\lambda,t),u(t))E_1^T G^T P + PGE_1\\ &\quad \times M(\eta(\lambda,t),u(t))N_1 + Q_1 + Q_2 + Q_3]d\lambda\, \tilde{x}(t)\\ &\quad - \tilde{x}^T(t-h_1)Q_1\tilde{x}(t-h_1)\\ &\quad - \tilde{x}^T(t-h_2)Q_2\tilde{x}(t-h_2) + \dot{\tilde{x}}^T(t)\Upsilon\dot{\tilde{x}}(t)\\ &\quad - (1-\mu)\tilde{x}^T(t-h(t))Q_3\tilde{x}(t-h(t))\\ &\quad + \begin{bmatrix}\tilde{x}^T(t) & \tilde{x}^T(t-h_1)\end{bmatrix}\begin{bmatrix}-R_1 & R_1\\ R_1 & -R_1\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-h_1)\end{bmatrix}\\ &\quad + \begin{bmatrix}\tilde{x}^T(t-h_1) & \tilde{x}^T(t-h_2)\end{bmatrix}\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}\tilde{x}(t-h_1)\\ \tilde{x}(t-h_2)\end{bmatrix}\\ &\quad + \tilde{x}^T(t)\int_0^1[PG_h E_2 M(\beta(\lambda,t),u(t))N_2 + PA_h]\\ &\quad \times d\lambda \tilde{x}(t-h(t)) + \tilde{x}^T(t-h(t))\int_0^1[A_h^T P +\\ &\quad N_2^T M^T(\beta(\lambda,t),u(t))E_2^T G_h^T P]d\lambda \tilde{x}(t).\\ &= \tilde{x}^T(t)\int_0^1[(A-LC)^T P + P(A-LC) +\\ &\quad N_1^T M^T(\eta(\lambda,t),u(t))E_1^T G^T P + PGE_1\\ &\quad \times M(\eta(\lambda,t),u(t))N_1 + Q_1 + Q_2 + Q_3]d\lambda\, \tilde{x}(t)\\ &\quad - \tilde{x}^T(t-h_1)Q_1\tilde{x}(t-h_1)\\ &\quad - \tilde{x}^T(t-h_2)Q_2\tilde{x}(t-h_2) - (1-\mu)\tilde{x}^T(t-h(t))\\ &\quad \times Q_3\tilde{x}(t-h(t)) + \begin{bmatrix}\tilde{x}^T(t) & \tilde{x}^T(t-h_1)\end{bmatrix}\\ &\quad \times\begin{bmatrix}-R_1 & R_1\\ R_1 & -R_1\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-h_1)\end{bmatrix}\\ &\quad + \begin{bmatrix}\tilde{x}^T(t-h_1) & \tilde{x}^T(t-h_2)\end{bmatrix}\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}\tilde{x}(t-h_1)\\ \tilde{x}(t-h_2)\end{bmatrix}\\ &\quad + [(A-LC)\tilde{x} + A_h\tilde{x}(t-h(t)) + G\Delta f_1 + G_h\Delta f_2]^T\\ &\quad \times \Upsilon[(A-LC)\tilde{x} + A_h\tilde{x}(t-h(t)) + G\Delta f_1 + G_h\Delta f_2]\\ &\quad + \tilde{x}^T(t)\int_0^1[PG_h E_2 M(\beta(\lambda,t),u(t))N_2 + PA_h]\\ &\quad \times d\lambda \tilde{x}(t-h(t)) + \tilde{x}^T(t-h(t))\int_0^1[A_h^T P\\ &\quad + N_2^T M^T(\beta(\lambda,t),u(t))E_2^T G_h^T P]d\lambda \tilde{x}(t).\end{aligned}$$

$$= \int_0^1 \xi^T \{ \begin{pmatrix} \Omega_1 - R_1 & R_1 & 0 & \Omega_{14} \\ * & \Omega_{22} & R_2 & 0 \\ * & * & -R_2 - Q_2 & 0 \\ * & * & * & -(1-\mu)Q_3 \end{pmatrix}$$
$$+\Theta^T(\lambda,t)\Upsilon\Theta(\lambda,t)\} \ \xi d\lambda, \tag{13}$$

where

$\Upsilon = h_1^2 R_1 + h_{12}^2 R_2,$

$\Omega_{22} = -R_1 - R_2 - Q_1,$

$\xi^T = [\tilde{x}^T(t) \ \ \tilde{x}^T(t-h_1) \ \ \tilde{x}^T(t-h_2) \ \ \tilde{x}^T(t-h(t))],$

$\Omega_1 = (A-LC)^T P + P(A-LC) + Q_1 + Q_2 + Q_3$
$\quad + N_1^T M^T(\eta(\lambda,t),u(t)) E_1^T G^T P$
$\quad +PGE_1 M(\eta(\lambda,t),u(t)) N_1,$

$\Omega_{14} = PG_h E_2 M(\beta(\lambda,t),u(t)) N_2 + PA_h,$

$\Theta = [(A-LC) + GE_1 M(\eta(\lambda,t),u(t)) N_1,$
$\quad 0, \ 0, \ A_h + G_h E_2 M(\beta(\lambda,t),u(t)) N_2].$

We conclude by Lemma 1 that $\dot{V} < 0$ if

$$\int_0^1 \begin{pmatrix} \Omega_1 - R_1 & R_1 & 0 & \Omega_{14} & \Omega_2^T\Upsilon \\ * & \Omega_{22} & R_2 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & \Omega_{44} & \Omega_{45}^T\Upsilon \\ * & * & * & * & -\Upsilon I \end{pmatrix} d\lambda < 0.$$

where

$\Omega_2 = (A-LC) + GE_1 M(\eta(\lambda,t),u(t)) N_1,$

$\Omega_{22} = -R_1 - R_2 - Q_1,$

$\Omega_{33} = -R_2 - Q_2$

$\Omega_{44} = -(1-\mu)Q_3,$

$\Omega_{45} = A_h + G_h E_2 M(\beta(\lambda,t),u(t)) N_2.$

The last integral inequality can be rewritten as

$$\begin{pmatrix} \Omega_0 - R_1 & R_1 & 0 & PA_h & (A-LC)^T\Upsilon \\ * & \Omega_{22} & R_2 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & \Omega_{44} & A_h^T\Upsilon \\ * & * & * & * & -\Upsilon I \end{pmatrix}$$
$$+\int_0^1 \begin{pmatrix} \Theta_1 & 0 & 0 & \Theta_{14} & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{pmatrix} d\lambda$$
$$+\int_0^1 \begin{pmatrix} 0 & 0 & 0 & 0 & \Theta_2 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & \Theta_{45} \\ * & * & * & * & 0 \end{pmatrix} d\lambda < 0, \tag{14}$$

where

$\Omega_0 = (A-LC)^T P + P(A-LC) + Q_1 + Q_2 + Q_3,$

$\Theta_1 = N_1^T M^T(\eta(\lambda,t),u(t)) E_1^T G^T P$
$\quad +PGE_1 M(\eta(\lambda,t),u(t)) N_1,$

$\Theta_{14} = PG_h E_2 M(\eta(\lambda,t),u(t)) N_2,$

$\Theta_2 = [GE_1 M(\eta(\lambda,t),u(t)) N_1]^T \Upsilon,$

$\Theta_{45} = [G_h E_2 M(\beta(\lambda,t),u(t)) N_2]^T \Upsilon.$

Set

$$\Pi_1 = \begin{pmatrix} PGE_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Pi_2 = \begin{pmatrix} N_1^T \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Pi_3 = \begin{pmatrix} PG_h E_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$
$$\Pi_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ N_2^T \\ 0 \end{pmatrix}, \quad \Pi_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Upsilon GE_1 \end{pmatrix}, \quad \Pi_6 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Upsilon G_h E_2 \end{pmatrix}.$$

We have

$$\int_0^1 \begin{pmatrix} \Theta_1 & 0 & 0 & \Theta_{14} & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{pmatrix} d\lambda$$
$$= \int_0^1 \{\Pi_1 M(\eta(\lambda,t),u(t)) \begin{pmatrix} N_1 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$+\Pi_2 M^T(\eta(\lambda,t),u(t))$$
$$\times \begin{pmatrix} E_1^T G^T P & 0 & 0 & 0 & 0 \end{pmatrix}\} d\lambda$$
$$+\int_0^1 \{\Pi_3 M(\beta(\lambda,t),u(t))$$
$$\times \begin{pmatrix} 0 & 0 & 0 & N_2 & 0 \end{pmatrix} + \Pi_4 M^T(\beta(\lambda,t),u(t))$$
$$\times \begin{pmatrix} E_2^T G_h^T P & 0 & 0 & 0 & 0 \end{pmatrix}\} d\lambda$$
$$\le \varepsilon_1^{-1}\Pi_1 \begin{pmatrix} E_1^T G^T P & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$+\varepsilon_1 \Pi_2 \begin{pmatrix} N_1 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$+\varepsilon_2^{-1}\Pi_3 \begin{pmatrix} E_2^T G_h^T P & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$
\begin{aligned}
&+\varepsilon_2\Pi_4\begin{pmatrix}0 & 0 & 0 & N_2 & 0\end{pmatrix}\\
&=\begin{pmatrix}
\mathrm{X} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \varepsilon_2 N_2^T N_2 & 0\\
0 & 0 & 0 & 0 & 0
\end{pmatrix},
\end{aligned} \tag{15}
$$

where

$\mathrm{X}=P(\varepsilon_1^{-1}GE_1E_1^TG^T+\varepsilon_2^{-1}G_hE_2E_2^TG_h^T)P+\varepsilon_1N_1^TN_1,$

and

$$
\begin{aligned}
&\int_0^1\begin{pmatrix}
0 & 0 & 0 & 0 & \Theta_2\\
* & 0 & 0 & 0 & 0\\
* & * & 0 & 0 & 0\\
* & * & * & 0 & \Theta_{45}\\
* & * & * & * & 0
\end{pmatrix}d\lambda\\
&=\int_0^1\{\Pi_2M^T(\eta(\lambda,t),u(t))\\
&\quad\times\begin{pmatrix}0 & 0 & 0 & 0 & E_1^TG^T\Upsilon\end{pmatrix}\\
&\quad+\Pi_5M(\eta(\lambda,t),u(t))\begin{pmatrix}N_1 & 0 & 0 & 0 & 0\end{pmatrix}\\
&\quad+\Pi_4M^T(\beta(\lambda,t),u(t))\begin{pmatrix}0 & 0 & 0 & 0 & E_2^TG_h^T\Upsilon\end{pmatrix}\\
&\quad+\Pi_6M(\beta(\lambda,t),u(t))\begin{pmatrix}0 & 0 & 0 & N_2 & 0\end{pmatrix}\}d\lambda\\
&\le\varepsilon_3\Pi_2\begin{pmatrix}N_1 & 0 & 0 & 0 & 0\end{pmatrix}\\
&\quad+\varepsilon_3^{-1}\Pi_5\begin{pmatrix}0 & 0 & 0 & 0 & E_1^TG^T\Upsilon\end{pmatrix}\\
&\quad+\varepsilon_4\Pi_4\begin{pmatrix}0 & 0 & 0 & N_2 & 0\end{pmatrix}\\
&\quad+\varepsilon_4^{-1}\Pi_6\begin{pmatrix}0 & 0 & 0 & 0 & E_2^TG_h^T\Upsilon\end{pmatrix}\\
&=\begin{pmatrix}
\varepsilon_3N_1^TN_1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \varepsilon_4N_2^TN_2 & 0\\
0 & 0 & 0 & 0 & \mathrm{E}
\end{pmatrix},
\end{aligned} \tag{16}
$$

where

$$\mathrm{E}=\Upsilon(\varepsilon_3^{-1}GE_1E_1^TG^T+\varepsilon_4^{-1}G_hE_2E_2^TG_h^T)\Upsilon.$$

The inequality (14) holds if the following inequality holds:

$$
\begin{pmatrix}
\bar{\Omega} & R_1 & 0 & PA_h & (A-LC)^T\Upsilon\\
* & \Omega_{22} & R_2 & 0 & 0\\
* & * & \Omega_{33} & 0 & 0\\
* & * & * & \bar{\Omega}_{44} & A_h^T\Upsilon\\
* & * & * & * & -\Upsilon I+\mathrm{E}
\end{pmatrix}<0, \tag{17}
$$

where

$$
\begin{aligned}
\bar{\Omega}&=\Omega_0-R_1+P(\varepsilon_1^{-1}GE_1E_1^TG^T+\varepsilon_2^{-1}G_hE_2E_2^TG_h^T)P\\
&\quad+\varepsilon_1N_1^TN_1+\varepsilon_3N_1^TN_1,\\
\bar{\Omega}_{44}&=\Omega_{44}+(\varepsilon_2+\varepsilon_4)N_2^TN_2.
\end{aligned}
$$

(17) is equivalent to (5) by Lemma 1. From (5) we get $\dot{V}<0$. This means that the system (4) is asymptotically stable. This ends of our proof.

Corollary 1. Suppose that Assumption 1 is satisfied. Consider system (1) with $h_1=0$, then the observer error dynamic (4) is asymptotically stable, if there exist a matrix *L*, positive definite matrices $Q_1,Q_2,R_1,$ $P\in R^{n\times n}$, and positive scalars $\varepsilon_i>0, i=1,2,3,4$ such that the following linear matrix inequality holds:

$$
\begin{pmatrix}
\Pi_{11} & R_1 & PA_h & \Pi_{14} & \Pi_{15} & \Pi_{16} & 0 & 0\\
* & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & \Pi_{33} & \Pi_{34} & 0 & 0 & 0 & 0\\
* & * & * & \Pi_{44} & 0 & 0 & \Pi_{47} & \Pi_{48}\\
* & * & * & * & -\varepsilon_1I & 0 & 0 & 0\\
* & * & * & * & * & -\varepsilon_2I & 0 & 0\\
* & * & * & * & * & * & -\varepsilon_3I & 0\\
* & * & * & * & * & * & * & -\varepsilon_4I
\end{pmatrix}<0, \tag{18}
$$

where

$$
\begin{aligned}
&\Pi_{11}=\Omega-R_1,\quad \Pi_{14}=(A-LC)^Th_2^2R_1,\\
&\Pi_{15}=PGE_1,\quad \Pi_{22}=-R_1-Q_1,\\
&\Pi_{16}=PG_hE_2,\\
&\Pi_{33}=-(1-\mu)Q_2+\varepsilon_2N_2^TN_2+\varepsilon_4N_2^TN_2,\\
&\Pi_{34}=A_h^Th_2^2R_1,\quad \Pi_{44}=-h_2^2R_1I,\\
&\Pi_{47}=h_2^2R_1GE_1,\quad \Pi_{48}=h_2^2R_1G_hE_2,\\
&\Omega=(A-LC)^TP+P(A-LC)+Q_1+Q_2\\
&\qquad+\varepsilon_1N_1^TN_1+\varepsilon_3N_1^TN_1.
\end{aligned}
$$

Proof. Let us choose the Lyapunov-Krasovskii functional candidate

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t),$$

where

$$
\begin{aligned}
&V_1(t)=\tilde{x}^T(t)P\tilde{x}(t),\quad P>0\\
&V_2(t)=\int_{t-h_2}^t\tilde{x}^T(s)Q_1\tilde{x}(s)ds,\quad Q_1>0,\\
&V_3(t)=h_2\int_{-h_2}^0\int_{t+v}^t\dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)dsdv,\quad R_1>0,\\
&V_4(t)=\int_{t-h(t)}^t\tilde{x}^T(s)Q_2\tilde{x}(s)ds,\quad Q_2>0.
\end{aligned}
$$

Using the similar method shown in the proof of Theorem 1, the inequality (18) can be obtained and the detailed proof is omitted.

Consider the nonlinear system with time-delay

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) + Gf_1(x(t), u(t)) \\ \qquad + G_h f_2(x(t-h), u(t)), \\ x(t) = \phi(t), \quad t \in [-h, 0], \\ y(t) = Cx(t), \end{cases} \tag{19}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the system input, $y \in R^p$ is the output, $G, G_h \in R^{n\times n}$, $C \in R^{p\times n}$ are all constant matrices. $\phi(t) \in C([-h,0], R^n)$ is the initial function. We assume that (A, C) is observable.

Consider the following nonlinear observer dynamical equation

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + A_h\hat{x}(t-h) + Gf_1(\hat{x}(t), u(t)) \\ &\quad + G_h f_2(\hat{x}(t-h), u(t)) + L(y(t) - C\hat{x}(t)). \end{aligned} \tag{20}$$

Let $\tilde{x}(t) = x(t) - \hat{x}(t)$. The estimation error dynamic is given by

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x} + A_h\tilde{x}(t-h) + G\Delta f_1 + G_h\Delta f_2, \tag{21}$$

where

$$\Delta f_1 = f_1(x(t), u(t)) - f_1(\hat{x}(t), u(t)),$$
$$\Delta f_2 = f_2(x(t-h), u(t)) - f_2(\hat{x}(t-h), u(t)).$$

Theorem 2. Suppose that Assumption 1 is satisfied. Then the observer error dynamics (21) is asymptotically stable, if there exist a matrix L, positive definite matrices $P, Q_1, R_1 \in R^{n\times n}$, and positive scalars $\varepsilon_i > 0, i = 1,2,3,4$ such that the following linear matrix inequality holds:

$$\begin{pmatrix} \Omega & \mathrm{T}_{12} & \mathrm{T}_{13} & \mathrm{T}_{14} & \mathrm{T}_{15} & 0 & 0 \\ * & \mathrm{T}_{22} & \mathrm{T}_{23} & 0 & 0 & 0 & 0 \\ * & * & -h^2R_1 & 0 & 0 & \mathrm{T}_{36} & \mathrm{T}_{37} \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_4 I \end{pmatrix} < 0, \tag{22}$$

where

$$\mathrm{T}_{12} = R_1 + PA_h, \quad \mathrm{T}_{13} = h^2(A - LC)^T R_1,$$
$$\mathrm{T}_{14} = PGE_1, \quad \mathrm{T}_{15} = PG_hE_2,$$
$$\mathrm{T}_{22} = -R_1 - Q_1 + \varepsilon_2 N_2^T N_2 + \varepsilon_4 N_2^T N_2,$$
$$\mathrm{T}_{23} = h^2A_h^T R_1, \quad \mathrm{T}_{36} = h^2R_1GE_1, \quad \mathrm{T}_{37} = h^2R_1G_hE_2,$$
$$\Omega = (A - LC)^T P + P(A - LC) + Q_1 - R_1 + \varepsilon_1 N_1^T N_1 + \varepsilon_3 N_1^T N_1.$$

Proof. Consider the Lyapunov-Krasovskii functional candidate

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$

where

$$V_1(t) = \tilde{x}^T(t)P\tilde{x}(t), \quad P > 0,$$
$$V_2(t) = \int_{t-h}^{t} \tilde{x}^T(s)Q_1\tilde{x}(s)ds, \quad Q_1 > 0,$$
$$V_3(t) = h\int_{-h}^{0}\int_{t+v}^{t} \dot{\tilde{x}}^T(s)R_1\dot{\tilde{x}}(s)dsdv, \quad R_1 > 0.$$

Using the similar method shown in the proof of Theorem 1, the inequality (22) can be obtained and the detailed proof is omitted.

3 Numerical example

Consider the nonlinear system (19) with

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -40 \end{pmatrix}, \quad A_h = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix},$$
$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad N_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix},$$
$$N_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$G_h = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \quad h = 0.5.$$

Choose

$$L = \begin{pmatrix} 30 & 1 \end{pmatrix}^T,$$
$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1.$$

Solving the inequality (22), we get the following positive definite matrices:

$$P = \begin{pmatrix} 0.2722 & -0.0031 \\ -0.0031 & 0.1968 \end{pmatrix},$$
$$Q_1 = \begin{pmatrix} 8.4987 & -0.0495 \\ -0.0495 & 8.1398 \end{pmatrix},$$
$$R_1 = \begin{pmatrix} 0.0138 & -0.0005 \\ -0.0005 & 0.008 \end{pmatrix}.$$

According to Theorem 2, the observer error dynamics (21) is asymptotically stable.

The observer dynamical equation is given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + A_h\hat{x}(t-h) + Gf_1(\hat{x}(t), u(t)) \\ &\quad + G_h f_2(\hat{x}(t-h), u(t)) + L(y(t) - C\hat{x}(t)). \end{aligned}$$

When

$$f_1 = \begin{pmatrix} \sin x_1 \\ \sin x_2 \end{pmatrix},$$

and

$$f_2 = \begin{pmatrix} 4\sin(x_1 - h) \\ 4\sin(x_2 - h) \end{pmatrix},$$

simulation results are shown in Fig. 1 and 2. It is seen from Fig. 1 and 2 that the observer error dynamic is asymptotically stable.

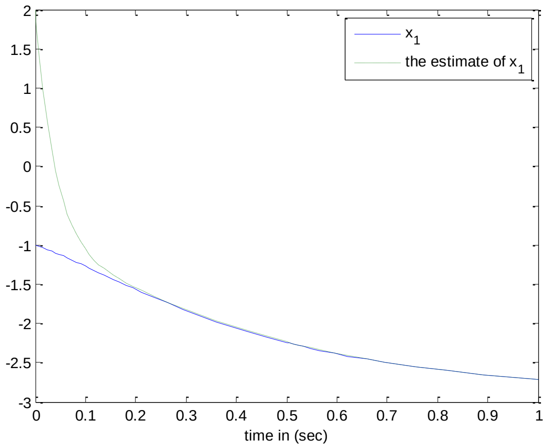

Fig. 1 The state x_1 and its estimate

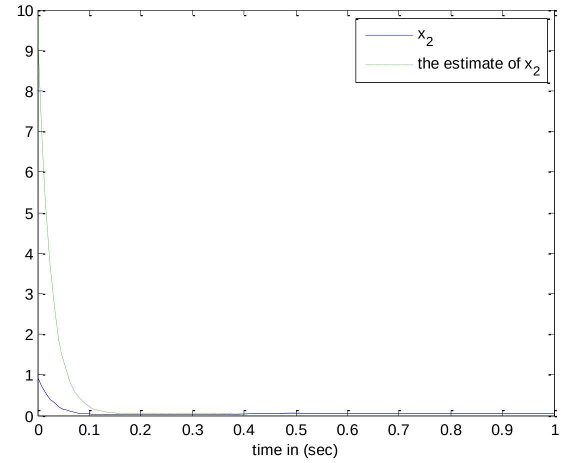

Fig.2 The state x_2 and its estimate

4. Conclusion

The main purpose of this paper is to offer a systematic algorithm for designing an observer for a class of nonlinear systems with unknown nonlinearity vector and time-varying delay. Firstly, the sufficient conditions are established in terms of a matrix inequality, which guarantee the nonlinear observer for a class of nonlinear systems with time-varying delay is an asymptotically stable observer. Then, the new sufficient conditions are presented, which ensure the convergence of the observer for a class of uncertain nonlinear systems with constant delay. Finally, an illustrative example is given to demonstrate the utilization of the results.

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